REFLECTIVE CENTERS OF MODULE CATEGORIES & QUANTUM K-MATRICES

ARXIV: 2307.14764

• CHELSEA WALTON •

JOINT W/ ROBERT LAUGWITZ
& MILEN YAKIMOV

EVERYBODY LIKES BRAIDINGS!

ONE MAJOR APPLICATION: STATISTICAL MECHANICS

QYBE: USED TO STUDY EXACTLY SOLVABLE MODELS

SOLUTIONS ↔ SOLUTIONS TO THE BRAID EQUATIONS ↔ QUANTUM R-MATRICES

MODERN WAY OF CAPTURING SOLUTIONS: USE BRAIDED MONOIDAL CATEGORIES

ESP. REPS OF QUASI Δ HOPF ALGS

EVERYBODY LIKES BRAIDINGS!

ONE MAJOR APPLICATION: STATISTICAL MECHANICS WITH BOUNDARY

BOUNDARY QYBE: USED TO STUDY EXACTLY SOLVABLE MODELS WITH BOUNDARY

(QUANTUM REFLECTION EQ)

SOLUTIONS ↔ SOLUTIONS TO THE BRAID EQUATIONS WITH ADDITIONAL EQS ↔ QUANTUM ~~R-MATRICES~~ K-MATRICES

MODERN WAY OF CAPTURING SOLUTIONS: USE BRAIDED ~~MONOIDAL~~ MODULE CATEGORIES

ESP. REPS OF QUASI Δ ~~HOPF ALGS~~ COMODULE ALGS

EVERYBODY LIKES BRAIDINGS!

FAVORITE EXAMPLE OF A BRAIDED MONOIDAL CATEGORY

$\mathcal{Z}(\mathcal{C})$	DRINFELD CENTER OF AN ARBITRARY MONOIDAL CATEGORY $\mathcal{C}$

ESP: $\mathcal{Z}(H\text{-Mod}) \simeq D(H)\text{-Mod}$

- H: HOPF ALG
- $D(H)$: DRINFELD DOUBLE OF H
- $D(H)$: QUASIΔ HOPF ALG

MODERN WAY OF CAPTURING SOLUTIONS TO THE ~~QYBE~~ Q.REF.EQ.:

USE BRAIDED ~~MONOIDAL~~ MODULE CATEGORIES

ESP. REPS OF QUASIΔ ~~HOPF ALGS~~ COMODULE ALGS

EVERYBODY LIKES BRAIDINGS!

FAVORITE EXAMPLE OF A BRAIDED MONOIDAL CATEGORY

$\mathcal{Z}(\mathcal{C})$	DRINFELD CENTER OF AN ARBITRARY MONOIDAL CATEGORY $\mathcal{C}$

ESP: $\mathcal{Z}(H\text{-Mod}) \simeq D(H)\text{-Mod}$

H: HOPF ALG

$D(H)$: DRINFELD DOUBLE OF H; QUASI△ HOPF ALG

GOAL:

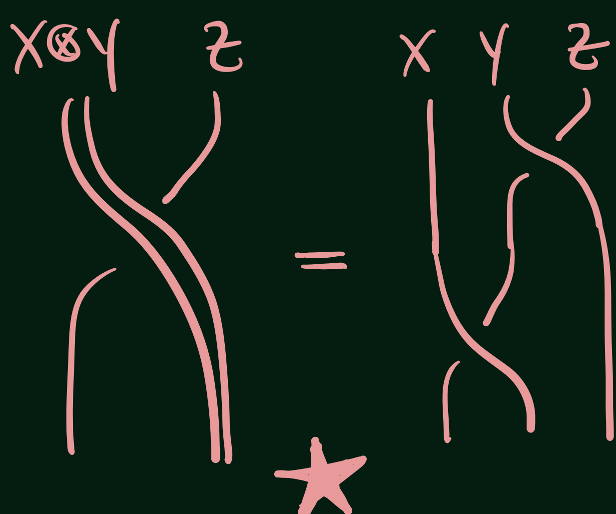
COOK UP BRAIDED MODULE CATEGORICAL ANALOGUES OF THE ABOVE

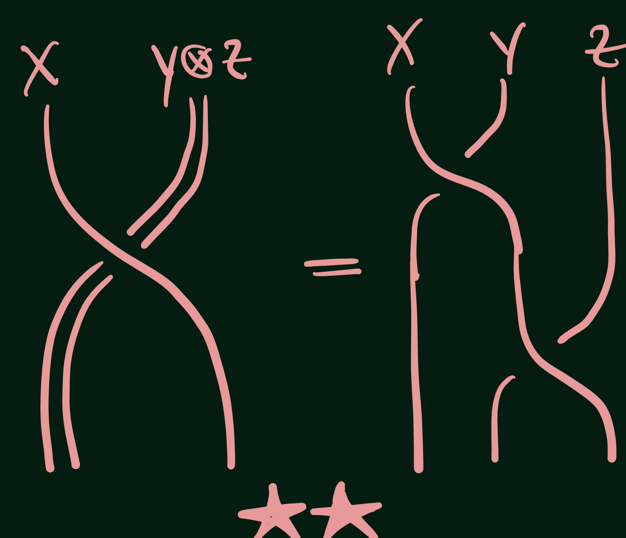
EVERYBODY LIKES BRAIDINGS!

A MONOIDAL CATEGORY $(\mathcal{C}, \otimes, \mathbb{1})$ IS **BRAIDED** IF $\exists$ NATURAL ISOM

$$c = \{ c_{X,Y} : X \otimes Y \xrightarrow{\sim} Y \otimes X \}_{X,Y \in \mathcal{C}}$$

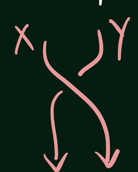
COMPATIBLE WITH $\otimes$ AS FOLLOWS:



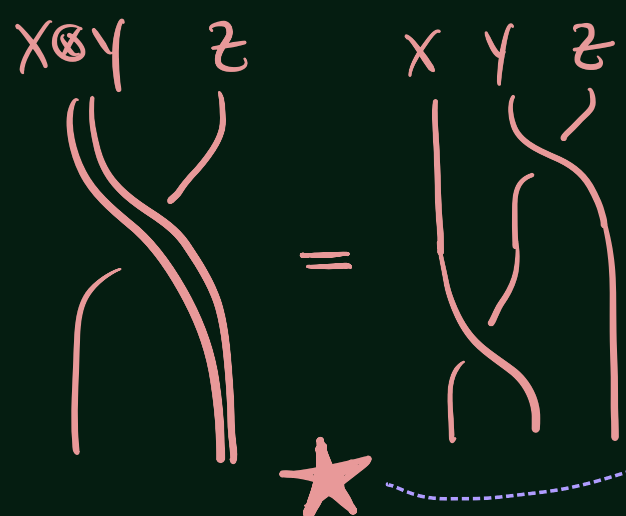


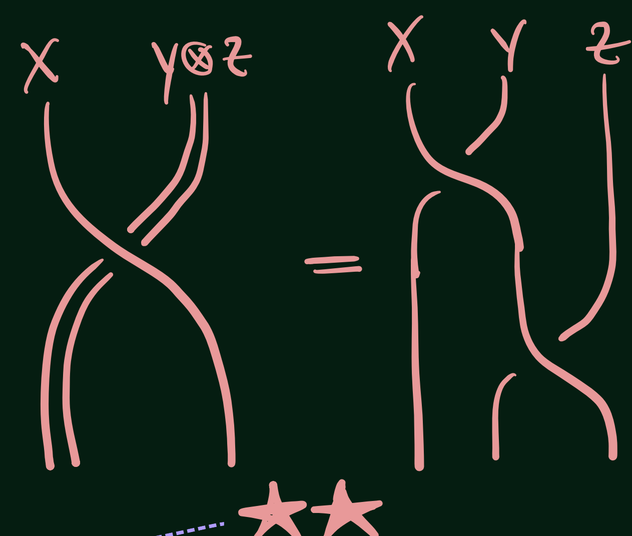
EVERYBODY LIKES BRAIDINGS!

