Math 401 HW#6, due Monday 3/7/22 NAME:

1. Exercise 3.24
   Let S be a regular quadric - see Example 2.16 - given implicitly by the second degree equation

$$0 = (1, p^t)A \begin{pmatrix} 1 \\ p \end{pmatrix} = \langle Bp, p \rangle + 2\langle b, p \rangle + c, \quad p \in \mathbb{R}^3,$$

   where A is a symmetric matrix of order four, B is a non-null symmetric matrix of order three, $b \in \mathbb{R}^3$, and $c \in \mathbb{R}$. Show that

$$N(p) = \frac{Bp + b}{|Bp + b|}, \quad \text{for all } p \in S,$$

   is a Gauss map defined on S. As a consequence, check that the corresponding second fundamental form is given by

$$\sigma_p(v, v) = -\frac{1}{|Bp + b|}\langle Bv, v \rangle, \quad p \in S,\ v \in T_pS.$$

   Conclude that an ellipsoid has positive Gauss curvature at each of its points.

2. Exercise §3 (3)
   Suppose that a compact connected surface S has the following property: for each $p_0 \in \mathbb{R}^3 \setminus A$, where A is a countable set, the square of the distance function at the point p_0 has at most two critical points. Show that this surface is a sphere.

3. Exercise §3 (8)
   Use the methods of Section 3.6 to compute the Gauss and mean curvatures of the torus of revolution described in Example 2.17. Find its elliptic, hyperbolic, and parabolic points.

* Assignment Reflections
How difficult was this assignment? How many hours did you spend on it? Which problems did you find to provide a worthwhile learning experience?