Math 401 HW#6, due Wednesday 10/23/19 NAME:

1. Exercise 5.12
   Show that the map $X : (0, \pi) \times (0, 2\pi) \to \mathbb{R}^3$ defined by

   $$X(u, v) = (a_1, a_2, a_3) + r(\sin u \cos v, \sin u \sin v, \cos u)$$

   is a parametrization of the sphere $S^2(r)$ with center $a = (a_1, a_2, a_3) \in \mathbb{R}^3$ and radiu $r > 0$. Use this together with Proposition 5.8 to prove that the area of the sphere is $4\pi r^2$.

2. Exercise §5 (2)
   Let S be a compact surface with non-vanishing Gauss curvature everywhere and an injective Gauss map. Show that

   $$\int_S K = 4\pi.$$

3. Exercise §5 (8)
   Show that, if S is a compact surface,

   $$\int_S \langle N, a \rangle = 0,$$

   where N is a Gauss map and a is an arbitrary vector.

4. Exercise 5.36
   We associate to each differentiable function $f : A \to \mathbb{R}$, defined on a subset A of Euclidean space, a vector field ∇f, also defined on A, by

   $$\langle (\nabla f)(p), v \rangle = (df)_p(v), \qquad \text{for all } v \in \mathbb{R}^3$$

   This vector field is called the *gradient* of f. Prove that the three components of ∇f are

   $$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right).$$

   Show that, if the gradient of a differentiable function f vanishes everywhere, then f is constant on each connected component of A.

* Assignment Reflections
How difficult was this assignment? How many hours did you spend on it? Which problems did you find to provide a worthwhile learning experience?