Math 401 HW#7, due Monday 3/28/22 NAME:

For full credit, pick 4 of the 5 problems to do. In gradescope please indicate which problem you have omitted, e.g. write #X Problem Omitted and tag this for #X in gradescope.

1. Exercise 7.3
   Prove that, if $f : S \to S'$ is an isometry between two surfaces, then the absolute value of the Jacobian of f is identically 1. As a consequence, if S and S' are compact, their areas are the same.

2. Exercise 7.4
   Let P be the plane with equation $z = 0$ in $\mathbb{R}^3$ and C the right unit cylinder given by the equation $x^2 + y^2 = 1$. Show that the map $f : P \to C$ given by $f(x, y, 0) = (\cos x, \sin x, y)$ for each $(x, y, 0) \in P$ is a local isometry.

3. Exercise 7.5
   Prove that the composition of local isometries between surfaces is also a local isometry and that the inverse map of an isometry is an isometry as well. Conclude that the set of all isometries from a surface onto itself is a group relative to the composition law. This is the *group of isometries of the surface.*

4. Exercise §7 (4)
   Consider the following differentiable maps X and X'.

$$\begin{aligned} X(u, v) &= (u \cos v, u \sin v, \log u) \quad \text{for all } (u, v) \in \mathbb{R}^+ \times (0, 2\pi) \\ X'(u, v) &= (u \cos v, u \sin v, v) \quad \text{for all } (u, v) \in \mathbb{R}^2 \end{aligned}$$

   Prove that $S = X(\mathbb{R}^+ \times (0, 2\pi))$ and $S' = X'(\mathbb{R}^2)$ are surfaces and that X and X' are parametrizations for each of them. Show that the map $f : S \to S'$ given by $f = X' \circ X^{-1}$ satisfies $K = K' \circ f$ but it is not a local isometry. This shows the "converse" of Gauss' Theorema Egregium is not true.

5. Do Carmo §4 (9)
   Justify why the surfaces below are not pairwise locally isometric. If you use previously established facts from Montiel and Ros, please reference and cite them as Theorem/Proposition/Corollary X.xx.

   - Sphere
   - Cylinder
   - Pringle $z = x^2 - y^2$

* Assignment Reflections
   How difficult was this assignment? How many hours did you spend on it? Which problems did you find to provide a worthwhile learning experience?