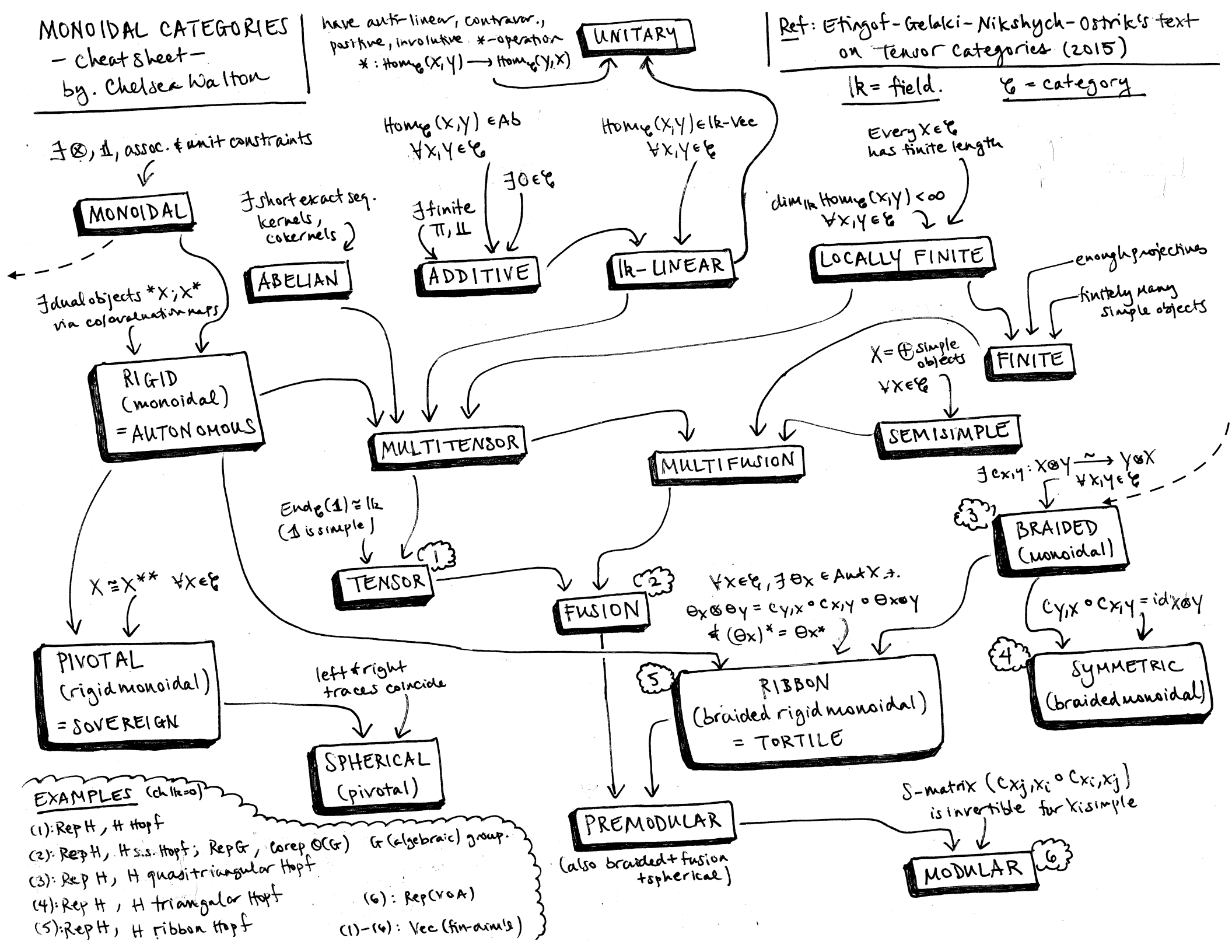
MONOIDAL CATEGORIES
– cheat sheet –
by. Chelsea Walton
Ref: Etingof-Gelaki-Nikshych-Ostrik's text on tensor categories (2015)
k = field.
C = category
have anti-linear, contravar., positive, involutive *-operation
* : Hom_C(X,Y) → Hom_C(Y,X)
UNITARY
∃ ⊗, 𝟙, assoc. & unit constraints
MONOIDAL
Hom_C(X,Y) ∈ Ab ∀X,Y∈C
∃0∈C
Hom_C(X,Y) ∈ k-Vec ∀X,Y∈C
Every X∈C has finite length
dim_k Hom_C(X,Y) < ∞ ∀X,Y∈C
∃ short exact seq. kernels, cokernels
ABELIAN
∃ finite (Π, ⊔)
ADDITIVE
k-LINEAR
LOCALLY FINITE
enough projectives
finitely many simple objects
FINITE
∃ dual objects *X; X* via coevaluation maps
RIGID (monoidal) = AUTONOMOUS
X = ⊕ simple objects ∀X∈C
SEMISIMPLE
MULTITENSOR
MULTIFUSION
End_C(𝟙) ≅ k (𝟙 is simple)
TENSOR
1
FUSION
2
∃ c_{X,Y}: X⊗Y → Y⊗X ∀X,Y∈C
3
BRAIDED (monoidal)
X ≅ X** ∀X∈C
∀X∈C, ∃θ_X ∈ Aut X s.t.
θ_X⊗θ_Y = c_{Y,X} ∘ c_{X,Y} ∘ θ_{X⊗Y}
& (θ_X)* = θ_{X*}
c_{Y,X} ∘ c_{X,Y} = id_{X⊗Y}
4
SYMMETRIC (braided monoidal)
PIVOTAL (rigid monoidal) = SOVEREIGN
left & right traces coincide
SPHERICAL (pivotal)
5
RIBBON (braided rigid monoidal) = TORTILE
PREMODULAR
(also braided + fusion + spherical)
S-matrix (c_{Xj,Xi} ∘ c_{Xi,Xj}) is invertible for X_i simple
MODULAR
6
EXAMPLES (ch k = 0)
(1): Rep H, H Hopf
(2): Rep H, H s.s. Hopf; Rep G, corep O(G) G (algebraic) group.
(3): Rep H, H quasitriangular Hopf
(4): Rep H, H triangular Hopf
(5): Rep H, H ribbon Hopf
(6): Rep(V_f A)
(1)–(6): Vec (fin-dim'l)