FROBENIUS ALGEBRAS

galore

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SETTING THE SCENE: ALGEBRAIC STRUCTURES...

SET ⟶ SEMIGROUP ⟶ MONOID ⟶ GROUP ⟶ ABELIAN GROUP

- SEMIGROUP: SET, + (ADDITION (ASSOCIATIVE))
- MONOID: SET, +, 0
- GROUP: SET, +, 0, − (ADDITIVE INVERSES)
- ABELIAN GROUP: SET, +, 0, − (COMMUTATIVE)

ABELIAN GROUP ⟶ RING, ABELIAN GROUP ⟶ MODULE

RING
SET, +, 0, −, X, 1
COM   MULTIPLICATION (ASSOCIATIVE) (COULD BE NON COM.)

MODULE OVER A RING R
SET, +, 0, −, *
COM   SCALAR (R-)MULTIP.

RING ⟶ FIELD, MODULE ⟶ VECTOR SPACE

FIELD
SET, +, 0, −, X, 1, ÷
COM   COM   MULTIP. INVERSES

VECTOR SPACE OVER A FIELD $\Bbbk$
SET, +, 0, −, *
COM   SCALAR ($\Bbbk$-)MULTIP.

RING ⇢ ALGEBRA ⇠ VECTOR SPACE

ALGEBRA OVER A FIELD $\Bbbk$
SET, +, 0, −, X, 1, *
COM ASSOC.   ↑ ASSOCIATIVE COULD BE NON COMMUTATIVE

A UNITAL RING R THAT IS GIVEN THE STRUCTURE OF AN $\Bbbk$-VS

$\Longleftrightarrow$

A $\Bbbk$-VECTOR SPACE V THAT IS GIVEN THE STRUCTURE OF A UNITAL RING

(SEE DUMMIT-FOOTE...)

(CATEGORICALLY, THIS IS NICER...)

CATEGORY
$\equiv$ COLLECTION OF OBJECTS
& MAPS BETWEEN THESE OBJECTS.
(SUBJECT TO ADD'L NICE CONDITIONS)

ALGEBRA/$\Bbbk$ IS A TRIPLE:

A, $\Bbbk$-VECTOR SPACE
$m: A\otimes A \longrightarrow A$, $\Bbbk$-LINEAR MAP "mult."
$u: \Bbbk \longrightarrow A$, $\Bbbk$-LINEAR MAP "unit" $\quad \otimes := \otimes_{\Bbbk}$

SUBJECT TO ASSOCIATIVITY & UNITALITY CONSTRAINTS

$$\begin{tikzcd}
(A\otimes A)\otimes A \arrow[rr, "\sim"] \arrow[d, "m\otimes id"'] & & A\otimes(A\otimes A) \arrow[d, "id\otimes m"] \\
A\otimes A \arrow[dr, "m"'] \arrow[rr, phantom, "2"] & & A\otimes A \arrow[dl, "m"] \\
& A &
\end{tikzcd}$$

$$\begin{tikzcd}
\Bbbk\otimes A \arrow[r, "\simeq"] \arrow[d, "u\otimes id"'] & A \arrow[d, phantom, "2"] & A\otimes \Bbbk \arrow[l, "\simeq"'] \arrow[d, "id\otimes u"] \\
A\otimes A \arrow[r, "m"'] & A & A\otimes A \arrow[l, "m"]
\end{tikzcd}$$

CAN REVERSE ARROWS
& DEFINE "CO" STRUCTURES

ALGEBRA/$\Bbbk$ IS A TRIPLE:

A, $\Bbbk$-VECTOR SPACE
$m: A\otimes A \longrightarrow A$, $\Bbbk$-LINEAR MAP
$u: \Bbbk \longrightarrow A$, $\Bbbk$-LINEAR MAP

SUBJECT TO ASSOCIATIVITY
& UNITALITY CONSTRAINTS

$$\begin{tikzcd}
(A\otimes A)\otimes A \arrow[rr, "\sim"] \arrow[d, "m\otimes id"'] & & A\otimes(A\otimes A) \arrow[d, "id\otimes m"] \\
A\otimes A \arrow[dr, "m"'] \arrow[rr, phantom, "2"] & & A\otimes A \arrow[dl, "m"] \\
& A &
\end{tikzcd}$$

$$\begin{tikzcd}
\Bbbk\otimes A \arrow[r, "\simeq"] \arrow[d, "u\otimes id"'] & A \arrow[d, phantom, "2"] & A\otimes \Bbbk \arrow[l, "\simeq"'] \arrow[d, "id\otimes u"] \\
A\otimes A \arrow[r, "m"'] & A & A\otimes A \arrow[l, "m"]
\end{tikzcd}$$