A MONOIDAL CATEGORY $(\mathcal{C}, \otimes, \mathbb{1})$ IS **BRAIDED** IF $\exists$ NATURAL ISOM $\{c_{X,Y} : X \otimes Y \xrightarrow{\sim} Y \otimes X\}_{X,Y \in \mathcal{C}}$

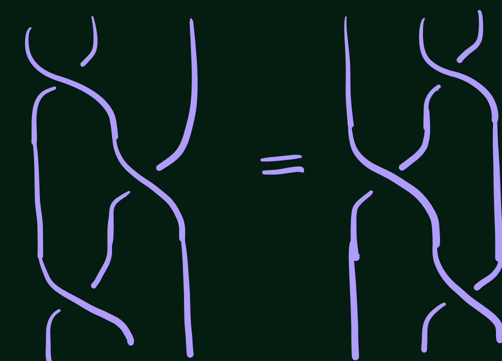


COMPATIBLE WITH $\otimes$ AS FOLLOWS:





YIELDING SOLUTIONS TO THE BRAID EQUATION

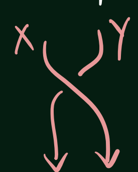


EVERYBODY LIKES DRINFELD CENTERS!

A MONOIDAL CATEGORY $(\mathcal{B}, \otimes, \mathbb{1})$ IS BRAIDED

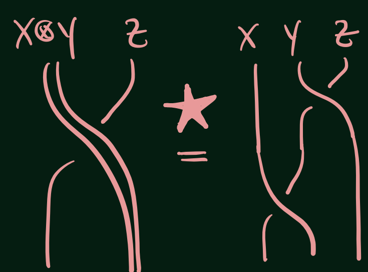
IF $\exists$ NATURAL ISOM

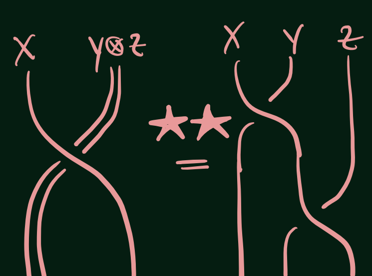
$\{c_{X,Y} : X \otimes Y \xrightarrow{\sim} Y \otimes X\}_{X,Y \in \mathcal{B}}$



COMPATIBLE WITH $\otimes$

AS FOLLOWS:





EXAMPLE : $\mathcal{Z}(\mathcal{C})$ ← $(\mathcal{C}, \otimes, \mathbb{1})$ MONOIDAL CAT.

OBJECTS : $(V \in \mathcal{C},\ c^V)$

w/ $c^V = \{c^V_X : X \otimes V \xrightarrow{\sim} V \otimes X\}_{X \in \mathcal{C}}$

WHERE $c^V_{X \otimes Y}$ SATISFIES ★

MORPHISMS $(V, c^V) \longrightarrow (W, c^W)$:

$f : V \to W \in \mathcal{C}$ w/

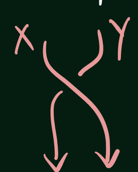
$$\begin{array}{ccc} X \otimes V & \xrightarrow{c^V} & V \otimes X \\ {\scriptstyle id \otimes f} \downarrow & \circlearrowleft & \downarrow {\scriptstyle f \otimes id} \\ X \otimes W & \xrightarrow{c^W} & W \otimes X \end{array}$$

EVERYBODY LIKES DRINFELD CENTERS!

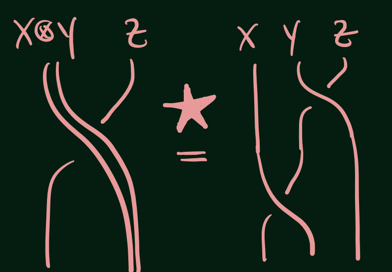
A MONOIDAL CATEGORY $(\mathcal{B}, \otimes, \mathbb{1})$ IS BRAIDED

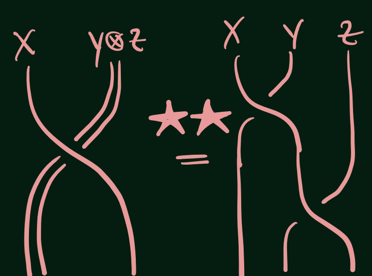
IF $\exists$ NATURAL ISOM

$\{c_{X,Y} : X \otimes Y \xrightarrow{\sim} Y \otimes X\}_{X,Y \in \mathcal{B}}$



COMPATIBLE WITH $\otimes$ AS FOLLOWS:





EXAMPLE: $\mathcal{Z}(\mathcal{C})$ ← $(\mathcal{C}, \otimes, \mathbb{1})$ MONOIDAL CAT.

OBJECTS: $(V \in \mathcal{C}, c^V)$

WHERE $c^V_{X \otimes Y}$ SATISFIES ★

MONOIDAL PRODUCT

$$(V, c^V) \otimes (W, c^W) \overset{\text{DEF}}{=} (V \otimes W, c^{V \otimes W})$$

WHERE $c^{V \otimes W}_X$ SATISFIES ★★

BRAIDING

$$\begin{array}{ccc} (V, c^V) \otimes (W, c^W) & & (W, c^W) \otimes (V, c^V) \\ \| & & \| \\ (V \otimes W, c^{V \otimes W}) & \xrightarrow{c^W_V} & (W \otimes V, c^{W \otimes V}) \end{array}$$

EVERYBODY ALSO LIKES REPRESENTATIONS!

GIVEN A MONOIDAL CATEGORY $(\mathcal{C}, \otimes, \mathbb{1})$,

A PAIR $(\mathcal{M}, \triangleright)$ IS A

LEFT $\mathcal{C}$-MODULE CATEGORY

WHEN $\mathcal{M}$ IS A CATEGORY

$\triangleright : \mathcal{C} \times \mathcal{M} \longrightarrow \mathcal{M}$ BIFUNCTOR

COMPATIBLE WITH $\otimes$, $\mathbb{1}$

EVERYBODY ALSO LIKES REPRESENTATIONS!

GIVEN $(\mathcal{C}, \otimes, \mathbb{1})$,

$(\mathcal{M}, \triangleright)$ IS A

LEFT $\mathcal{C}$-MODULE CATEGORY

WHEN

$\mathcal{M}$ IS A CATEGORY

$\triangleright : \mathcal{C} \times \mathcal{M} \longrightarrow \mathcal{M}$

BIFUNCTOR

COMPATIBLE WITH $\otimes, \mathbb{1}$

GOAL

$\forall (\mathcal{C}, \otimes, \mathbb{1})$ MONOIDAL

CANONICAL BRAIDED CONSTRUCTION

$\mathcal{Z}(\mathcal{C}) =: \mathcal{D}$ BRAIDED MONOIDAL

TAKE REP'N

TAKE REP'N

$(\mathcal{M}, \triangleright) \in \mathcal{C}\text{-Mod}$

CANONICAL BRAIDED CONSTRUCTION

???

