# LONG PROJECT SUGGESTIONS, MATH 191 SPRING 2018

This document will be updated over time. These are intended to be suggestions only – please feel free to search for and/or come up with your own ideas! Many are inspired by other sources.

(1) Investigate the uniqueness of satellite decompositions and compute some of your own. A good place to start is Ryan Budney's "JSJ-decompositions of knot and link complements in $S^3$" as well as the Soma reference in the textbook.

(2) Which of the invariants we've talked about work to distinguish mutants? Which don't? Why? Investigate other invariants.

(3) Think about unknotting algorithms in the context of braids – can you come up with a fast "unbraiding" algorithm? What about a fast algorithm using only Markov moves to recognize the unknot?

(4) Birman and Brendle have a great survey "Braids: a survey" which contains background and problems about braids. If you're interested in the intersection of word problems (e.g. recognizing the identity in a group) and knots, braids are a great place to get started. You can also check out Farb's list "Problems on mapping class groups and related topics."

(5) There are lots of problems about unknotting number on page 61 of Adams. Also: what is $u(10_{11})$?

(6) Explain the computation of the Alexander polynomial using Fox calculus and the fundamental group. Prove some lemmas and do some examples. Explore other uses of Fox calculus or other knot invariants computed in similar ways. Compare and contrast computational methods for the Alexander polynomial. References: `http://www.ucl.ac.uk/~ucbpeal/alexandermac.pdf` and `https://etd.ohiolink.edu/rws_etd/document/get/bgsu1394199102/inline`

(7) Check out `https://arxiv.org/pdf/1511.06329.pdf` for an intrinsic (!) characterization of alternating knots. What can you do with this? Can you re-prove any of the results we've proved in class about alternating knots (which aren't already proved in the paper)?

(8) Investigate left-orderability of the fundamental group of a knot complement.