REPRESENTATIONS OF BRAIDED CATEGORIES

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·SLIDES·

REPRESENTATIONS

$\boxed{\text{REPRESENTATIONS}}$

ALLOW US TO REALIZE STRUCTURES AS SYMMETRIES ...

$$G \rightsquigarrow \mathrm{Sym}(V)$$

GROUP

SYMMETRIES OF A VECTOR SPACE V OVER A FIELD $\mathbb{k}$

$\boxed{\text{REPRESENTATIONS}}$

ALLOW US TO REALIZE STRUCTURES AS SYMMETRIES ...

$$G \rightsquigarrow \mathrm{Sym}(V)$$

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$\boxed{\text{REPRESENTATIONS}}$

ALLOW US TO REALIZE STRUCTURES AS SYMMETRIES ...

$$G \rightsquigarrow \mathrm{End}_{\mathbb{k}}(V)$$

GROUP

$\mathbb{k}$-LINEAR ENDOMORPHISMS

VECTOR SPACE V

$\boxed{\text{REPRESENTATIONS}}$

ALLOW US TO REALIZE STRUCTURES AS SYMMETRIES ...

GROUP MAP

$$G \xrightarrow{\rho} \mathrm{End}_{\mathbb{k}}(V)$$

GROUP

$\mathbb{k}$-LINEAR ENDOMORPHISMS

VECTOR SPACE V

$\boxed{\text{REPRESENTATIONS}}$

ALLOW US TO REALIZE STRUCTURES AS SYMMETRIES ...

$$\underset{\text{GROUP}}{G} \xrightarrow[\text{GROUP MAP}]{\rho} \underset{\substack{\Bbbk\text{-LINEAR ENDOMORPHISMS} \\ \text{VECTOR SPACE } V}}{\mathrm{End}_{\Bbbk}(V)}$$

NEEDS TO BE A GROUP

$\boxed{\text{REPRESENTATIONS}}$

ALLOW US TO REALIZE STRUCTURES AS SYMMETRIES ...

$$\underset{\text{GROUP}}{G} \xrightarrow[\text{GROUP MAP}]{\rho} \underset{\substack{\mathbb{k}\text{-LINEAR AUTOMORPHISMS} \\ \text{VECTOR SPACE } V}}{\mathrm{Aut}_{\mathbb{k}}(V) =: GL(V)}$$

$\boxed{\text{REPRESENTATIONS}}$

ALLOW US TO REALIZE STRUCTURES AS SYMMETRIES ...

$$\underset{\text{GROUP}}{G} \xrightarrow[\text{GROUP MAP}]{\rho} \underset{\substack{\mathbb{k}\text{-LINEAR AUTOMORPHISMS} \\ \text{VECTOR SPACE } V}}{\mathrm{Aut}_{\mathbb{k}}(V)} =: GL(V)$$

$\boxed{\text{REPRESENTATIONS}}$

↑ ALLOW US TO REALIZE STRUCTURES AS SYMMETRIES ...

$$\underset{\text{GROUP}}{G} \xrightarrow[\text{GROUP MAP}]{\rho} \underset{\substack{\mathbb{k}\text{-LINEAR AUTOMORPHISMS} \\ \text{VECTOR SPACE } V}}{\mathrm{Aut}_{\mathbb{k}}(V)} =: GL(V)$$

& AS MATRICES HERE...

$$G \xrightarrow{\rho} GL(\mathbb{k}^n) \cong GL_n(\mathbb{k})$$

$\boxed{\text{REPRESENTATIONS}}$

↑ ALLOW US TO REALIZE STRUCTURES AS SYMMETRIES ...

$$\underset{\text{GROUP}}{G} \xrightarrow[\text{GROUP MAP}]{\rho} \underset{\substack{\mathbb{k}\text{-LINEAR AUTOMORPHISMS} \\ \text{VECTOR SPACE } V}}{\mathrm{Aut}_{\mathbb{k}}(V)} =: GL(V)$$

& AS MATRICES HERE...

$$G \xrightarrow{\rho} GL(\mathbb{k}^n) \cong GL_n(\mathbb{k})$$

⇝ TRANSPORT GROUPS TO LINEAR ALGEBRA LAND

REPRESENTATIONS

↑ ALLOW US TO REALIZE STRUCTURES AS SYMMETRIES ...