COALGEBRA/$\Bbbk$ IS A TRIPLE:

A, $\Bbbk$-VECTOR SPACE
$\Delta: A \longrightarrow A\otimes A$, $\Bbbk$-LINEAR MAP "comult."
$\varepsilon: A \longrightarrow \Bbbk$, $\Bbbk$-LINEAR MAP "counit"

SUBJECT TO COASSOCIATIVITY & COUNITALITY CONSTRAINTS

$$\begin{tikzcd}
(A\otimes A)\otimes A \arrow[rr, "\sim"] & & A\otimes(A\otimes A) \\
A\otimes A \arrow[u, "\Delta\otimes id"] \arrow[rr, phantom, "2"] & & A\otimes A \arrow[u, "id\otimes\Delta"'] \\
& A \arrow[ul, "\Delta"] \arrow[ur, "\Delta"'] &
\end{tikzcd}$$

$$\begin{tikzcd}
\Bbbk\otimes A \arrow[r, "\simeq"] & A \arrow[d, phantom, "2"] & A\otimes \Bbbk \arrow[l, "\simeq"'] \\
A\otimes A \arrow[u, "\varepsilon\otimes id"] & A \arrow[l, "\Delta"'] \arrow[r, "\Delta"] & A\otimes A \arrow[u, "id\otimes\varepsilon"']
\end{tikzcd}$$

CAN EASILY COMBINE STRUCTURES TO FORM NEW & COOL ONES

FROBENIUS ALGEBRA/$\Bbbk$

IS A 5-TUPLE:

FORMS AN ALGEBRA/$\Bbbk$

- A, $\Bbbk$-VECTOR SPACE
- $m : A \otimes A \longrightarrow A$, $\Bbbk$-LINEAR MAP
- $u : \Bbbk \longrightarrow A$, $\Bbbk$-LINEAR MAP
- $\Delta : A \longrightarrow A \otimes A$, $\Bbbk$-LINEAR MAP
- $\varepsilon : A \longrightarrow \Bbbk$, $\Bbbk$-LINEAR MAP

FORMS A COALGEBRA/$\Bbbk$

NOT THE ORIGINAL DEFINITION

SUBJECT TO COMPATIBILITY CONDITION:

$$\begin{tikzcd}
 & A \otimes A \arrow[dl, "\Delta \otimes id"'] \arrow[d, "m"] \arrow[dr, "id \otimes \Delta"] & \\
(A \otimes A) \otimes A \arrow[d, "s"'] & A \arrow[dd, "\Delta"] & A \otimes (A \otimes A) \arrow[d, "s"] \\
A \otimes (A \otimes A) \arrow[dr, "id \otimes m"'] & & (A \otimes A) \otimes A \arrow[dl, "m \otimes id"] \\
 & A \otimes A &
\end{tikzcd}$$

BUT WHY CARE??

OUTLINE FOR REST OF TALK

I. HISTORY & THE VARIOUS DEFNS OF FROBENIUS ALGEBRAS/$\Bbbk$

II. FROBENIUS ALGEBRAS IN MONOIDAL CATEGORIES

III. RECENT WORK INVOLVING FROBENIUS ALGEBRAS

I. HISTORY & THE VARIOUS DEFN'S OF FROBENIUS ALGEBRAS/$\Bbbk$

A 1903-STYLE GAME —

DUE TO F.G. FROBENIUS (1849–1917)

- START WITH A FINITE-DIM'L $\Bbbk$-ALGEBRA A
- PICK BASIS $\{v_1, \dots, v_n\}$ OF A
- GET SCALARS $\beta_{ij}^{(a)}$, $\gamma_{ij}^{(a)} \in \Bbbk$ SUCH THAT

$$v_i a = \sum_{j=1}^{n} \beta_{ij}^{(a)} v_j \quad \& \quad a v_i = \sum_{j=1}^{n} \gamma_{ji}^{(a)} v_j \qquad \forall a \in A$$

