1. Show that $\mathbb{RP}^2 \# \mathbb{RP}^2 \# \mathbb{RP}^2$ is homeomorphic to $T^2 \# \mathbb{RP}^2$. 
2. Using Theorem 4.1 (Massey IV §5) compute $\pi_1(T^2 \# T^2 \# T^2)$. 
3. If $(X, A)$ is a CW pair consisting of a CW complex $X$ and a contractible subcomplex $A$, show that the quotient map $X \to X/A$ is a homotopy equivalence.
4. Construct a 2-dimensional cell complex that contains both an annulus $S^1 \times I$ and a Möbius band as deformation retracts.
5. (Put a CW complex on $\mathbb{R}P^n$)

The $n$-dimensional real projective space, denoted by $\mathbb{R}P^n$, is the set of all 1-dimensional subspaces of $\mathbb{R}^{n+1}$. It may be topologized as a quotient space of $\mathbb{R}^{n+1} \setminus \{0\}$ or of the unit sphere $S^n$. Each 1-dimensional subspace of $\mathbb{R}^{n+1}$ intersects $S^n$ in a pair of antipodal points. The inclusions

$$\mathbb{R}^1 \subset \mathbb{R}^2 \subset ... \subset \mathbb{R}^{n+1}$$

give rise to corresponding inclusions of real projective spaces:

$$\mathbb{R}P^0 \subset \mathbb{R}P^1 \subset ... \subset \mathbb{R}P^n.$$ 

(a) What common spaces are $\mathbb{R}P^0$ and $\mathbb{R}P^1$ respectively?

(b) Show that $\mathbb{R}P^k$ is obtained from $\mathbb{R}P^{k-1}$ by attaching a single cell of dimension $k$.

In (b), using homogenous coordinates, the map $f : B^k \to \mathbb{R}P^k$, defined by

$$f_k(x_1, ..., x_k) = (x_1, ..., x_k, \sqrt{1 - ||x||^2})$$

where $x = (x_1, ..., x_k)$, may prove to be helpful.
The mapping torus $T_f$ of a map $f : X \to X$ is the quotient of $X \times I$ obtained by identifying each point $(x, 0)$ with $(f(x), 1)$. In the case $X = S^1 \lor S^1$ with $f$ basepoint-preserving, compute a presentation for $\pi_1(T_f)$ in terms of the induced map $f_* : \pi_1(X) \to \pi_1(X)$. [One way to do this is to regard $T_f$ as built from $X \lor S^1$ by attaching cells.]
The Hawaiian earring $H$ is the topological space defined by the union of circles in the Euclidean plane $\mathbb{R}^2$ with center $\left(\frac{1}{n}, 0\right)$ and radius $\frac{1}{n}$ for $n = 1, 2, 3,...$. The space $H$ is homeomorphic to the one-point compactification of the union of a countably infinite family of open intervals. The Hawaiian earring can be given a complete metric and it is compact. It is path connected but not semilocally simply connected.

At first glance the Hawaiian earring looks very similar to the wedge sum of countably infinitely many circles, but the two spaces are not homeomorphic.

(a) Show that $H$ is not homeomorphic to $\bigvee_{i=1}^{\infty} S^1$.

(b) Show that $\pi_1(H)$ is uncountably generated.

(c) Tell me your favorite peculiar feature of $\pi_1(H)$.

Before googling the Hawaiian earring (you inevitably will need to do so) try to sort out what is going on with its fundamental group by talking amongst each other. For homework 5 one of the two grad problems will be to explain the Hawaiian earring to an undergraduate student in this class.
everyone: How difficult was this assignment? How many hours did you spend on it?