

# FALL 2021 AG READING GROUP: BRAUER GROUPS

TONY VÁRILLY-ALVARADO

## 1. INTRODUCTION

The goal of this reading group is to work through some parts of the upcoming book by Colliot-Thélène and Skorobogatov [CTS21]. The goal of any given lecture is not to go through the details of every technical proof. Rather, it is to give a big picture overview of the topic, with definitions, examples, and theorem statements. Brief sketches of proofs to indicate logical dependencies of the material are OK.

## 2. FALL 2021

**Lecture 1 (08/25).** The language of Central Simple Algebras: corresponding to §1.2 of [CTS21]. This will be mostly warm-up review.

**Lecture 2 (09/08).** The language of Galois Cohomology: corresponding to §1.3 of [CTS21]. This will also be mostly warm-up review.

**Lecture 3 (09/22).** Galois Cohomology of discretely valued fields: corresponding to §1.4 of [CTS21]. This will likely contain material many in the audience have not seen before.

**Lecture 4 (10/06).** Étale cohomology I: corresponding to §2.1 of [CTS21]. Grothendieck topologies, presheaves and sheaves, points and stalks, morphisms of sites.

**Lecture 5 (10/20).** Étale cohomology II: corresponding to §2.2 and §2.4 of [CTS21]. Definition of cohomology. Relation to Galois cohomology; some standard spectral sequences and computation of a few cohomology groups.

**Lecture 6 (11/03–Tony out of town).** Étale cohomology III: corresponding to §2.5 of [CTS21]. Construction of the Picard Scheme.

**Lecture 7 (11/17).** Smooth Varieties I: corresponding to §4.1–§4.2 of [CTS21].

## 3. SPRING 2022

Exact dates to be determined.

**Lecture 8.** Smooth Varieties II: corresponding to §4.3–§4.4 of [CTS21].

**Lecture 9.** Smooth Varieties III: corresponding to §4.5–§4.6 of [CTS21].

---

*Date:* August 20th, 2021.

**Lecture 10.** Unramified Brauer Groups: corresponding to §5.2 of [CTS21].

**Lecture 11.** Severi–Brauer varieties: corresponding to §6.1 of [CTS21].

**Lecture 12.** The Brauer–Manin Set I: corresponding to §12.1–12.2 of [CTS21].

**Lecture 13.** The Brauer–Manin Set II: corresponding to §12.3–12.4 of [CTS21].

**Lecture 14.** The Brauer–Manin Set III: corresponding to §12.5–12.6 of [CTS21].

#### REFERENCES

- [CTS21] J.-L. Colliot-Thélène and A. N. Skorobogatov, *The Brauer–Grothendieck group*, 2021. ↑1, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3
- [Gro68a] A. Grothendieck, *Le groupe de Brauer. I. Algèbres d’Azumaya et interprétations diverses*, Dix exposés sur la cohomologie des schémas, Adv. Stud. Pure Math., vol. 3, North-Holland, Amsterdam, 1968, pp. 46–66 (French). ↑
- [Gro68b] ———, *Le groupe de Brauer. II. Théorie cohomologique*, Dix exposés sur la cohomologie des schémas, Adv. Stud. Pure Math., vol. 3, North-Holland, Amsterdam, 1968, pp. 67–87 (French). ↑
- [Gro68c] ———, *Le groupe de Brauer. III. Exemples et compléments*, Dix exposés sur la cohomologie des schémas, Adv. Stud. Pure Math., vol. 3, North-Holland, Amsterdam, 1968, pp. 88–188 (French). MR244271 ↑
- [Mil80] J. S. Milne, *Étale cohomology*, Princeton Mathematical Series, No. 33, Princeton University Press, Princeton, N.J., 1980. ↑

ANTHONY VÁRILLY-ALVARADO, DEPARTMENT OF MATHEMATICS MS 136, RICE UNIVERSITY, 6100 S. MAIN ST., HOUSTON, TX 77005, USA

*E-mail address:* [av15@rice.edu](mailto:av15@rice.edu)

*URL:* <http://math.rice.edu/~av15>