Math 401 Midterm, due Monday 2/21/22 at 11pm   NAME:

This is a 6 hour open notes exam. You may use the listed course textbooks, Zoom class recordings, course canvas materials, and your notes, but you are not permitted to use the internet or any other materials. You are not permitted to communicate with anyone about this exam except Leo and Prof Jo. Write and sign the Rice honor pledge at the end of the exam.

Honor pledge: On my honor, I have neither given nor received any unauthorized aid on this exam.

1. Exercise 1.31 (10 points)
   Let $\alpha : I \to \mathbb{R}^3$ be a curve p.b.a.l. Prove that α is a segment of a straight line if and only if the curvature of α vanishes everywhere. (Do each direction separately. Recall as before, that I is a connected open subset of $\mathbb{R}$.)

2. (12 points)

   (a) Prove that $S := \{(x, y, z) \in \mathbb{R}^3 \mid z = ax^2 + by^2\}$ is a surface for constants $a, b \in \mathbb{R}$.

   (b) Sketch and provide a description of the general shape of the surface S in each of the following cases. Describe how changing a and b within the given ranges impacts the shape of the surface. Show some work by drawing traces of curves in 3D at appropriate x, y, and z heights; these curves are obtained by setting one of the variables equal to a constant. Use these to build your surface.

      i. $a > b > 0$
      ii. $a > b = 0$
      iii. $a > 0 > b$
      iv. $a = b = 0$

3. **Exercise 2.62** (10 points)
   Let $\phi : S_1 \to S_2$ be a diffeomorphism between surfaces and $p \in S_1$.
   Prove that $(d\phi)_p : T_p S_1 \to T_{\phi(p)} S_2$ is a linear isomorphism and that $(d\phi)_p^{-1} = (d\phi^{-1})_{\phi(p)}$.

4. (10 points)
   Given a differentiable function $g : \mathbb{R}^2 \to \mathbb{R}$, prove that $v = (v_1, v_2, v_3) \in \mathbb{R}^3$ is tangent to $S := \{(x, y, z) \in \mathbb{R}^3 \mid z = g(x, y)\}$ at a regular point $p \in S$ if and only if

   $$v_3 = \frac{\partial g}{\partial x}(p_1, p_2)v_1 + \frac{\partial g}{\partial y}(p_1, p_2)v_2$$

   In order to receive full credit, Example 2.51 cannot be used without an intermediate step, which you should explain with 1-2 sentences.

   * Midterm Reflections
   How difficult was this midterm? How is the pace of class? What topic have you enjoyed the most?