EVERYBODY ALSO LIKES REPRESENTATIONS!

[BROCHIER 2013] (MODIFIED IN [LWY])

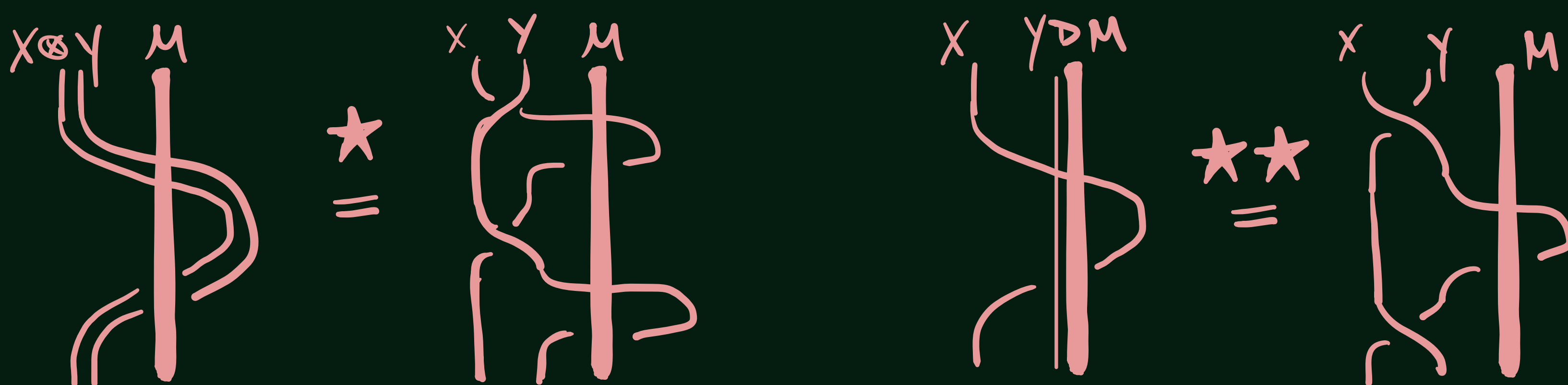
GIVEN A BRAIDED CATEGORY $(\mathcal{B}, \otimes, \mathbb{1}, c)$,

SAY $(\mathcal{M}, \triangleright) \in \mathcal{B}$-mod IS BRAIDED

IF $\exists$ NATURAL ISOM

$$e := \{ e_{X,M} : X \triangleright M \xrightarrow{\sim} X \triangleright M \}_{X \in \mathcal{B},\, M \in \mathcal{M}}$$

COMPATIBLE WITH $\otimes, \triangleright$ AS FOLLOWS:



$\mathcal{B}$

BRAIDED MONOIDAL

TAKE REP'N

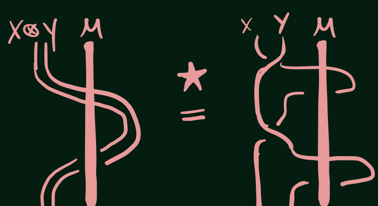
???

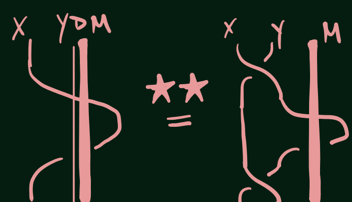
EVERYBODY ALSO LIKES REPRESENTATIONS!

GIVEN A BR. CAT. $(\mathcal{B}, \otimes, \mathbb{1}, c)$, $(\mathcal{M}, \triangleright) \in \mathcal{B}$-Mod IS BRAIDED IF ∃ NATURAL ISOM

$\{e_{X,M} : X \triangleright M \xrightarrow{\sim} X \triangleright M\}$ $\quad X \in \mathcal{B},\ M \in \mathcal{M}$

COMPATIBLE WITH $\otimes, \triangleright$ AS FOLLOWS:





THEOREM [CWY 2023] $\mathcal{B} := (\mathcal{B}, \otimes, \mathbb{1}, c)$

GIVEN $(\mathcal{M}, \triangleright) \in \mathcal{B}$-Mod, ∃ CANONICAL BRAIDED LEFT $\mathcal{B}$-MODULE CATEGORY

$\mathcal{E}_{\mathcal{B}}(\mathcal{M}) \in \mathcal{B}$-BrMod $\quad$ REFLECTIVE CENTER OF $\mathcal{M} \in \mathcal{B}$-Mod

WHERE

<u>OBJECTS</u>: $(M \in \mathcal{M}, e^M)$ ∋. $e^M_{X \otimes Y}$ SATISFIES ⋆

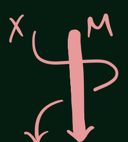
<u>MORPHISMS</u>: $(M, e^M) \longrightarrow (N, e^N)$:

$f : M \to N \in \mathcal{M}$ COMPATIBLE W/ e^M, e^N

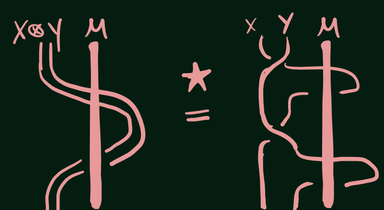
EVERYBODY ALSO LIKES REPRESENTATIONS!

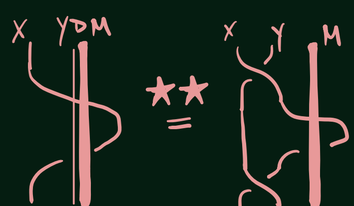
GIVEN A BR. CAT. $(\mathcal{B}, \otimes, \mathbb{1}, c)$, $(\mathcal{M}, \triangleright) \in \mathcal{B}$-mod IS BRAIDED IF ∃ NATURAL ISOM

$\{e_{X,M} : X \triangleright M \xrightarrow{\sim} X \triangleright M\}$ $X \in \mathcal{B}$, $M \in \mathcal{M}$



COMPATIBLE WITH ⊗, ▷ AS FOLLOWS:





THEOREM [LWY 2023] $\mathcal{B} := (\mathcal{B}, \otimes, \mathbb{1}, c)$

GIVEN $(\mathcal{M}, \triangleright) \in \mathcal{B}$-Mod, ∃ CANONICAL BRAIDED LEFT $\mathcal{B}$-MODULE CATEGORY

$\mathcal{E}_{\mathcal{B}}(\mathcal{M}) \in \mathcal{B}$-BrMod REFLECTIVE CENTER OF $\mathcal{M} \in \mathcal{B}$-Mod

WHERE

OBJECTS: $(M \in \mathcal{M}, e^M)$ s.t. $e^M_{X \otimes Y}$ SATISFIES ★

ACTION: $Y \triangleright (M, e^M) \overset{\text{DEF}}{=} (Y \triangleright M, e^{Y \triangleright M})$ s.t. $e^{Y \triangleright M}_X$ SATISFIES ★★

BRAIDING GIVEN BY e^M_Y

EVERYBODY ALSO LIKES REPRESENTATIONS!

