Math 401 HW#4, due Wednesday 9/25/19 NAME:

1. Exercise 2.62
   Let $\phi : S_1 \to S_2$ be a diffeomorphism between surfaces and $p \in S_1$. Prove that $(d\phi)_p : T_pS_1 \to T_{\phi(p)}S_2$ is a linear isomorphism and that $(d\phi)_p^{-1} = (d\phi^{-1})_{\phi(p)}$.

2. Exercise §2 (4)
   Let S be a compact surface, and let there be a differentiable function $f : S \to \mathbb{R}$ with at most three critical points. Show that S is connected.

3. Exercise §2 (9)
   Let $\phi : S \to \mathbb{R}^3$ be a differentiable map defined on a surface that satisfies

   - $(d\phi)_p : T_pS \to \mathbb{R}^3$ is injective for all $p \in S$.
   - $\phi : S \to \phi(S)$ is a homeomorphism.

   Prove that $\phi(S)$ is a surface and that $\phi : S \to \phi(S)$ is a diffeomorphism. If S is compact, show that the second requirement is equivalent to requiring that ϕ be injective.

4. Exercise 3.16
   Consider two diffeomorphic surfaces S_1 and S_2. Show that S_1 is orientable if and only if S_2 is orientable.

5. Exercise 3.17
   If S is an oriented surface, N is the corresponding unit normal field, and $p \in S$, then we say that a basis $\{a, b\}$ of the tangent plane T_pS is *positively oriented* when $\det(a, b, N(p)) > 0$. Otherwise we say that it is *negatively oriented*. If S_1 and S_2 are two oriented surfaces, we say that a local diffeomorphism $f : S_1 \to S_2$ *preserves orientation* if its differential at each point of S_1 takes positively oriented bases on S_1 into positively oriented bases on S_2. We define a function

   $$\mathrm{Jac} f : S_1 \to \mathbb{R}$$

   that is called the *Jacobian* of f - compare with HW #3 - by the equation

   $$(\mathrm{Jac} f)(p) = \det((df)_p(e_1), (df)_p(e_2), N_2(f(p))),$$

   where $\{e_1, e_2\}$ is a positively oriented orthonormal basis of T_pS_1.
   Prove that, if S_1 and S_2 are connected, f preserves orientation if and only if its Jacobian is positive everywhere.

* Assignment Reflections
How difficult was this assignment? How many hours did you spend on it? Which problems did you find to provide a worthwhile learning experience?