Math 444/539 HW#4, due Monday 9/23/19 **NAME:**

1. Show that $\mathbb{RP}^2\#\mathbb{RP}^2\#\mathbb{RP}^2$ is homeomorphic to $T^2\#\mathbb{RP}^2$.

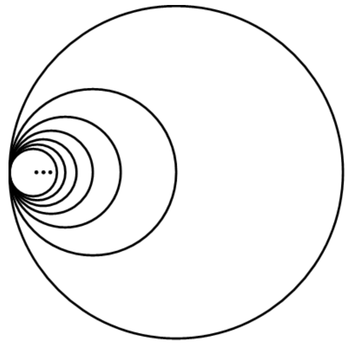
2. Let $X \subset \mathbb{R}^3$ be the union of n lines through the origin. Compute $\pi_1(\mathbb{R}^3 \setminus X)$.

3. Show that the free product $G * H$ of nontrivial groups G and H has trivial center, and that the only elements of $G * H$ of finite order are the conjugates of finite-order elements of G and H.

4. Give an example of a local homeomorphism $f : X \to Y$ and a subset $A \subset X$ such that $f|_A$ is not a local homeomorphism of A onto $f(A)$.

*math: π_1(The Hawaiian Earring)

The Hawaiian earring H is the topological space defined by the union of circles in the Euclidean plane $\mathbb{R}^2$ with center $(\frac{1}{n}, 0)$ and radius $\frac{1}{n}$ for $n = 1, 2, 3....$ The space H is homeomorphic to the one-point compactification of the union of a countably infinite family of open intervals. The Hawaiian earring can be given a complete metric and it is compact. It is path connected but not semilocally simply connected.



At first glance the Hawaiian earring looks very similar to the wedge sum of countably infinitely many circles, but the two spaces are not homeomorphic.

(a) Show that H is not homeomorphic to $\bigvee_{i=1}^{\infty} S^1$.

(b) Show that $\pi_1(H)$ is uncountably generated.

(c) Tell me your favorite peculiar feature of $\pi_1(H)$.

Before googling the Hawaiian earring (you inevitably will need to do so) try to sort out what is going on with its fundamental group by talking amongst each other.

everyone: How difficult was this assignment? How many hours did you spend on it?