ALGEBRA MAP

$$A \xrightarrow{\rho} \mathrm{End}_{\mathbb{k}}(V)$$

$\mathbb{k}$-ALGEBRA

$\mathbb{k}$-LINEAR ENDOMORPHISMS

VECTOR SPACE V

& AS MATRICES HERE...

$$A \xrightarrow{\rho} \mathrm{End}_{\mathbb{k}}(\mathbb{k}^n) \cong \mathrm{Mat}_n(\mathbb{k})$$

⇝ TRANSPORT $\mathbb{k}$-ALGS TO LINEAR ALGEBRA LAND

REPRESENTATIONS

ABSTRACTLY...

$$\mathcal{S} \rightsquigarrow \mathrm{Sym}(\mathcal{U})$$

STRUCTURE

FOR SOME OBJECT $\mathcal{U}$

REPRESENTATIONS

ABSTRACTLY...

STRUCTURE MAP

$$\mathcal{S} \xrightarrow{\rho} \mathrm{Sym}(\mathcal{U})$$

STRUCTURE

SAME KIND OF STRUCTURE FOR SOME OBJECT $\mathcal{U}$

REPRESENTATIONS

ABSTRACTLY...

$$\mathcal{S} \xrightarrow{\rho} \mathrm{Sym}(\mathcal{U})$$

STRUCTURE MAP

STRUCTURE

SAME KIND OF STRUCTURE FOR SOME OBJECT $\mathcal{U}$

REPRESENTATIONS

ABSTRACTLY...

$$\mathcal{S} \xrightarrow[\text{STRUCTURE MAP}]{\rho} \mathrm{Sym}(\mathcal{U})$$

STRUCTURE

SAME KIND OF STRUCTURE FOR SOME OBJECT $\mathcal{U}$

Ex.

ME

REPRESENTATIONS

ABSTRACTLY...

$$\mathcal{S} \xrightarrow[\text{STRUCTURE MAP}]{\rho} \mathrm{Sym}(\mathcal{U})$$

STRUCTURE

SAME KIND OF STRUCTURE FOR SOME OBJECT $\mathcal{U}$

Ex.

$\xrightarrow{\rho}$ $\mathrm{Sym}\Big($  $\Big)$

ME

REPRESENTATIONS

ABSTRACTLY...

$$\mathcal{S} \xrightarrow[\text{STRUCTURE MAP}]{\rho} \mathrm{Sym}(U)$$

STRUCTURE

SAME KIND OF STRUCTURE FOR SOME OBJECT U

Ex.

$$\xrightarrow{\rho} \mathrm{Sym}\Big($$

$$\Big)$$

ME

✓ WORKS ... ALTHOUGH A BIT BORING

REPRESENTATIONS

ABSTRACTLY...

STRUCTURE MAP

$$\mathcal{S} \xrightarrow{\rho} \mathrm{Sym}(\mathcal{U})$$

STRUCTURE

SAME KIND OF STRUCTURE FOR SOME OBJECT $\mathcal{U}$

Ex.

$\xrightarrow{\rho}$ $\mathrm{Sym}\Big($  $\Big)$

ME

✓ WORKS ... ALTHOUGH A BIT BORING... OR REGULAR

REPRESENTATIONS

ABSTRACTLY...

$$\mathcal{S} \xrightarrow[\text{STRUCTURE MAP}]{\rho} \mathrm{Sym}(\mathcal{U})$$

STRUCTURE

SAME KIND OF STRUCTURE FOR SOME OBJECT $\mathcal{U}$

Ex.

ME

$$\xrightarrow{\rho} \mathrm{Sym}\left(\text{ANY OF YOU}\right)$$

REPRESENTATIONS

ABSTRACTLY...

$$\mathcal{S} \xrightarrow[\text{STRUCTURE MAP}]{\rho} \mathrm{Sym}(\mathcal{U})$$

STRUCTURE

SAME KIND OF STRUCTURE FOR SOME OBJECT $\mathcal{U}$

Ex.

$$\xrightarrow{\rho} \mathrm{Sym}\left(\text{ANY OF YOU}\right)$$

ME

WE ALL LIKE REP THEORY SO THIS SHOULD WORK

REPRESENTATIONS

ABSTRACTLY...

$$\mathcal{S} \xrightarrow[\text{STRUCTURE MAP}]{\rho} \mathrm{Sym}(\mathcal{U})$$

STRUCTURE

SAME KIND OF STRUCTURE FOR SOME OBJECT $\mathcal{U}$

Ex.