- GET $\Bbbk$-LINEAR MAPS

$$\beta : A \longrightarrow \mathrm{Mat}_n(\Bbbk), \quad a \longmapsto (\beta_{ij}^{(a)}) \qquad \& \qquad \gamma : A \longrightarrow \mathrm{Mat}_n(\Bbbk), \quad a \longmapsto (\gamma_{ij}^{(a)})$$

Q: WHEN DOES $\exists\, P \in GL_n(\Bbbk)$ $\ni$. $P\,\beta(a) = \gamma(a)\,P \quad \forall a \in A$?

IN THE MODERN LANGUAGE OF REP. THEORY:

Q $\Leftrightarrow$ WHEN ARE THE REG. REPS β, γ OF A EQUIVALENT?

BACK IN 1903:

NO REPS, NO MODULES,

& "ALGEBRAS" = "HYPERCOMPLEX SYSTEMS"

CAN ANSWER QUESTION VIA MATRICES...

FINITE-DIM'L $\Bbbk$-ALGEBRA A

BASIS $\{v_1, \dots, v_n\}$ OF A

$\beta_{ij}^{(a)}, \gamma_{ij}^{(a)} \in \Bbbk$ SUCH THAT

$$v_i a = \sum_{j=1}^{n} \beta_{ij}^{(a)} v_j$$

$$\& \quad a v_j = \sum_{i=1}^{n} v_i \gamma_{ij}^{(a)} \quad \forall a \in A$$

GET $\Bbbk$-LINEAR MAPS

$$\beta: A \longrightarrow \mathrm{Mat}_n(\Bbbk), \quad a \longmapsto (\beta_{ij}^{(a)})$$

$$\gamma: A \longrightarrow \mathrm{Mat}_n(\Bbbk), \quad a \longmapsto (\gamma_{ij}^{(a)})$$

Q: WHEN DOES $\exists P \in GL_n(\Bbbk)$ $\ni$.
$P\,\beta(a) = \gamma(a)\,P \quad \forall a \in A$?

GET SCALARS $p_{ij}^{k} \in \Bbbk$ SUCH THAT

$$v_i v_j = \sum_{k=1}^{n} p_{ij}^{k} v_k$$

FOR $\underline{c} = (c_1, \dots, c_n) \in \Bbbk^n$, DEFINE
PARATROPHIC MATRIX OF A AT $\underline{c}$

$$P_{\underline{c}} = \left(\textstyle\sum_{k=1}^{n} c_k p_{ij}^{k} \right)_{i,j} \in \mathrm{Mat}_n(\Bbbk)$$

A: [FROBENIUS, 1903]:
WHEN $\exists \underline{c} \in \Bbbk^n$ $\ni$. $P_{\underline{c}}$ IS INVERTIBLE
(IN THIS CASE, $P_{\underline{c}} = P$ ABOVE)

DEFINITION!

A FINITE DIM'L $\Bbbk$-ALG. A
IS FROBENIUS*

$$\Updownarrow$$

$$\exists\, \underline{c} \in \Bbbk^n \ \ni. \ \det(P_{\underline{c}}) \neq 0$$

* NOT SELF NAMED

(NON)EXAMPLES

TRY!

- $\Bbbk[x,y]/(x^2, y^2)$ IS FROB.
- $\Bbbk[x,y]/(x^2, xy, y^2)$ IS <u>NOT</u> FROB

FINITE-DIM'L $\Bbbk$-ALGEBRA A

BASIS $\{v_1, \dots, v_n\}$ OF A

Q: WHEN DOES $\exists P \in GL_n(\Bbbk)$ $\ni$.
$P\,\beta(a) = \gamma(a)\,P \quad \forall a \in A$?