PROPOSITION [LWY]

$\mathcal{B}$ BRAIDED TENS. CAT.

- $\mathcal{E}_{\mathcal{B}}(\mathcal{M})$ ABELIAN WHEN $\mathcal{M}$ EXACT, FINITE
- $\mathcal{E}_{\mathcal{B}}(\mathcal{M})$ FINITE WHEN $\mathcal{M}$ EXACT, FINITE & $\mathcal{B}$ FINITE
- $\mathcal{E}_{\mathcal{B}}(\mathcal{M})$ SEMISIMPLE WHEN $\mathcal{B}$ AND $\mathcal{M}$ FINITE, SEMISIMPLE

THEOREM [LWY 2023] $\mathcal{B} := (\mathcal{B}, \otimes, \mathbb{1}, c)$

GIVEN $(\mathcal{M}, \triangleright) \in \mathcal{B}\text{-Mod}$, $\exists$ CANONICAL BRAIDED LEFT $\mathcal{B}$-MODULE CATEGORY

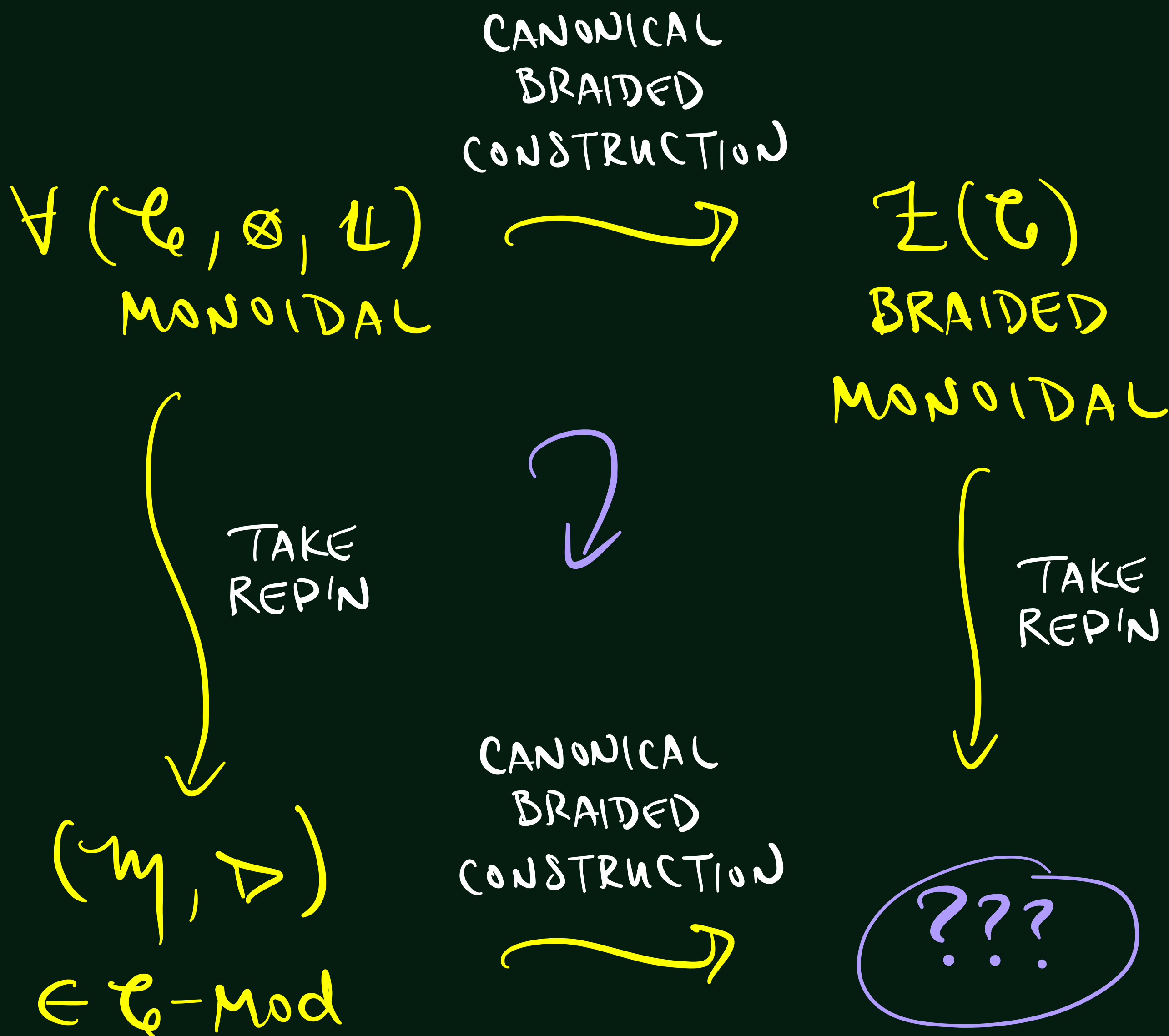
$$\mathcal{E}_{\mathcal{B}}(\mathcal{M}) \in \mathcal{B}\text{-BrMod}$$