$$\xrightarrow{\rho} \mathrm{Sym}(\qquad)$$

ME

REPRESENTATIONS

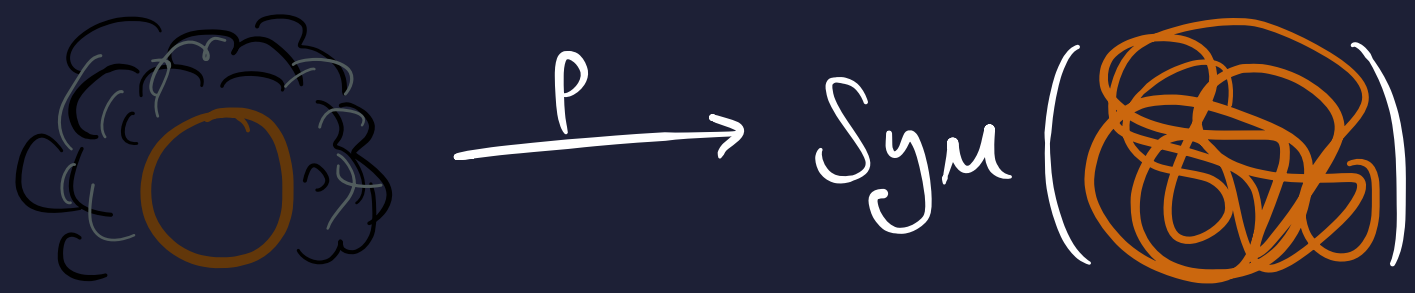
ABSTRACTLY...

$$\mathcal{S} \xrightarrow[\text{STRUCTURE MAP}]{\rho} \mathrm{Sym}(\mathcal{U})$$

STRUCTURE

SAME KIND OF STRUCTURE FOR SOME OBJECT $\mathcal{U}$

Ex.



ME

REPRESENTATIONS

ABSTRACTLY...

$$\mathcal{S} \xrightarrow[\text{STRUCTURE MAP}]{\rho} \mathrm{Sym}(\mathcal{U})$$

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SAME KIND OF STRUCTURE FOR SOME OBJECT $\mathcal{U}$

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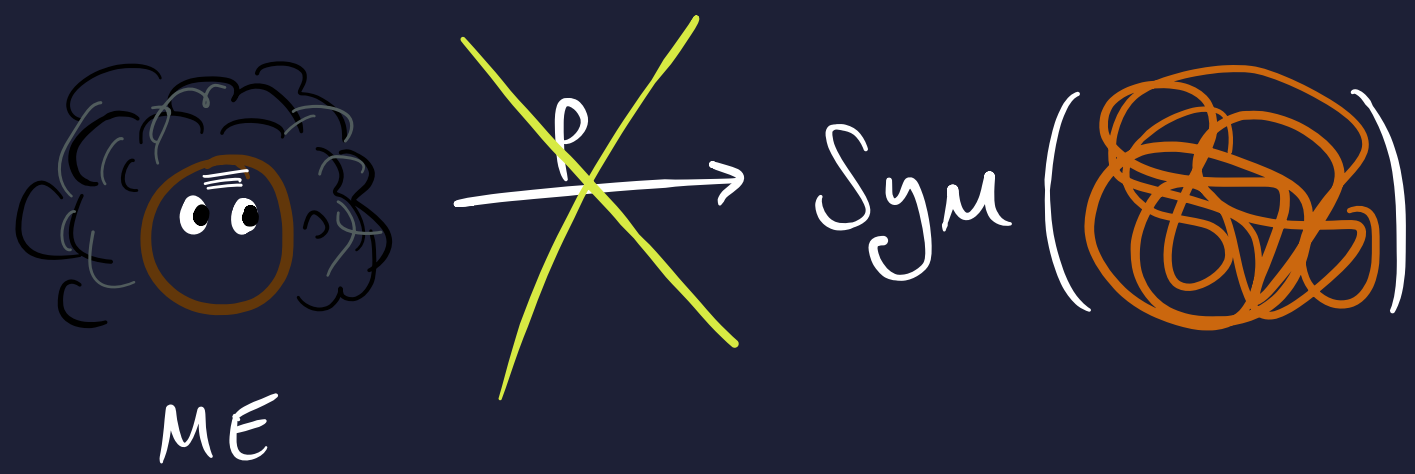

$\boxed{\text{REPRESENTATIONS}}$

STRUCTURE MAP

$$\mathcal{S} \xrightarrow{\rho} \mathrm{Sym}(\mathcal{U})$$

STRUCTURE

SAME KIND OF STRUCTURE FOR SOME OBJECT $\mathcal{U}$

$\boxed{\text{REPRESENTATIONS}}$

BRAIDED $\otimes$ CAT.

STRUCTURE MAP

$$\mathcal{S} \xrightarrow{\rho} \mathrm{Sym}(\mathcal{U})$$

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SAME KIND OF STRUCTURE FOR SOME OBJECT $\mathcal{U}$

REPRESENTATIONS

BRAIDED $\otimes$ CAT.

