Typos from Fortney

Please let me know if you find any typos and I will add them here.

Page 25 In the definition of the a_{ij} at the bottom of the page, the last offset equation should read

$$Df(x) \cdot e_j = \sum_{i=1}^{m} a_{ij} \tilde{e}_i = \begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{bmatrix}$$

Page 26 In the last three offset equations on the page, each x_{i_0} should be replaced with x_{j_0}.

Page 45 In Definition 2.3.2, the first input $t_0 + ta$ of f should be $x_0 + ta$.

Pages 50-52 The notation switches between $\left.\frac{\partial f}{\partial x_i}\right|_p$ and $\left.\frac{\partial f}{\partial x_i}\right|_{p_i}$; it should always be the former. In particular, in the computation at the bottom of page 50, each $\frac{\partial f}{\partial x_i}$ should be evaluated at $(x_1(0), \ldots, x_n(0))$ (in the third line) or at p (in the fourth line).

Page 67 In Question 2.31, both ω and α indicate the same one-form.

Page 67 In Question 2.33, it isn't possible to evaluate df_p for some of the fs of Question 2.32 (specifically (e), (g), (h), and (i), as those involve roots and ln, which cannot be applied to negative numbers). It also isn't possible to evaluate $df[v_p]$ when $v_p \in T_p\mathbb{R}^3$ for (a)-(c) of Question 2.32, because in those cases $f : \mathbb{R}^2 \to \mathbb{R}$.

Page 86 All discussion of counting the number of two-dimensional subspaces refers to the two-dimensional subspaces spanned by combinations of the standard basis vectors.

Page 91 In the first equation, dx^2 should be dx_2.

Page 91 In the formula for the wedge product of two arbitrary forms, the sum on the right hand side is over all $I \in \mathcal{I}_{k,n}$ and $J \in \mathcal{I}_{\ell,n}$.

Page 96 In the first line after the equations, "indice" should be "index."

Page 99 In the third line of the computation of $\iota_v(\alpha \wedge \beta)$ in the middle of the page, dx_p should be dx_k.

Page 99 The last two lines on the page are missing some minus signs. They should read, after the equals sign:

$$= \sum_{i=1}^{k-1} (-1)^{i-1} dx_i(v) \sum_{j=i+1}^{k} (-1)^{(j-1)-1} dx_j(u) (dx_1 \wedge \cdots \widehat{dx_i} \cdots \widehat{dx_j} \cdots \wedge dx_k)$$
$$+ \sum_{i=2}^{k} (-1)^{i-1} dx_i(v) \sum_{j=1}^{i-1} (-1)^{j-1} dx_j(u) (dx_1 \wedge \cdots \widehat{dx_j} \cdots \widehat{dx_i} \cdots \wedge dx_k)$$

Page 176 The descriptor of Figure 5.33 should read "As an example, a generic one-form α on $\mathbb{R}^2$ is shown in Figure 5.33."

Page 147 In Question 4.17, you should be asked to show

$$d(\phi \wedge \psi) = d\phi \wedge \psi - \phi \wedge d\psi$$

Page 149 Not exactly a typo, but a note: the result of Question 4.33 holds for $\mathbb{R}^n$, not just $\mathbb{R}^2$.

Page 228 In Question 6.27, α is a two-form, not a one-form.

Integration and orientations The formula

$$\int_R f\, dx_1 \wedge \cdots \wedge dx_n = \int_{\phi(R)} \left(\phi^{-1}\right)^* \left(f\, dx_1 \wedge \cdots \wedge dx_n\right)$$

does not work in general – it only works if ϕ "preserves orientation." If ϕ "reverses orientation," then we have to add a minus sign:

$$\int_R f\, dx_1 \wedge \cdots \wedge dx_n = -\int_{\phi(R)} \left(\phi^{-1}\right)^* \left(f\, dx_1 \wedge \cdots \wedge dx_n\right).$$

Similarly, when integrating a k-form over a k-manifold,

$$\int_{\Sigma_k} \alpha = \int_{\phi(\Sigma_k)} \left(\phi^{-1}\right)^* \alpha$$

only if ϕ preserves orientation (this requries choosing an orientation for Σ_k!), and

$$\int_{\Sigma_k} \alpha = -\int_{\phi(\Sigma_k)} \left(\phi^{-1}\right)^* \alpha$$

if ϕ reverses orientation.

Page 235 In the first line of the displayed equations at the bottom of the page, the integral with respect to x should be from zero to $y+2$, not from $y+2$ to zero. You can check this because the answer he gets for the integral is negative, while the function he's integrating is positive over the region R of integration.

Page 238-9 The formula $\int_{\phi(R)} f\, du \wedge dv = \iint_{\phi(R)} f\, dudv$ only works if $du \wedge dv$ is a properly oriented volume form. In this case it's not, which we know because $(\phi^{-1})^* du \wedge dv = -\frac{1}{2} dx \wedge dy$. So we can't compute

$$\int_{\phi(R)} f\, du \wedge dv = \iint_{\phi(R)} f\, dudv;$$

instead we have to compute

$$-\int_{\phi(R)} f\, dudv.$$

Page 243 There is a sign error in this parameterization; the correct integrand is the negative of Fortney's.

page 253 In Example Two (b), the parameterization should be

$$(\theta, \phi) \mapsto (\sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi).$$