REFLECTIVE CENTER OF $\mathcal{M} \in \mathcal{B}\text{-Mod}$

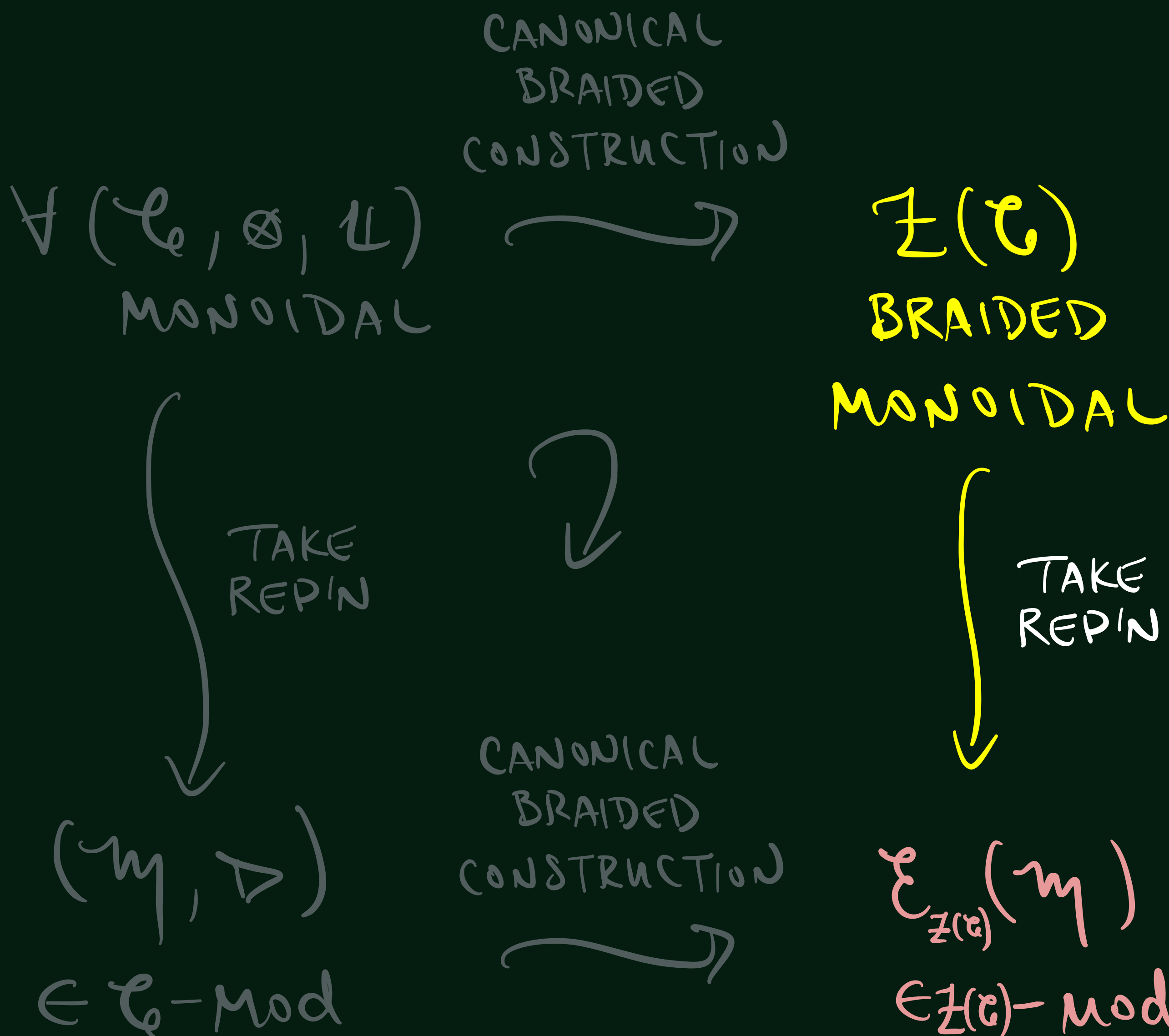
ACHIEVED BY REALIZING $\mathcal{E}_{\mathcal{B}}(\mathcal{M})$ AS A CENTER OF A BIMODULE CATEGORY (WORK OF GREENOUGH 2010)

HAVE $\mathcal{E}_{\mathcal{Z}(\mathcal{C})}(\mathcal{M}) \in \mathcal{Z}(\mathcal{C})\text{-Mod}$

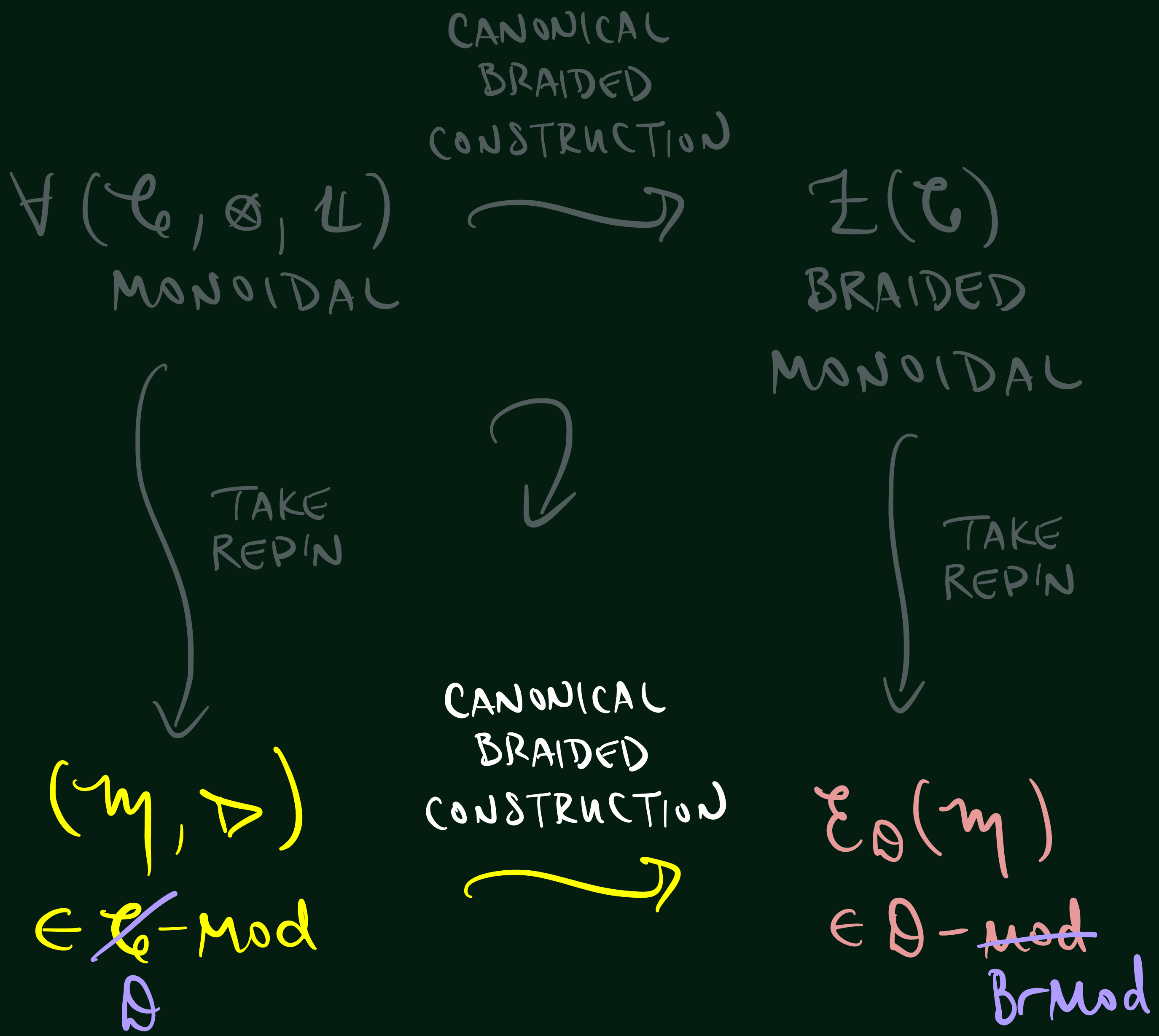
EVERYBODY LIKES WHEN GOALS ARE ACHIEVED!