STRUCTURE MAP

$$\mathcal{S} \xrightarrow{\rho} \mathrm{Sym}(\mathcal{U})$$

STRUCTURE

SAME KIND OF STRUCTURE FOR SOME OBJECT $\mathcal{U}$

SHIFT FRAMEWORK

$$\mathcal{S} \times \mathcal{U} \xrightarrow{\triangleright} \mathcal{U}$$

MODULES

$\boxed{\text{REPRESENTATIONS}}$

BRAIDED $\otimes$ CAT.

STRUCTURE MAP

$$\mathcal{S} \xrightarrow{\rho} \mathrm{Sym}(\mathcal{U})$$

$$s \longmapsto \rho_s : \mathcal{U} \to \mathcal{U}$$

STRUCTURE

SAME KIND OF STRUCTURE FOR SOME OBJECT $\mathcal{U}$

SHIFT FRAMEWORK

$$\mathcal{S} \times \mathcal{U} \xrightarrow{\triangleright} \mathcal{U}$$

$$(s, u) \longmapsto \rho_s(u)$$

$\boxed{\text{MODULES}}$

REPRESENTATIONS

BRAIDED $\otimes$ CAT.

STRUCTURE MAP

$$\mathcal{S} \xrightarrow{\rho} \mathrm{Sym}(\mathcal{U})$$

$$s \longmapsto \rho_s : \mathcal{U} \to \mathcal{U}$$

STRUCTURE

SAME KIND OF STRUCTURE FOR SOME OBJECT $\mathcal{U}$

SHIFT FRAMEWORK

$$\mathcal{S} \times \mathcal{U} \xrightarrow{\triangleright} \mathcal{U}$$

$$(s, u) \longmapsto \rho_s(u)$$

MODULES

ALWAYS LEFT HERE

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{S}$ = MONOIDAL CATEGORY

$$\left(\mathcal{C} \ , \ \otimes : \mathcal{C} \times \mathcal{C} \longrightarrow \mathcal{C} \ , \ \mathbb{1} \right)$$

REPRESENTATIONS OF BRAIDED CATEGORIES

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CATEGORY

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CATEGORY

BIFUNCTOR

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CATEGORY

BIFUNCTOR

DISTINGUISHED OBJECT IN $\mathcal{C}$

REPRESENTATIONS OF BRAIDED CATEGORIES

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CATEGORY

BIFUNCTOR

DISTINGUISHED OBJECT IN $\mathcal{C}$

... THAT MIMICS A MONOID

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{S}$ = MONOIDAL CATEGORY

$$\left(\mathcal{C}\,,\ \otimes : \mathcal{C}\times\mathcal{C} \longrightarrow \mathcal{C}\,,\ \mathbb{1}\right)$$

CATEGORY

BIFUNCTOR

DISTINGUISHED OBJECT IN $\mathcal{C}$

... THAT MIMICS A MONOID

Ex. $\left(H\text{-Mod}_{\Bbbk}\,,\ \otimes = \otimes_{\Bbbk}\,,\ \mathbb{1} = \Bbbk\right)$

FOR: $H = (H, m, u, \Delta, \varepsilon)$ BIALGEBRA

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{S}$ = MONOIDAL CATEGORY

$$\left(\mathcal{C}\,,\ \otimes : \mathcal{C}\times\mathcal{C} \longrightarrow \mathcal{C}\,,\ \mathbb{1}\right)$$

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COMULTIPLIN

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{S}$ = MONOIDAL CATEGORY

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BIFUNCTOR

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COUNIT

REPRESENTATIONS OF BRAIDED CATEGORIES

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CATEGORY

BIFUNCTOR

DISTINGUISHED OBJECT IN $\mathcal{C}$

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Ex. $\left(H\text{-Mod}_{\Bbbk}\,,\ \otimes = \otimes_{\Bbbk}\,,\ \mathbb{1} = \Bbbk\right)$

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REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{S}$ = RIGID MONOIDAL CATEGORY

$$\left(\mathcal{C}\,,\ \otimes : \mathcal{C}\times\mathcal{C}\longrightarrow\mathcal{C}\,,\ \mathbb{1}\right)$$

CATEGORY — BIFUNCTOR — DISTINGUISHED OBJECT IN $\mathcal{C}$

... WITH DUAL OBJECTS

$$\left(X^*\,,\ ev_X^L : X^*\otimes X\longrightarrow\mathbb{1}\,,\ coev_X^L : \mathbb{1}\longrightarrow X\otimes X^*\right)$$

$$\left({}^*X\,,\ ev_X^R : X\otimes {}^*X\longrightarrow\mathbb{1}\,,\ coev_X^R : \mathbb{1}\longrightarrow {}^*X\otimes X\right)$$

