Math 444/539 HW#5, due Tuesday 10/1/19 NAME:

1. Let a and b be the generators of $\pi_1(S^1 \vee S^1)$ corresponding to the two S^1 summands. Draw a picture of the covering space of $S^1 \vee S^1$ corresponding to the normal subgroup generated by a^2, b^2, and $(ab)^4$, and prove that this covering space is indeed the correct one.

2. Construct a simply-connected covering space of the space $X \subset \mathbb{R}^3$ that is the union of a 2-sphere and a diameter. Do the same when X is the union of a sphere and a circle intersecting it in two points.

3. (Massey V Exercise 7.2) Determine the group of automorphisms of the covering spaces described in Examples 2.2, 2.4, 2.7, and 2.9 of Massey V Section 2.

4. Let $X := T^n = \mathbb{R}^n/\mathbb{Z}^n$. Let $\tilde{X}$ be a path connected covering space of X. Show that $\tilde{X}$ is homeomorphic to $T^m \times \mathbb{R}^{n-m}$ for some $m \in \{0, ..., n\}$.

*math: Find all the connected covering space of $\mathbb{RP}^2 \vee \mathbb{RP}^2$.

everyone: How difficult was this assignment? How many hours did you spend on it?