Math 444/539 UPDATED HW#7, due Tuesday 10/29/19 NAME:

1. Lee 3-4 [SECOND] = 4-1 [FIRST].
   Show that TS^1 is diffeomorphic to $S^1 \times \mathbb{R}$.

2. Show that if M and N are smooth manifolds and if $p \in M$ and $q \in N$, then there is a canonical isomorphism
$$T_{(p,q)}(M \times N) = T_pM \oplus T_qN.$$
   Describe this isomorphism in terms of (a) [math grads] derivations and (b) [everyone] linear combinations of partial derivatives with respect to coordinate charts.

3. The *zero section* of the tangent bundle TM is the set of zero tangent vectors,
$$Z = \{(p, 0)\} \subset TM = \{(p, V) \mid p \in M, V \in T_pM\}.$$
   (a) Show that Z is a submanifold of TM which is diffeomorphic to M.
   (b) Show that if $(p, 0) \in Z$, then there is a canonical (not depending on a choice of coordinates) isomorphism
$$T_{(p,0)}TM = T_pM \oplus T_pM.$$

4. Lee 8-10 [SECOND]
   Let M be the open submanifold of $\mathbb{R}^2$ where both x and y are positive and let $F : M \to N$ be the map
$$F(x, y) = \left(xy, \frac{y}{x}\right).$$
   Show that F is a diffeomorphism, and compute F_*X and F_*Y where
$$X = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}; \qquad Y = y\frac{\partial}{\partial x}$$

5. Lee 8-11 [SECOND] = 4-5 [FIRST]
   For each of the following vector fields on the plane, compute its coordinate representation in polar coordinates on the right half-plane $\{(x, y) \in \mathbb{R}^2 \mid x > 0\}$.
$$\text{(a)} \quad X = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}$$
$$\text{(b)} \quad Y = x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}$$

6. Lee 8-16 [SECOND] = 4-11 [FIRST]
   For each of the following pairs of vector fields $X,\ Y$ defined on $\mathbb{R}^3$, compute the Lie bracket $[X, Y]$.
$$\text{(a)} \quad X_1 = y\frac{\partial}{\partial z} - 2xy^2\frac{\partial}{\partial y}; \qquad Y_1 = \frac{\partial}{\partial y}$$
$$\text{(b)} \quad X_2 = x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x} \qquad Y_2 = y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}$$

everyone: How difficult was this assignment? How many hours did you spend on it?