Ex. $\left(H\text{-Mod}_{\Bbbk}\,,\ \otimes=\otimes_{\Bbbk}\,,\ \mathbb{1}=\Bbbk\right)$

FOR: $H=(H,m,u,\Delta,\varepsilon)$ BIALGEBRA

REPRESENTATIONS OF BRAIDED CATEGORIES

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CATEGORY — BIFUNCTOR — DISTINGUISHED OBJECT IN $\mathcal{C}$

... WITH DUAL OBJECTS

$$\left({}^{*}X\,,\ ev^{L}_{X} : X^{*}\otimes X\longrightarrow\mathbb{1}\,,\ coev^{L}_{X} : \mathbb{1}\longrightarrow X\otimes X^{*}\right)$$

$$\left({}^{*}X\,,\ ev^{R}_{X} : X\otimes {}^{*}X\longrightarrow\mathbb{1}\,,\ coev^{R}_{X} : \mathbb{1}\longrightarrow {}^{*}X\otimes X\right)$$

Ex. $\left(H\text{-Mod}_{\Bbbk}\,,\ \otimes = \otimes_{\Bbbk}\,,\ \mathbb{1} = \Bbbk\right)$ + DUALS

FOR: $H = (H, m, u, \Delta, \varepsilon)$ $\mathcal{S}$ ~~B~~IALGEBRA HOPF

REPRESENTATIONS OF BRAIDED CATEGORIES

• STRICT CASE •

$\mathcal{S}$ = MONOIDAL CATEGORY

$$\left(\mathcal{C}\ ,\ \otimes : \mathcal{C} \times \mathcal{C} \longrightarrow \mathcal{C}\ ,\ \mathbb{1}\right)$$

GRAPHICAL CALCULUS

Ex. $\left(H\text{-Mod}_{\Bbbk}\ ,\ \otimes = \otimes_{\Bbbk}\ ,\ \mathbb{1} = \Bbbk\right)$

FOR: $H = (H, m, u, \Delta, \varepsilon)$ BIALGEBRA

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GRAPHICAL CALCULUS

$X \in \mathcal{C}$

Ex. $\left(H\text{-Mod}_{\Bbbk}\ ,\ \otimes = \otimes_{\Bbbk}\ ,\ \mathbb{1} = \Bbbk\right)$

FOR: $H = (H, m, u, \Delta, \varepsilon)$ BIALGEBRA

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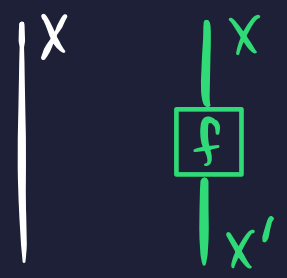
GRAPHICAL CALCULUS

(READ DOWN)



$X \in \mathcal{C}$

$f : X \to X' \in \mathcal{C}$

Ex. $\left(H\text{-Mod}_{\Bbbk}\ ,\ \otimes = \otimes_{\Bbbk}\ ,\ \mathbb{1} = \Bbbk\right)$

FOR: $H = (H, m, u, \Delta, \varepsilon)$ BIALGEBRA

REPRESENTATIONS OF BRAIDED CATEGORIES

• STRICT CASE •

$\mathcal{S}$ = MONOIDAL CATEGORY

$$\left(\mathcal{C}\ ,\ \otimes : \mathcal{C}\times\mathcal{C} \longrightarrow \mathcal{C}\ ,\ \mathbb{1}\right)$$

GRAPHICAL CALCULUS (READ DOWN)

$X, Y \in \mathcal{C}$

$f : X \to X' \in \mathcal{C}$

Ex. $\left(H\text{-Mod}_{\Bbbk}\ ,\ \otimes = \otimes_{\Bbbk}\ ,\ \mathbb{1} = \Bbbk\right)$

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REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{S}$ = MONOIDAL CATEG.

$$\left(\mathcal{C},\ \otimes : \mathcal{C}\times\mathcal{C} \longrightarrow \mathcal{C},\ \mathbb{1}\right)$$

GRAPHICAL CALCULUS

$X, Y \in \mathcal{C}$ $\quad f : X \to X' \in \mathcal{C}$

Ex.

$$\left(H\text{-Mod}_{\Bbbk},\ \otimes_{\Bbbk},\ \Bbbk\right)$$

$H = (H, m, u, \Delta, \varepsilon)$

BIALGEBRA

$\mathcal{U}$ = $\mathcal{C}$-MODULE CATEGORY

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{S}$ = MONOIDAL CATEG.

