Math 402/500 HW#1, due Friday 3/5/20 **NAME:**

1. Consider spherical coordinates on $\mathbb{R}^3$ (not including the line $x = y = 0$) ρ, ϕ, θ defined in terms of the Euclidean coordinates x, y, z by

$$x = \rho \sin\phi \cos\theta, \quad y = \rho \sin\phi \sin\theta, \quad z = \rho \cos\phi.$$

   (a) Express $\partial/\partial\rho$, $\partial/\partial\phi$, and $\partial/\partial\theta$ as linear combinations of $\partial/\partial x$, $\partial/\partial y$, and $\partial/\partial z$. (The coefficients in these linear combinations will be functions on $\mathbb{R}^3 \setminus (x = y = 0)$.)

   (b) Express $d\rho$, $d\phi$, and $d\theta$ as linear combinations of dx, dy, and dz.

2. Lee 8-10 [SECOND]
Let M be the open submanifold of $\mathbb{R}^2$ where both x and y are positive and let $F : M \to M$ be the map

$$F(x, y) = \left(xy, \frac{y}{x}\right).$$

Show that F is a diffeomorphism, and compute F_*X and F_*Y where

$$X = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}; \qquad Y = y\frac{\partial}{\partial x}$$

Note: The definition of the pushforward yields $(F_*Z)_{(s,t)} = dF_{F^{-1}(s,t)} Z_{F^{-1}(s,t)}$.

everyone: How difficult was this assignment? How many hours did you spend on it?