GET SCALARS $p_{ij}^{k} \in \Bbbk$ SUCH THAT

$$v_i v_j = \sum_{k=1}^{n} p_{ij}^{k} v_k$$

FOR $\underline{c} = (c_1, \dots, c_n) \in \Bbbk^n$, DEFINE
PARATROPHIC MATRIX OF A AT $\underline{c}$

$$P_{\underline{c}} = \left(\textstyle\sum_{k=1}^{n} c_k p_{ij}^{k} \right)_{i,j} \in \mathrm{Mat}_n(\Bbbk)$$

A: [FROBENIUS, 1903]:
WHEN $\exists \underline{c} \in \Bbbk^n$ $\ni$. $P_{\underline{c}}$ IS INVERTIBLE
(IN THIS CASE, $P_{\underline{c}} = P$ ABOVE)

≡ AND THEN THE GAME WAS OVER FOR A WHILE ≡

≡ UNTIL ~35 YEARS LATER ≡

DEFINITION 1

A FINITE DIM'L $\Bbbk$-ALG. A IS FROBENIUS *

$\Updownarrow$

$\exists\, \underline{c} \in \Bbbk^n \ni.\ \det(P_{\underline{c}}) \neq 0$

PARATROPHIC MATRIX

* NOT SELF NAMED

MORE TOOLS OF ABSTRACT ALG. NOW RINGS, ALGEBRAS, MODULES ...

≡ ABLE TO WORK BASIS-FREE ≡

(THANKS, EMMY NOETHER!)

BRAUER, NAKAYAMA, & NESBITT REVIVED FROBENIUS ALGEBRAS

FROM LAM'S BOOK —

C. Nesbitt wrote: "The writer, in collaborating with T. Nakayama, adopted the term Frobeniusean algebra, but now, quailing before our critics, we return to simply Frobenius algebra." So apparently, people were not attracted by a six-syllable word.

DEFINITION 1

FINITE DIM. $\Bbbk$-ALG. A IS FROBENIUS

$\Updownarrow$

$\exists\, \underline{c} \in \Bbbk^n \ni.\ \det(P_{\underline{c}}) \neq 0$

PARATROPHIC MATRIX

EXAMPLES

- MATRIX ALGEBRAS $\mathrm{Mat}_n(\Bbbk)$
- EXTERIOR ALGEBRAS $\Lambda(V)$ (CALLED "CARTAN'S ALGEBRA OF OUTER MULTIPLICATION" IN 1941)
- GROUP ALGEBRAS $\Bbbk G$
- COHOMOLOGY ALGEBRAS OF ORIENTED MANIFOLDS $H^*(X)$

DEFINITIONS galore

THEOREM [BRAUER-NESBITT, NAKAYAMA, 1937-1942]

LET A BE A FINITE DIM'L $\Bbbk$-ALGEBRA

THEN THE FOLLOWING ARE EQUIVALENT:

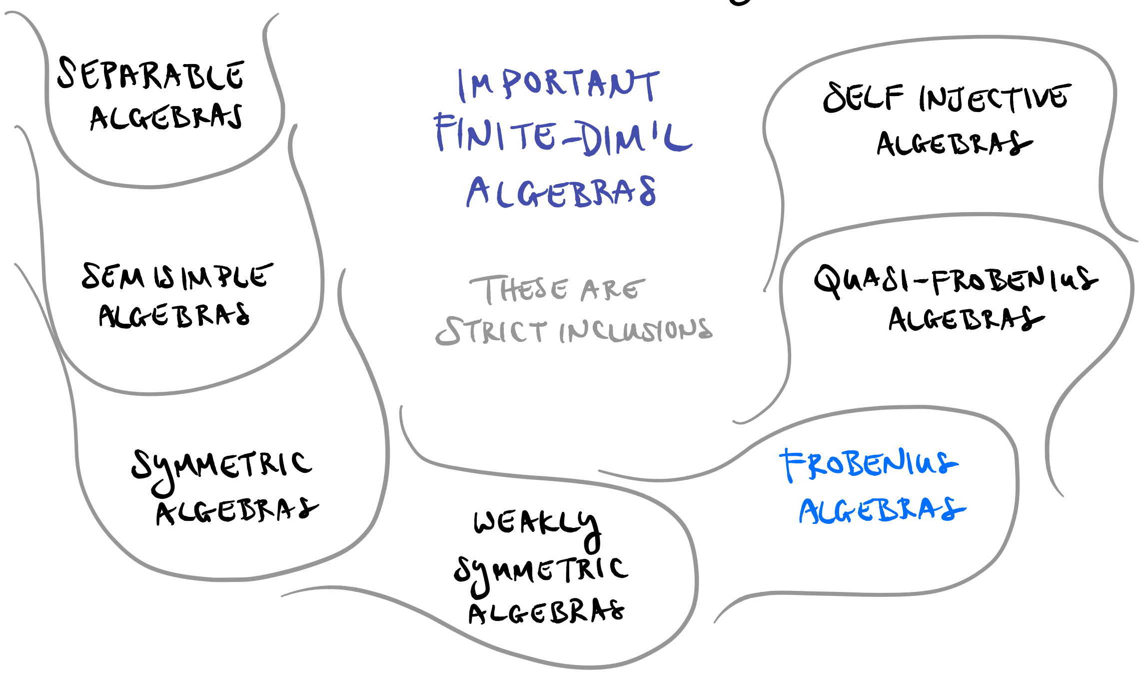
1. A IS FROBENIUS
2. $\exists$ NONDEG. ASSOC. $\Bbbk$-BILINEAR FORM $(-,-): A \times A \longrightarrow \Bbbk$
3. $\exists$ ISOM. OF LEFT A-MODULES : $A \cong A^*$