$$(\mathcal{C}, \otimes : \mathcal{C} \times \mathcal{C} \longrightarrow \mathcal{C}, \mathbb{1})$$

GRAPHICAL CALCULUS

$X, Y \in \mathcal{C}$ $\quad f : X \to X' \in \mathcal{C}$

Ex.

$$(H\text{-}\mathrm{Mod}_{\Bbbk}, \otimes_{\Bbbk}, \Bbbk)$$

$H = (H, m, u, \Delta, \varepsilon)$

BIALGEBRA

$\mathcal{U}$ = $\mathcal{C}$-MODULE CATEGORY

$$(\mathcal{M}, \triangleright : \mathcal{C} \times \mathcal{M} \longrightarrow \mathcal{M})$$

CATEGORY

BIFUNCTOR

... LIKE A MODULE OVER A MONOID

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{S}$ = MONOIDAL CATEG.

$$(\mathcal{C}, \otimes : \mathcal{C} \times \mathcal{C} \longrightarrow \mathcal{C}, \mathbb{1})$$

GRAPHICAL CALCULUS

$X, Y \in \mathcal{C}$ $\quad f : X \to X' \in \mathcal{C}$

Ex.

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GRAPHICAL CALCULUS

$X, Y \in \mathcal{C}$ $\quad f : X \to X' \in \mathcal{C}$

Ex.

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CATEGORY $\qquad$ BIFUNCTOR

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REPRESENTATIONS OF BRAIDED CATEGORIES

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$$(\mathcal{C}, \otimes : \mathcal{C} \times \mathcal{C} \longrightarrow \mathcal{C}, \mathbb{1})$$

GRAPHICAL CALCULUS

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BIALGEBRA

$\mathcal{U}$ = $\mathcal{C}$-MODULE CATEGORY

$$(\mathcal{M}, \triangleright : \mathcal{C} \times \mathcal{M} \longrightarrow \mathcal{M})$$

CATEGORY $\qquad$ BIFUNCTOR

...LIKE A MODULE OVER A MONOID

Ex. $(A\text{-Mod}_{\Bbbk}, \triangleright = \otimes_{\Bbbk})$

FOR: $A = (A, m, u, \delta : A \longrightarrow H \otimes A)$

LEFT H-COMODULE ALGEBRA

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{S}$ = MONOIDAL CATEG.

$$(\mathcal{C}, \otimes : \mathcal{C} \times \mathcal{C} \longrightarrow \mathcal{C}, \mathbb{1})$$

GRAPHICAL CALCULUS

$X, Y \in \mathcal{C}$ $\quad f : X \to X' \in \mathcal{C}$

Ex.

$$(H\text{-Mod}_{\Bbbk}, \otimes_{\Bbbk}, \Bbbk)$$

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BIALGEBRA

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CATEGORY $\qquad$ BIFUNCTOR

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LEFT H-COMODULE ALGEBRA

REPRESENTATIONS OF BRAIDED CATEGORIES

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GRAPHICAL CALCULUS

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$\mathcal{M}$ = $\mathcal{C}$-MODULE CATEGORY

$$(\mathcal{M}, \triangleright : \mathcal{C} \times \mathcal{M} \longrightarrow \mathcal{M})$$

CATEGORY $\qquad$ BIFUNCTOR

...LIKE A MODULE OVER A MONOID

Ex. $(A\text{-Mod}_{\Bbbk}, \triangleright = \otimes_{\Bbbk})$

FOR: $A = (A, m, u, \delta : A \longrightarrow H \otimes A)$

COACTION

LEFT H-COMODULE ALGEBRA

REPRESENTATIONS OF BRAIDED CATEGORIES

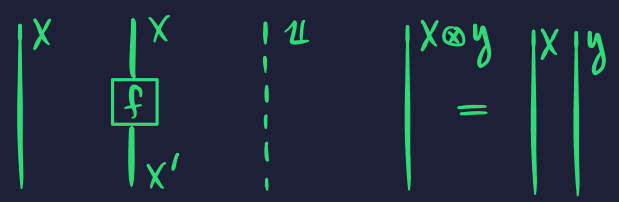
$\mathcal{S}$ = MONOIDAL CATEG.

$$(\mathcal{C}, \otimes : \mathcal{C} \times \mathcal{C} \longrightarrow \mathcal{C}, \mathbb{1})$$

$\mathcal{U}$ = $\mathcal{C}$-MODULE CATEGORY

$$(\mathcal{M}, \triangleright : \mathcal{C} \times \mathcal{M} \longrightarrow \mathcal{M})$$

