Math 401 HW#4, due Wednesday 9/25/24 **NAME:**

UPDATED: For full credit turn in solutions to 3 out of 4 problems

1. **Exercise §2 (8)**

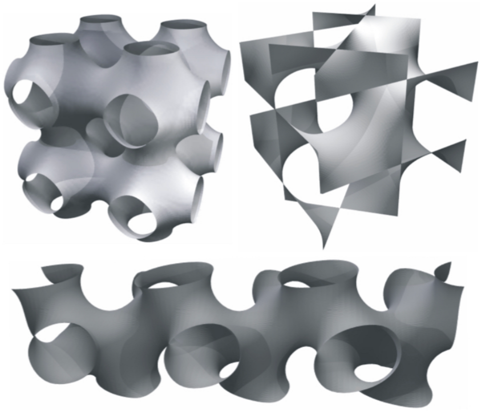

Figure 2.8. *Triply periodic surfaces*

(8) A surface is said to be *triply periodic* when it is invariant under three linearly independent translations of $\mathbb{R}^3$. This type of surface plays an important role in crystallography. Prove that the sets $\mathcal{P}$, $\mathcal{D}$, and $\mathcal{G}$, given, respectively, by the equations

$$\cos x + \cos y + \cos z = 0,$$
$$\cos(x - y - z) + \cos(x + y - z) - \cos(x + y + z) = 0,$$
$$\sin(x + y) + \sin(x - y) + \sin(y + z)$$
$$+ \sin(y - z) + \sin(x + z) + \sin(x - z) = 0$$

are examples of triply periodic surfaces. They are called, respectively, *primitive*, *diamond*, and *gyroid*. Show also that, for one of these examples, there are rigid motions of $\mathbb{R}^3$ interchanging the two regions determined by the surface.

Figure 1: Hint: Appeal to trig identities to get linearly independent translations of $\mathbb{R}^3$.

https://math.stackexchange.com/questions/3589185/a-rigid-motion-that-interchanges-the-the-two-regions-determined-by-the-gyroid

2. **Exercise 2.43**
If $X : U \to S$ is a parametrization of a surface, show that $X : U \to X(U)$ is a diffeomorphism.

3. **Exercise §2 (9)**
Let $\phi : S \to \mathbb{R}^3$ be a differentiable map defined on a surface that satisfies

- $(d\phi)_p : T_pS \to \mathbb{R}^3$ is injective for all $p \in S$.
- $\phi : S \to \phi(S)$ is a homeomorphism.

Prove that $\phi(S)$ is a surface and that $\phi : S \to \phi(S)$ is a diffeomorphism. If S is compact, show that the second requirement is equivalent to requiring that ϕ be injective.

4. **Exercise 2.55**
Let
$$S = \{p \in \mathbb{R}^3 \mid |p|^2 - \langle p, a\rangle^2 = r^2\},$$
with $|a| = 1$ and $r > 0$, be a right cylinder of radius r whose axis is the line passing through the origin with direction a. Prove that
$$T_pS = \{v \in \mathbb{R}^3 \mid \langle p, v\rangle - \langle p, a\rangle\langle a, v\rangle = 0\}.$$
Conclude that all the normal lines of S cut the axis perpendicularly.

* Assignment Reflections
How difficult was this assignment? How many hours did you spend on it? Which problems did you like?