3'. $\exists$ ISOM. OF RIGHT A-MODULES : $A \cong A^*$

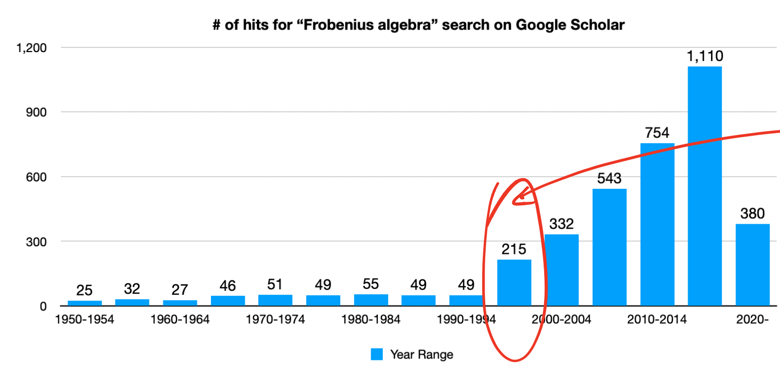
4. $\exists\ \Bbbk$-LINEAR FORM $\nu: A \longrightarrow \Bbbk \ni.\ \ker(\nu)$ DOES NOT CONTAIN $\neq 0$ LEFT IDEAL OF A

4'. $\exists\ \Bbbk$-LINEAR FORM $\nu: A \longrightarrow \Bbbk \ni.\ \ker(\nu)$ DOES NOT CONTAIN $\neq 0$ RIGHT IDEAL OF A

≡ RELATED GAMES OF THE 1940's & BEYOND INVOLVE THE PLAYERS... ≡

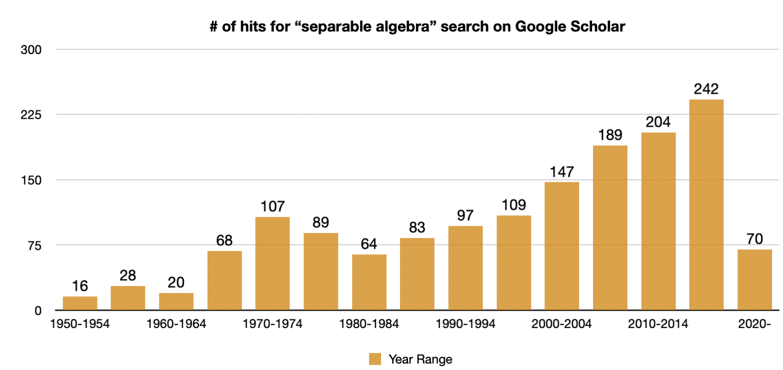


POPULARITY THROUGHOUT THE YEARS



SOMETHING HAPPENED HERE

(JUST FOR COMPARISION)



(EXAMPLES INCLUDE IMPORTANT FROB. ALGS LIKE $\mathrm{Mat}_n(\Bbbk)$, $\Bbbk G$, $|G| \in \Bbbk^\times$)

II. FROBENIUS ALGEBRAS IN MONOIDAL CATEGORIES

A LATE 1980's / 1990's STYLE GAME –

DUE TO G. SEGAL, E. WITTEN, M. ATIYAH ORIGINALLY...

