1. Lee 3-4 [SECOND] = 4-1 [FIRST].
   Show that \( TS^1 \) is diffeomorphic to \( S^1 \times \mathbb{R} \).

2. Show that if \( M \) and \( N \) are smooth manifolds and if \( p \in M \) and \( q \in N \), then there is a canonical isomorphism
   \[
   T_{(p,q)}(M \times N) = T_pM \oplus T_qN. 
   \]
   Describe this isomorphism in terms of (a) [math grads] derivations and (b) [everyone] linear combinations of partial derivatives with respect to coordinate charts.

3. The zero section of the tangent bundle \( TM \) is the set of zero tangent vectors,
   \[
   Z = \{(p,0)\} \subset TM = \{(p,V) \mid p \in M, V \in T_pM\}. 
   \]
   (a) Show that \( Z \) is a submanifold of \( TM \) which is diffeomorphic to \( M \).
   (b) Show that if \( (p,0) \in Z \), then there is a canonical (not depending on a choice of coordinates) isomorphism
   \[
   T_{(p,0)}TM = T_pM \oplus T_pM. 
   \]

4. Lee 8-10 [SECOND]
   Let \( M \) be the open submanifold of \( \mathbb{R}^2 \) where both \( x \) and \( y \) are positive and let \( F : M \to N \) be the map
   \[
   F(x,y) = \left( xy, \frac{y}{x} \right). 
   \]
   Show that \( F \) is a diffeomorphism, and compute \( F_*X \) and \( F_*Y \) where
   \[
   X = x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y}; \quad Y = y \frac{\partial}{\partial x}
   \]

5. Lee 8-11 [SECOND] = 4-5 [FIRST]
   For each of the following vector fields on the plane, compute its coordinate representation in polar coordinates on the right half-plane \( \{(x,y) \in \mathbb{R}^2 \mid x > 0\} \).
   (a) \( X = x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} \)
   (b) \( Y = x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \)

6. Lee 8-16 [SECOND] = 4-11 [FIRST]
   For each of the following pairs of vector fields \( X, Y \) defined on \( \mathbb{R}^3 \), compute the Lie bracket \([X,Y]\).
   (a) \( X_1 = y \frac{\partial}{\partial z} - 2xy^2 \frac{\partial}{\partial y}; \quad Y_1 = \frac{\partial}{\partial y} \)
   (b) \( X_2 = x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x}; \quad Y_2 = y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y} \)
7. (Lee Second 4-6) Let $M$ be a nonempty smooth compact manifold. Show that there is no smooth submersion $F : M \to \mathbb{R}^k$ for any $k > 0$.

Math* The Hopf fibration is the map $f : S^3 \to \mathbb{C}P^1$ sending $(z^1, z^2) \in \mathbb{C}^2$ with $|z^1|^2 + |z^2|^2 = 1$ to $[z^1 : z^2] \in \mathbb{C}P^1$. Show that the Hopf fibration is a submersion.

everyone: How difficult was this assignment? How many hours did you spend on it?