Math 401 HW#7, due Wednesday 10/30/24 NAME:

* Exercise §3 (3)
Read through the hint in the book, but don't turn in a solution
Suppose that a compact connected surface S has the following property: for each $p_0 \in \mathbb{R}^3 \setminus A$, where A is a countable set, the square of the distance function at the point p_0 has at most two critical points. Show that this surface is a sphere.

1. Exercise 3.40 *For full credit, you must verify every step in the hint in the book, and write a self contained exposition, summarizing the utility of the Hessian (Prop. 3.35) and the Hessian of the "scary" height function as it relates to your solution.*
   Show that there are no compact surfaces that have negative Gauss curvature everywhere.

2. Exercise 7.3
   Prove that, if $f : S \to S'$ is an isometry between two surfaces, then the absolute value of the Jacobian of f is identically 1. As a consequence, if S and S' are compact, their areas are the same.

3. Exercise 7.4
   Let P be the plane with equation $z = 0$ in $\mathbb{R}^3$ and C the right unit cylinder given by the equation $x^2 + y^2 = 1$. Show that the map $f : P \to C$ given by $f(x, y, 0) = (\cos x, \sin x, y)$ for each $(x, y, 0) \in P$ is a local isometry.

* Assignment Reflections
How difficult was this assignment? How many hours did you spend on it? Which problems did you find to provide a worthwhile learning experience?