BUILD A CATEGORICAL MACHINE

TO PRODUCE INVARIANTS OF LOW-DIM'L MANIFOLDS

CATEGORY $\mathcal{C}$

≡ COLLECTION OF OBJECTS
& MAPS BETWEEN THESE OBJECTS.
(SUBJECT TO ADD'L NICE CONDITIONS)

MONOIDAL CATEGORY $(\mathcal{C}, \otimes, \mathbb{1})$

≡ CATEGORY $\mathcal{C}$ EQUIPPED WITH
BIFUNCTOR $\otimes : \mathcal{C} \times \mathcal{C} \longrightarrow \mathcal{C}$
DISTINGUISHED OBJECT $\mathbb{1}$
THAT MIMIC THE STRUCTURE OF A MONOID
(SUBJECT TO COMPATIBILITY CONDITIONS)

SYMMETRIC MON'L CAT. $(\mathcal{C}, \otimes, \mathbb{1}, c)$

≡ MONOIDAL CATEGORY EQUIPPED WITH
NATURAL ISOMORPHISM : $\forall X, Y \in \mathcal{C}$
$c_{X,Y} : X \otimes Y \xrightarrow{\sim} Y \otimes X \ \ni. \ c^2 = id$

FLIP MAP

EX. $(Vec_{\Bbbk}, \otimes_{\Bbbk}, \Bbbk, c)$
IS A SYM. MON'L CATEGORY

TOP'L QUANTUM FIELD THY

d-TQFT ≡ A SYMMETRIC MONOIDAL FUNCTOR

$$Z : d\text{-}Cobord \longrightarrow Vec_{\Bbbk}$$

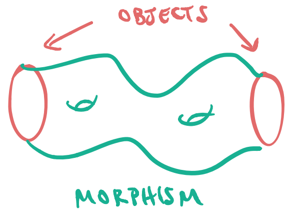
OBJECTS: ORIENTED, CLOSED SMOOTH $(d-1)$-DIM. MFLDS

MORPHISMS: (BOUNDARY & ORIENTATION PRES. DIFFEOM. CLASSES OF) COMPACT d-DIM. MFLDS.

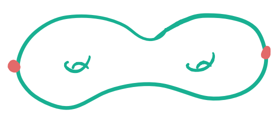
OBJECTS: $\Bbbk$-VECTOR SPACES

MORPHISMS: $\Bbbk$-LINEAR MAPS

$\underline{d=2}$



$\xrightarrow{\quad Z \quad}$ $V \xrightarrow[\Bbbk\text{-LIN. MAP}]{} V$ ($\Bbbk$-VS) — TOP'L INV. ||| MATRIX



$\xrightarrow{\quad Z \quad}$ $\Bbbk \longrightarrow \Bbbk$ — TOP'L INV. ||| SCALAR

d-TQFTs PRODUCE $\Bbbk$-LINEAR ALGEBRAIC INVARIANTS OF d-MFLDS

WHEN $\mathrm{Vec}_{\Bbbk} \rightsquigarrow \mathscr{C}$ (SYM⊗ CAT.) : " $\mathscr{C}$-ALGEBRAIC " " "

THEOREM ["FOLKLORE": A. VORONOV (1994)
"FOLKLORE NO MORE!" : L. ABRAMS (1996)]

> THERE IS AN EQUIVALENCE OF MONOIDAL CATEGORIES
>
> MON. CATEGORY OF 2-TQFTS WITH VALUE IN $\mathrm{VEC}_{\Bbbk}$ & MON. CATEGORY OF COMMUTATIVE FROBENIUS ALGEBRAS / $\Bbbk$

AND THE DEF'N OF A FROBENIUS ALG. USED HERE WAS THE ONE FROM THE VERY BEG.!