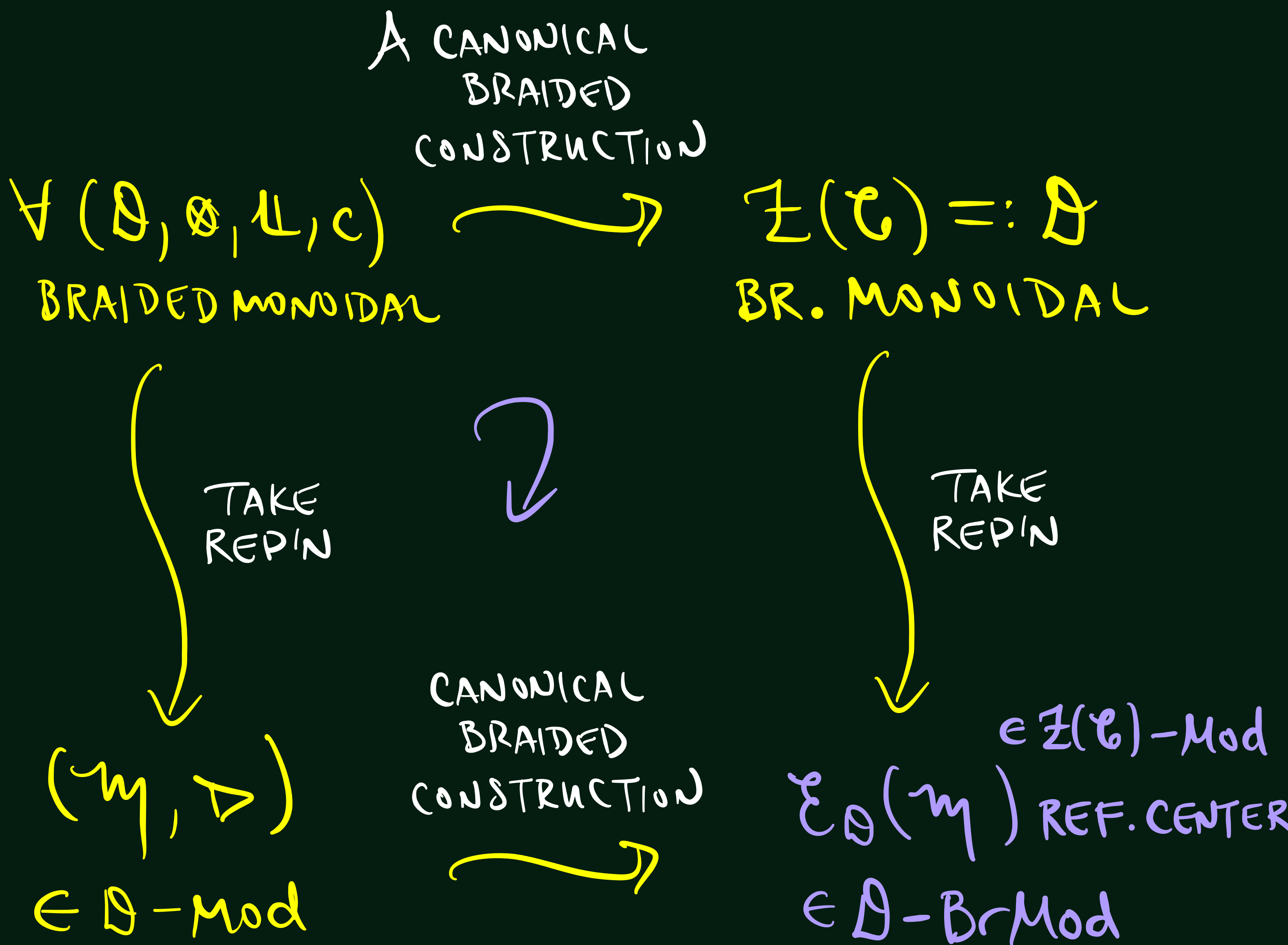
EVERYBODY LIKES WHEN GOALS ARE ACHIEVED!



EVERYBODY LIKES WHEN GOALS ARE ACHIEVED!



EVERYBODY LIKES WHEN GOALS ARE ACHIEVED!



THE HOPF CASE ... EVERYBODY LIKES IT!

RECALL FOR H = HOPF ALG., GET:

$$\underbrace{\mathcal{Z}(H\text{-Mod})}_{\text{DRINFELD CENTER OF H-Mod}} \overset{\text{BR}\otimes}{\simeq} \overbrace{{}^H_H\mathcal{YD}}^{\text{H-YETTER DRINFELD MODULES}} \overset{\text{BR}\otimes}{\simeq} \overbrace{D(H)}^{\text{DRINFELD DOUBLE OF H}}\text{-Mod}$$

THE HOPF CASE... EVERYBODY LIKES IT!

RECALL FOR H = HOPF ALG., GET:

DRINFELD CENTER OF H-MOD

H-YETTER DRINFELD MODULES

DRINFELD DOUBLE OF H

$$\mathcal{Z}(H\text{-Mod}) \overset{BR\otimes}{\simeq} {}^{H}_{H}\mathcal{YD} \overset{BR\otimes}{\simeq} D(H)\text{-Mod}$$

$(M, \bullet, \partial)$

$(M, \bullet)$ $\in H$-mod

(M, ∂) $\in H$-comod

+ COMPATIBILITY

THE HOPF CASE... EVERYBODY LIKES IT!

RECALL FOR H = HOPF ALG., GET:

DRINFELD CENTER OF H-MOD — H-YETTER DRINFELD MODULES — DRINFELD DOUBLE OF H

$$\mathcal{Z}(H\text{-Mod}) \overset{BR\otimes}{\simeq} {}^{H}_{H}\mathcal{YD} \overset{BR\otimes}{\simeq} D(H)\text{-Mod}$$

$(M, \bullet, \partial)$

$\bullet \in H$-mod

$\partial \in H$-comod

+ COMPATIBILITY

$D(H) \cong (H^*)^{op} \otimes H$ AS COALGS

SUBJECT TO RELATIONS

QUASI Δ HOPF ALG.

THE HOPF CASE... EVERYBODY LIKES IT!

RECALL FOR H = HOPF ALG., GET:

DRINFELD CENTER OF H-Mod

$$\mathcal{Z}(H\text{-Mod}) \overset{\text{BR}\otimes}{\simeq} {}^H_H\mathcal{YD} \overset{\text{BR}\otimes}{\simeq} D(H)\text{-Mod}$$

$^H_H\mathcal{YD}$: H-YETTER DRINFELD MODULES

$(M, \bullet, \partial)$

$(M, \bullet)$ $\in$ H-Mod

$(M, \bullet, \partial)$ $\in$ H-Comod + COMPATIBILITY

$D(H)$: DRINFELD DOUBLE OF H

$D(H) \cong (H^*)^{op} \otimes H$ AS COALGS SUBJECT TO RELATIONS

QUASI Δ HOPF ALG.

WHAT ABOUT FOR REFLECTIVE CENTERS?

THE HOPF CASE... EVERYBODY LIKES IT!

$\mathcal{L}$ = HOPF ALG w/ R-MATRIX R ($\mathcal{L}$ IS QUASI Δ)

$\underbrace{\text{R-MATRIX } R}_{\equiv}$

$\Updownarrow$

$R \in \mathcal{L} \otimes \mathcal{L}$ INVERTIBLE $\ni$.

$$(\Delta \otimes id)(R) = R_{13} R_{23}$$
$$(id \otimes \Delta)(R) = R_{13} R_{12}$$
$$R \Delta(\ell) = \Delta^{op}(\ell) R \quad \forall \ell \in \mathcal{L}$$

$\mathcal{D}$ = $\mathcal{L}$-Mod IS A $\otimes$ CATEG. THAT IS BRAIDED

$$R = \sum_i s_i \otimes t_i \longmapsto c_{x,y}(x \otimes y) = \sum_i (t_i \cdot y) \otimes (s_i \cdot x)$$

WHAT ABOUT FOR REFLECTIVE CENTERS?

$$\mathcal{Z}(H\text{-Mod}) \overset{BR\otimes}{\simeq} {}^H_H\mathcal{YD} \overset{BR\otimes}{\simeq} D(H)\text{-Mod}$$

THE HOPF CASE ... EVERYBODY LIKES IT!