GRAPHICAL CALCULUS



$X, Y \in \mathcal{C}$ $\quad f : X \to X' \in \mathcal{C}$

Ex.

$$(H\text{-Mod}_{\Bbbk}, \otimes_{\Bbbk}, \Bbbk)$$

$H = (H, m, u, \Delta, \varepsilon)$

BIALGEBRA

Ex. $(A\text{-Mod}_{\Bbbk}, \triangleright = \otimes_{\Bbbk})$

FOR: $A = (A, m, u, \delta : A \longrightarrow H \otimes A)$

LEFT H-COMODULE ALGEBRA

REPRESENTATIONS OF BRAIDED CATEGORIES

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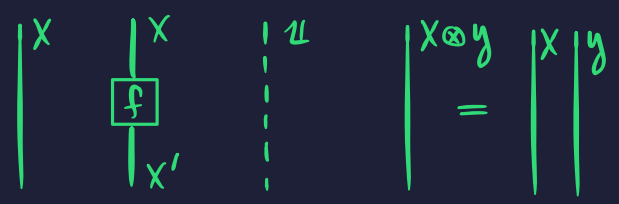
$(\mathcal{M}, \triangleright : \mathcal{C} \times \mathcal{M} \longrightarrow \mathcal{M})$

GRAPHICAL CALCULUS



$X, Y \in \mathcal{C}$ $\quad f : X \to X' \in \mathcal{C}$

$M \in \mathcal{M}$

Ex.

$(\mathrm{H\text{-}Mod}_{\Bbbk}, \otimes_{\Bbbk}, \Bbbk)$

$H = (H, m, u, \Delta, \varepsilon)$

BIALGEBRA

Ex. $(\mathrm{A\text{-}Mod}_{\Bbbk}, \triangleright = \otimes_{\Bbbk})$

FOR: $A = (A, m, u, \delta : A \longrightarrow H \otimes A)$

LEFT H-COMODULE ALGEBRA

REPRESENTATIONS OF BRAIDED CATEGORIES

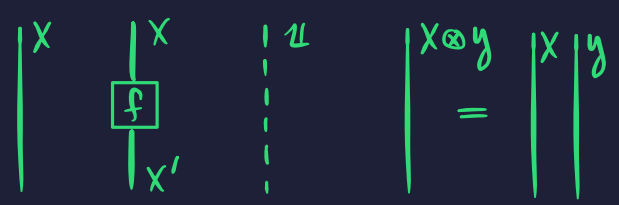
$\mathcal{S}$ = MONOIDAL CATEG.

$(\mathcal{C}, \otimes : \mathcal{C} \times \mathcal{C} \longrightarrow \mathcal{C}, \mathbb{1})$

$\mathcal{U}$ = $\mathcal{C}$-MODULE CATEGORY

$(\mathcal{M}, \triangleright : \mathcal{C} \times \mathcal{M} \longrightarrow \mathcal{M})$

GRAPHICAL CALCULUS



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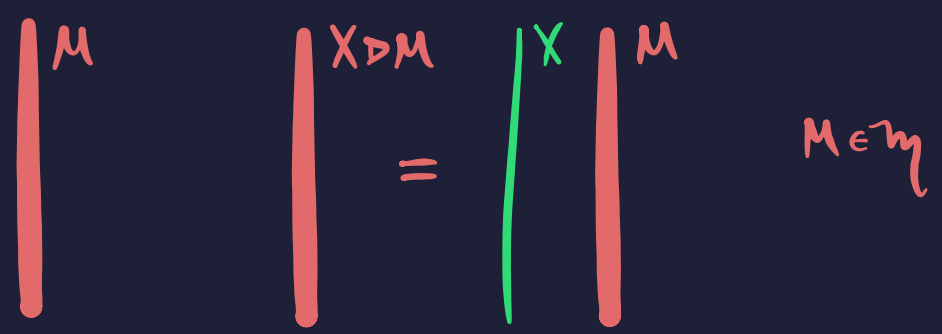

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REPRESENTATIONS OF BRAIDED CATEGORIES

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NAT'L $\cong$: BRAIDING

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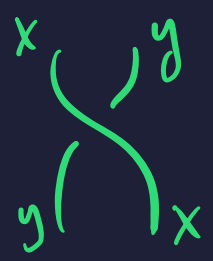
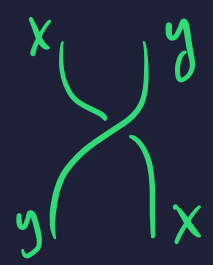
$\boxed{c}$



SUBJECT TO

$\boxed{c^{-1}}$



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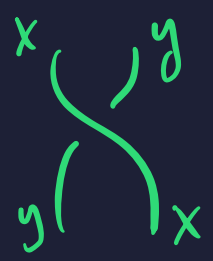