DEFINITION 5 [L. ABRAMS] SHOWED EQUIV. TO DEF 2 INVOLVING NONDEG PAIRING

FROBENIUS ALGEBRA / $\Bbbk$ IS A 5-TUPLE:

FORMS AN ALG / $\Bbbk$:
- A, $\Bbbk$-VECTOR SPACE
- $m : A \otimes A \to A$, $\Bbbk$-LINEAR MAP
- $u : \Bbbk \to A$, $\Bbbk$-LINEAR MAP

FORMS A COALG / $\Bbbk$ (with A):
- $\Delta : A \to A \otimes A$, $\Bbbk$-LINEAR MAP
- $\varepsilon : A \to \Bbbk$, $\Bbbk$-LINEAR MAP

WHERE:

$$
\begin{tikzcd}
 & A\otimes A \arrow[dl, "\Delta\otimes id"'] \arrow[dr, "id\otimes\Delta"] \arrow[dd, "m"] & \\
(A\otimes A)\otimes A \arrow[dd, "s"] & & A\otimes(A\otimes A) \arrow[dd, "s"] \\
 \arrow[r, phantom, "2"] & A \arrow[dd] \arrow[r, phantom, "2"] & \\
A\otimes(A\otimes A) \arrow[dr, "id\otimes m"'] & & (A\otimes A)\otimes A \arrow[dl, "m\otimes id"] \\
 & A\otimes A &
\end{tikzcd}
$$

CATEGORICAL UPGRADE TAKE $(\mathcal{C}, \otimes, \mathbb{1})$ A MONOIDAL CATEGORY

FROBENIUS ALGEBRA IN $\mathcal{C}$ IS A 5-TUPLE:

$A \in \mathcal{C}$ OBJECT

$m: A \otimes A \longrightarrow A \in \mathcal{C}$

$u: \mathbb{1} \longrightarrow A \in \mathcal{C}$

$\Delta: A \longrightarrow A \otimes A \in \mathcal{C}$

$\varepsilon: A \longrightarrow \mathbb{1} \in \mathcal{C}$

WITH (A, m, u) IS AN ALGEBRA IN $\mathcal{C}$

(A, Δ, ε) IS A COALGEBRA IN $\mathcal{C}$

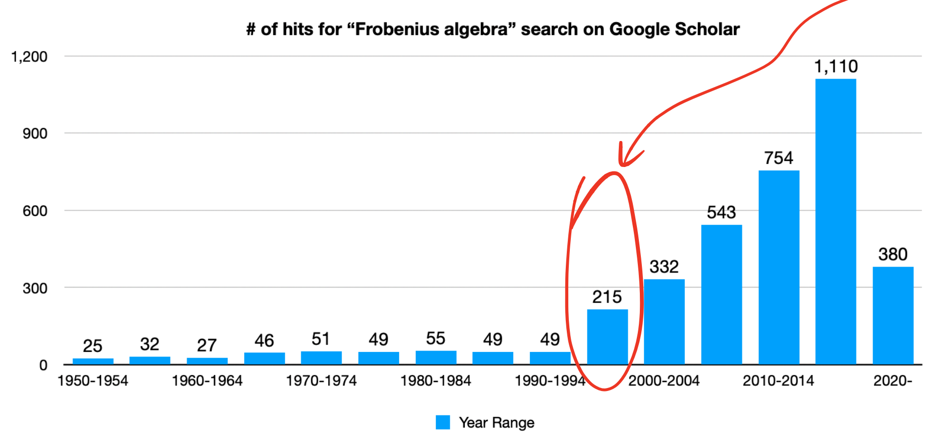
SATISFYING SIMILAR DIAGRAMS AS FOR $\mathcal{C} = \mathrm{Vec}_{\mathbb{k}}$

THEOREM FOR A SYMMETRIC MONOIDAL CATEG. $(\mathcal{C}, \otimes, \mathbb{1}, c)$

THERE IS AN EQUIVALENCE OF MONOIDAL CATEGORIES

MON. CATEGORY OF 2-TQFTS WITH VALUE IN $\mathcal{C}$ & MON. CATEGORY OF COMMUTATIVE FROBENIUS ALGEBRAS IN $\mathcal{C}$

(WITH SAME PROOF AS FOR $\mathcal{C} = \mathrm{Vec}_{\mathbb{k}}$)



2D-TQFTS MOSTLY CAUSED THIS

DUBROVIN'S (1996) "FROBENIUS MANIFOLDS" WAS ALSO A CAUSE ... QUANTUM COHOMOLOGY... (SKIPPING HERE...)

... AND FROBENIUS ALGEBRAS IN MONOIDAL CATEGORIES HAVE BEEN OF GREAT INTEREST EVER SINCE ...