$\mathcal{L}$ = HOPF ALG w/ $\underbrace{\text{R-MATRIX } R}_{\equiv}$ (L IS QUASI Δ)

$\Updownarrow$

$R \in L \otimes L$ INVERTIBLE s.t.

$$(\Delta \otimes id)(R) = R_{13} R_{23}$$

$$(id \otimes \Delta)(R) = R_{13} R_{12}$$

$$R\,\Delta(\ell) = \Delta^{op}(\ell)\,R \quad \forall \ell \in \mathcal{L}$$

$\mathcal{D}$ = L-Mod IS A $\otimes$ CATEG. THAT IS BRAIDED

$$R = \sum_i s_i \otimes t_i \longmapsto c_{x,y}(x \otimes y) = \sum_i (t_i \cdot y) \otimes (s_i \cdot x)$$

WHAT ABOUT FOR REFLECTIVE CENTERS?

$$\underbrace{\mathcal{E}_{L\text{-Mod}}}_{=:\,\mathcal{D}}(\mathcal{M}) \overset{\text{BR. } \mathcal{D}\text{-MOD}}{\simeq} ?? \text{ LIKE } {}^H_H\mathcal{YD}\ ?? \overset{\text{BR. } \mathcal{D}\text{-MOD}}{\simeq}\ ?? \text{ SOME}^{??} \text{ ALG-Mod}$$

THE HOPF CASE... EVERYBODY LIKES IT!

L = HOPF ALG w/ R-MATRIX R (L IS QUASI Δ)

$\mathcal{B}$ = L-Mod IS A $\otimes$ CATEG. THAT IS BRAIDED

TAKE $\mathcal{M}$ = A-Mod FOR A = LEFT L-COMODULE ALG.

VIA $\delta : A \longrightarrow L \otimes A$

$a \longmapsto a_{[-1]} \otimes a_{[0]}$

$\triangleright : L\text{-mod} \times A\text{-mod} \longrightarrow A\text{-mod}$

$$(X, \cdot), (M, *) \longmapsto (X \otimes M, a \tilde{*} (x \otimes m)$$
$$= (a_{[-1]} \cdot x) \otimes (a_{[0]} * m))$$

WHAT ABOUT FOR REFLECTIVE CENTERS?

$$\mathcal{E}_{L\text{-mod}}(A\text{-Mod}) \overset{\text{BR. } \mathcal{B}\text{-MOD}}{\simeq} ?? \text{ LIKE } {}^{H}_{H}\mathcal{YD} ?? \overset{\text{BR. } \mathcal{B}\text{-MOD}}{\simeq} \text{SOME}^{??}\ ??\ \text{ALG}-\text{Mod}$$

(A: LEFT L-COMOD ALG.; $L\text{-mod} =: \mathcal{B}$)

THE HOPF CASE... EVERYBODY LIKES IT!

L = HOPF ALG w/ R-MATRIX R (L IS QUASI Δ)

$\mathcal{D}$ = L-Mod IS A $\otimes$ CATEG. THAT IS BRAIDED

$\mathcal{M}$ = B-Mod FOR B = LEFT L-COMODULE ALG.

[KOLB 2020] (MODIFIED IN [LWY]) → GET $\mathcal{M} \in \mathcal{D}$-BrMod w/ $K \leftrightsquigarrow$ BRAIDING e

SAY THAT (B,δ) IS A QUASI Δ LEFT L-COMODULE ALG.

IF $\exists$ INV ELT $K \in L \otimes B$ (K-MATRIX) s.t.

$$(\Delta \otimes \mathrm{id})(K) = K_{23} R_{21} K_{13} R_{21}^{-1}, \quad K\,\delta(b) = \delta(b)\,K \quad \forall\, b \in B$$
$$(\mathrm{id} \otimes \delta)(K) = R_{21} K_{13} R_{12},$$

WHAT ABOUT FOR REFLECTIVE CENTERS?

$$\mathcal{E}_{L\text{-Mod}}(A\text{-Mod}) \;\overset{\text{BR. } \mathcal{D}\text{-MOD}}{\simeq}\; ?? \;\text{ LIKE } {}^H_H\mathcal{YD} \;??\; \overset{\text{BR. } \mathcal{D}\text{-MOD}}{\simeq} \; B\text{-Mod}$$

(A: LEFT L-COMOD ALG.; L-Mod $=: \mathcal{D}$; B: SOME QUASI Δ L-COMOD ALG)

THE HOPF CASE... EVERYBODY LIKES IT!

THEOREM [LWY 2023]

TAKE L = FINITE DIM QUASI Δ HOPF ALG

A = LEFT L-COMODULE ALG.

GET AS BRADED $\mathcal{D}=L$-Mod MODULE CATEGORIES:

$$\mathcal{E}_{L\text{-Mod}}(A\text{-Mod}) \simeq {}^{L}_{A}\hat{}\,DH(L) \simeq R_L(A)\text{-Mod}$$

${}^{L}_{A}DH(L)$: HOPF-DOI MODULES

$R_L(A)$: REFLECTIVE ALG (QUASI Δ L-COMOD. ALG.)

WHAT ABOUT FOR REFLECTIVE CENTERS?

$$\underbrace{\mathcal{E}_{L\text{-Mod}}}_{=:\,\mathcal{D}}(A\text{-Mod}) \overset{\text{BR. } \mathcal{D}\text{-MOD}}{\simeq} ?? \text{ LIKE } \#\, y\mathcal{D}\ ?? \overset{\text{BR. } \mathcal{D}\text{-MOD}}{\simeq} B\text{-Mod}$$

(A: LEFT L-COMOD ALG.; B: SOME QUASI Δ L-COMOD ALG)

THE HOPF CASE ... EVERYBODY LIKES IT!

THEOREM [LWY 2023]

TAKE L = FINITE DIM QUASI Δ HOPF ALG

A = LEFT L-COMODULE ALG.

GET AS BRADED $\mathcal{D} = L\text{-Mod}$ MODULE CATEGORIES:

$$\mathcal{E}_{L\text{-Mod}}(A\text{-Mod}) \simeq {}^{\hat{L}}_{A}DH(L) \simeq R_L(A)\text{-Mod}$$

HOPF-DOI MODULES

REFLECTIVE ALG (QUASI Δ L-COMOD. ALG.)

$\hat{L}$ = L-MODULE COALG

= L AS VS

RELATED TO MAJID'S "TRANSMUTED HOPF ALG"

THE HOPF CASE... EVERYBODY LIKES IT!

THEOREM [LWY 2023]

TAKE L = FINITE DIM QUASI Δ HOPF ALG

A = LEFT L-COMODULE ALG.

GET AS BRADED $\mathcal{D} = L\text{-Mod}$ MODULE CATEGORIES:

$$\mathcal{E}_{L\text{-Mod}}(A\text{-Mod}) \simeq {}^{\hat{L}}_{A}DH(L) \simeq R_L(A)\text{-Mod}$$

HOPF-DOI MODULES

REFLECTIVE ALG (QUASI Δ L-COMOD. ALG.)

$\hat{L}$ = L-MODULE COALG

= L AS VS

RELATED TO MAJID'S "TRANSMUTED HOPF ALG"

$(M, *, \varphi)$

$\in A\text{-Mod}$

$\in \hat{L}\text{-comod}$

+ COMPATIBILITY

THE HOPF CASE... EVERYBODY LIKES IT!

