Math 401 Midterm, due Wednesday 10/9/24 at 11pm **NAME:**

This is a 4 hour open book and open notes exam. (You make take 6 hours if you need the extra time.) Once you start a problem, you must finish it, but otherwise you do not need to complete the exam in a single sitting. You may use the listed course textbooks, course canvas materials, and your notes, but you are not permitted to use the internet, AI, or any other materials. You are not permitted to communicate with anyone about this exam except Prof Jo.

To receive full credit, write your solutions in complete sentences and provide justification in a way that another student in the course could follow. If you are using results we previously established please mention them and indicate how the hypotheses are verified when you use them.

Write and sign the Rice honor pledge at the end of the exam.

Honor pledge: On my honor, I have neither given nor received any unauthorized aid on this exam.

1. Exercise 1.31 (10 points)
   Let $\alpha : I \to \mathbb{R}^3$ be a curve p.b.a.l. Prove that α is a segment of a straight line if and only if the curvature of α vanishes everywhere. (Do each direction separately. Recall as before, that I is a connected open subset of $\mathbb{R}$.)

2. (5 points)
   Given a differentiable function $g : \mathbb{R}^2 \to \mathbb{R}$, prove that $v = (v_1, v_2, v_3) \in \mathbb{R}^3$ is tangent to $S := \{(x, y, z) \in \mathbb{R}^3 \mid z = g(x, y)\}$ at a regular point $p \in S$ if and only if

   $$v_3 = \frac{\partial g}{\partial x}(p_1, p_2)v_1 + \frac{\partial g}{\partial y}(p_1, p_2)v_2$$

   In order to receive full credit, Example 2.51 cannot be used without an intermediate step, which you should explain with 1-2 sentences.

3. **Exercise 2.62** (10 points)
   Let $\phi : S_1 \to S_2$ be a diffeomorphism between surfaces and $p \in S_1$.
   Prove that $(d\phi)_p : T_pS_1 \to T_{\phi(p)}S_2$ is a linear isomorphism and that $(d\phi)_p^{-1} = (d\phi^{-1})_{\phi(p)}$.

4. **Exercise §2 (13)** (10 points)
   Let $f : S_1 \to S_2$ be a differentiable map between surfaces. If $p \in S_1$ and $\{e_1, e_2\}$ is an orthonormal basis of T_pS_1, we define the **absolute value of the Jacobian of f at p** by

   $$|\text{Jac}f|(p) = |(df)_p(e_1) \wedge (df)_p(e_2)|.$$

   (a) Prove that this definition does not depend on the orthonormal basis.
   (b) Prove that $|\text{Jac}f|(p) \neq 0$ if and only if $(df)_p$ is an isomorphism.

* Midterm Reflections
   How difficult was this midterm? How is the pace of class? What topic have you enjoyed the most?