INCLUDING APPLICATIONS IN

- MORITA THEORY
- QFTS & CFTS
- COMPUTER SCIENCE
- RECENTLY: 3D-TQFTS ARXIV: 2105.04613 [KMRS]

CATEGORICAL FROBENIUS ALGEBRAS *galore*

LET A BE A FINITE DIM'L $\Bbbk$-ALGEBRA
THEN THE FOLLOWING ARE EQUIVALENT:

1. $\exists$ INVERTIBLE PARATROPHIC MATRIX

2. $\exists$ NONDEG. ASSOC. $\Bbbk$-BILINEAR FORM
$(-,-): A \times A \longrightarrow \Bbbk$

3. $\exists$ ISOM. OF LEFT A-MODULES: $A \simeq A^*$

3'. $\exists$ ISOM. OF RIGHT A-MODULES: $A \simeq A^*$

4. $\exists$ $\Bbbk$-LINEAR FORM $\nu: A \longrightarrow \Bbbk$ $\ni$. $\ker(\nu)$ DOES NOT CONTAIN $\neq 0$ LEFT IDEAL OF A

4'. $\exists$ $\Bbbk$-LINEAR FORM $\nu: A \longrightarrow \Bbbk$ $\ni$. $\ker(\nu)$ DOES NOT CONTAIN $\neq 0$ RIGHT IDEAL OF A

5. $\exists$ $\Bbbk$-LINEAR MAPS $\Delta: A \to A \otimes_{\Bbbk} A$, $\varepsilon: A \to \Bbbk$ $\ni$. (A, Δ, ε) IS A $\Bbbk$-COALG. + COMPATIBILITY

LET $\mathcal{C}$ BE A RIGID MONOIDAL CATEG.
$\exists$ DUAL OBJECTS$\Big)$. TAKE AN ALG IN $\mathcal{C}$.

1. NO CATEGORICAL ANALOGUE TO DATE

2.

3.
3'

$\exists$ CATEGORICAL ANALOGUE.

[FUCHS-STIGNER]
2009

4. $\exists$ CATEGORICAL ANALOGUE
4'. ...NOW! [W-YADAV]
2021, IN PREP.

NEED $\mathcal{C}$ ABELIAN
& $\otimes$ BIEXACT

5.

III. RECENT WORK INVOLVING FROBENIUS ALGEBRAS

GENERALIZES WORK OF
BONGALE (1967): $\mathcal{C} = \mathrm{Vec}_{\Bbbk}$

THEOREM [W-YADAV, 2021 (IN PREP.)]
TAKE $\mathcal{C}$ ABELIAN, RIGID MONOIDAL CATEG. W/ $\otimes$ BIEXACT
IF A IS A "FILTERED" ALGEBRA IN $\mathcal{C}$
& ITS ASSOC. GRADED ALGEBRA $gr(A)$ IS FROBENIUS IN $\mathcal{C}$
THEN SO IS A.

MAIN EXAMPLE $A =$ "BRAIDED CLIFFORD ALGEBRA"

$gr(A) =$ "BRAIDED EXTERIOR ALGEBRA"

OTHER RECENT WORKS:

[MMPRTW] (WINARTZ GROUP) (ARXIV: 2001.03837)

USES "SPECIAL" FROBENIUS ALGEBRAS TO STUDY MORITA EQUIVALENCE OF ALGEBRAS IN NICE "FUSION CATEGORIES"

BOTH WORKS ARE WRITTEN FOR NEWCOMERS

PUBLISHED VERS. WILL BE SHORTER.

[W-WICKS-WON] (ARXIV: 1911.12847)

DEVELOPS / RECONCILES FORMULAIC VS CATEGORICAL NOTIONS OF { ALGEBRAS, COALGEBRAS, FROB. ALGS } WITH "WEAK HOPF ALG." SYMMETRY...

NOW ACCEPTING APPLICATIONS

It's going to be galore-nios

WINART3 workshop

Women in Noncommutative Algebra and Representation Theory workshop 3.
Banff International Research Station (BIRS).
April 3 – 8, 2022.

GROUP LEADERS:

BAUR + TODOROV	BENKART + KIRKMAN
CANACKI + GARCIA ELSENER	CARBONE + JURISICH
FABER + MARSH	REDONDO + ROSSI BERTONE
SCHROLL + SOLOTAR	WALTON + WICKS

≡ THANKS FOR LISTENING ≡