THEOREM [LWY 2023]

TAKE L = FINITE DIM QUASI Δ HOPF ALG

A = LEFT L-COMODULE ALG.

GET AS BRAIDED $\mathcal{D} = L\text{-mod}$ MODULE CATEGORIES:

$$\mathcal{E}_{L\text{-mod}}(A\text{-mod}) \simeq {}^{\hat{L}}_{A}DH(L) \simeq R_L(A)\text{-Mod}$$

HOPF-DOI MODULES

REFLECTIVE ALG (QUASI Δ L-COMOD. ALG.)

$\hat{L}$ = L-MODULE COALG

= L AS VS

RELATED TO MAJID'S "TRANSMUTED HOPF ALG"

$(M, *, \varphi)$

$\in A\text{-mod}$

$\in \hat{L}\text{-comod}$

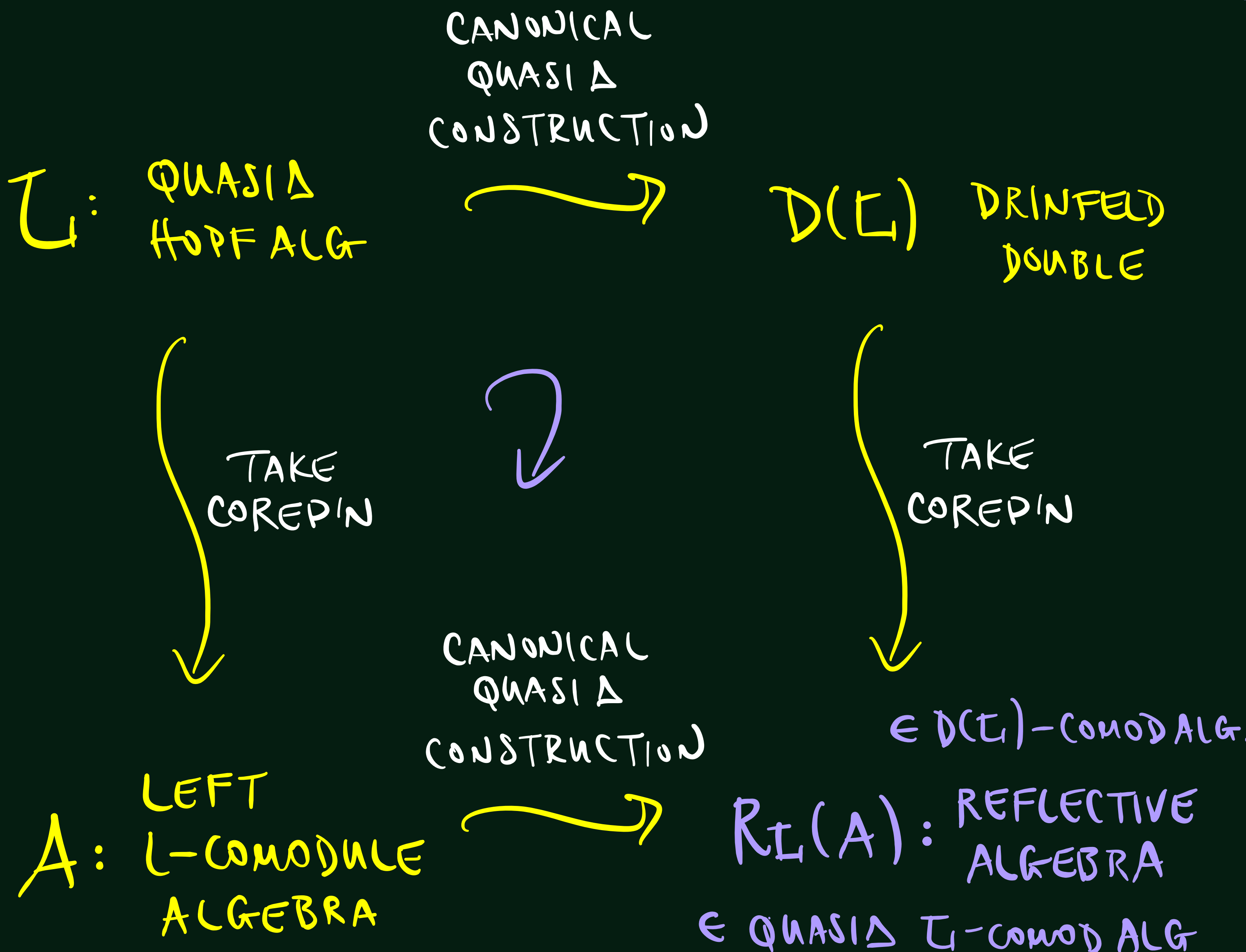
+ COMPATIBILITY

$= A \rtimes_L (\hat{L}^*)^{op}$

CROSSED PRODUCT ALG.

$= A \otimes L^*$ AS VS.

EVERYBODY LIKES WHEN GOALS ARE ACHIEVED AGAIN!



MORE ON REFLECTIVE ALGEBRAS —

H : QUASI Δ HOPF ALG

↓ TAKE COREP'N

$\Bbbk$: LEFT H-COMODULE ALGEBRA

CANONICAL QUASI Δ CONSTRUCTION ⟶

$(\hat{H}^*)^{op}$

$\cong$

$R_H(\Bbbk)$: REFLECTIVE ALGEBRA

$\in$ QUASI Δ H-COMOD ALG

THEOREM [LWY 2023]

TAKE THE CATEGORY ${}^{H}QT$ OF QUASI Δ LEFT H-COMOD ALGS. THEN $R_H(\Bbbk)$ IS AN INITIAL OBJECT OF ${}^{H}QT$

⇝ $R_H(\Bbbk) \equiv$ "UNIV. QT ENVELOPE" LIKE $D(H)$

REFLECTIVE CENTERS OF MODULE CATEGORIES & QUANTUM K-MATRICES

ARXIV: 2307.14764

Thanks for listening!

$\forall\, (\mathcal{B}, \otimes, \mathbb{1}, c)$ BRAIDED MONOIDAL —(A CANONICAL BRAIDED CONSTRUCTION)→ $\mathcal{Z}(\mathcal{C}) =: \mathcal{B}$ BR. MONOIDAL

↓ TAKE REP'N ↻ ↓ TAKE REP'N

$(\mathcal{M}, \triangleright)$ $\in \mathcal{B}$-Mod —(CANONICAL BRAIDED CONSTRUCTION)→ $\mathcal{E}_{\mathcal{B}}(\mathcal{M})$ REF. CENTER $\in \mathcal{Z}(\mathcal{C})$-Mod, $\in \mathcal{B}$-BrMod

L: QUASI Δ HOPF ALG —(CANONICAL QUASI Δ CONSTRUCTION)→ $D(L)$ DRINFELD DOUBLE

↓ TAKE COREP'N ↻ ↓ TAKE COREP'N

A: LEFT L-COMODULE ALGEBRA —(CANONICAL QUASI Δ CONSTRUCTION)→ $R_L(A)$: REFLECTIVE ALGEBRA $\in D(L)$-COMOD ALG, $\in$ QUASI Δ L-COMOD ALG