Math 401 HW #5, due Wednesday 10/2/24 NAME:
UPDATED: # 3 is about orientability, not the hyperbolic paraboloid

1. Guided Jones 11-10,  d'oh
   Suppose $0 \le a \le b$. Find the surface area of the torus obtained by revolving the circle $(x-b)^2+z^2 = a^2$ in the xz-plane about the z-axis.
   Suggestion: Show that the torus admits the parametrization $0 \le \varphi, \theta \le 2\pi$ by

$$\begin{aligned} x &= (b + a\cos\varphi)\cos\theta \\ y &= (b + a\cos\varphi)\sin\theta \\ z &= a\sin\varphi \end{aligned}$$

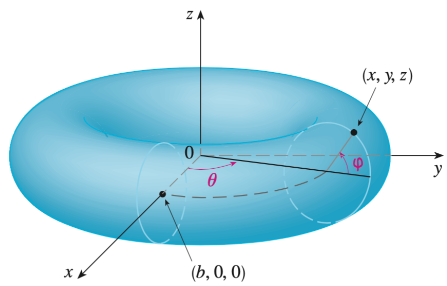

Figure 1: The hollow blue donut

2. Exercise 3.17
   If S is an oriented surface, N is the corresponding unit normal field, and $p \in S$, then we say that a basis $\{a, b\}$ of the tangent plane T_pS is *positively oriented* when $\det(a, b, N(p)) > 0$. Otherwise we say that it is *negatively oriented*. If S_1 and S_2 are two oriented surfaces, we say that a local diffeomorphism $f : S_1 \to S_2$ *preserves orientation* if its differential at each point of S_1 takes positively oriented bases on S_1 into positively oriented bases on S_2. We define a function

$$\mathrm{Jac} f : S_1 \to \mathbb{R}$$

   that is called the *Jacobian* of f by the equation

$$(\mathrm{Jac} f)(p) = \det((df)_p(e_1), (df)_p(e_2), N_2(f(p))),$$

   where $\{e_1, e_2\}$ is a positively oriented orthonormal basis of T_pS_1.
   Prove that, if S_1 and S_2 are connected, f preserves orientation if and only if its Jacobian is positive everywhere.

3. Exercise 3.16
   Consider two diffeomorphic surfaces S_1 and S_2. Show that S_1 is orientable if and only if S_2 is orientable.

* Assignment Reflections
   How difficult was this assignment? How many hours did you spend on it? Which problems did you find to provide a worthwhile learning experience?