GeoTop Midterm, due Friday 10/11/19 at 5pm NAME:

This is a 6 hour graduate/5 hour undergraduate open notes exam. You may use any of the listed course textbooks but are not permitted to use the internet or material beyond the course textbooks. You are not permitted to speak with or email anyone about this exam except me. Write and sign the Rice honor pledge at the end of the exam.

Honor pledge: On my honor, I have neither given nor received any unauthorized aid on this exam.

1. (10 points) Let X be a topological space. Under what conditions will two path classes γ and γ' from $x \in X$ to $y \in X$ give rise to the same isomorphism of $\pi_1(X, x)$ to $\pi_1(X, y)$.

2. (10 points) Show that the subspace of $\mathbb{R}^3$ that is the union of spheres of radius $1/n$ and center $(1/n, 0, 0)$ for $n = 1, 2, ...$ is simply connected.

3. (20 points, qual) Let $W = S^1 \vee S^1$ be the wedge of two circles.

   (a) Let S be the subgroup of $\pi_1(W)$ generated by $a^3, b^3, a^{-1}b, ba^{-1}$. Describe the covering space corresponding to S. Is this covering space regular? What is its group of deck transformations?

   (b) Let C be the commutator subgroup of $\pi_1(W)$. Recall that C is the subgroup of $\pi_1(W)$ generated by $aba^{-1}b^{-1}$ where $a, b \in \pi_1(W)$. Describe the covering space corresponding to C. Is this covering space regular? What is its group of deck transformations?

4. (10 points) Find all the connected 2-sheeted and 3-sheeted covering spaces of $S^1 \vee S^1$, up to isomorphism of covering spaces without basepoints.

*math: (10 points) Let T be the torus $S^1 \times S^1$ and let T' be T with a small open disk removed. Let X be obtained from T by attaching two copies of T', identifying their boundary circles with longitude and meridian circles $S^1 \times \{x_0\}$ and $\{x_0\} \times S^1$ in T. Compute $\pi_1(X)$.