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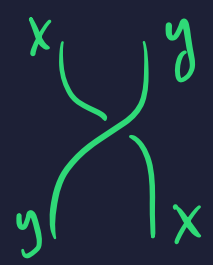

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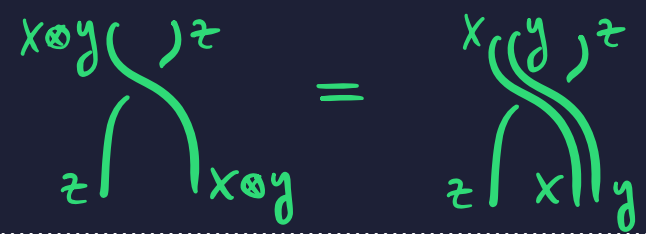

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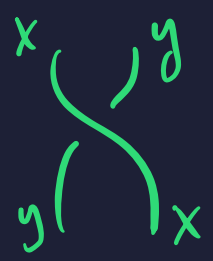

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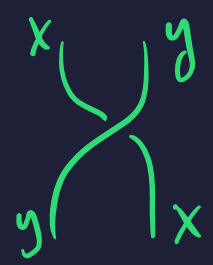

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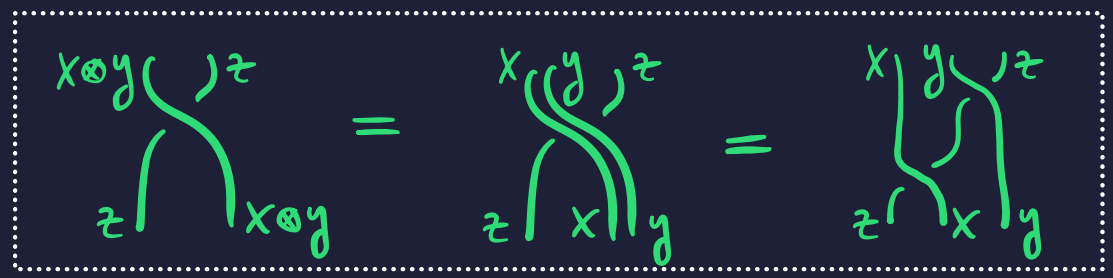

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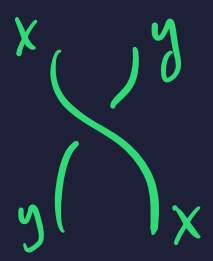
$\boxed{c}$



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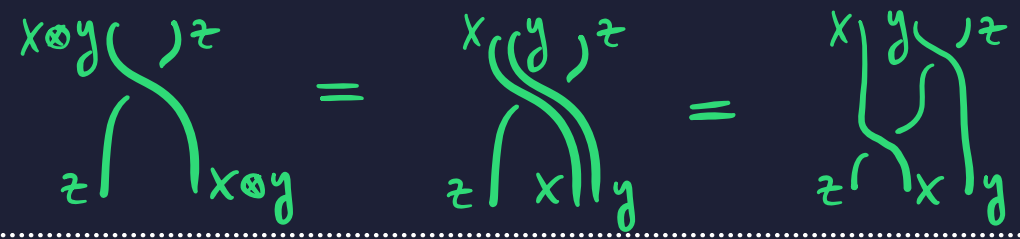

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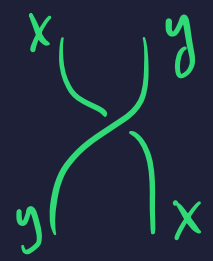

$$\begin{tikzcd}
X\otimes Y\otimes Z \arrow[rr, "c_{X\otimes Y, Z}"] \arrow[dr, "\mathrm{id}\otimes c_{Y,Z}"'] & \arrow[d, phantom, "\circlearrowright"] & Z\otimes X\otimes Y \\
& X\otimes Z\otimes Y \arrow[ur, "c_{X,Z}\otimes \mathrm{id}"'] &
\end{tikzcd}$$

REPRESENTATIONS OF BRAIDED CATEGORIES

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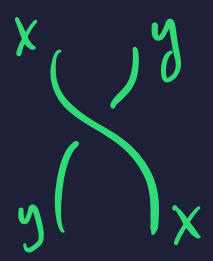
$x, y \in \mathcal{C}$

$\boxed{c}$



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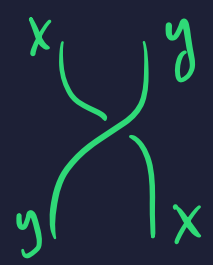

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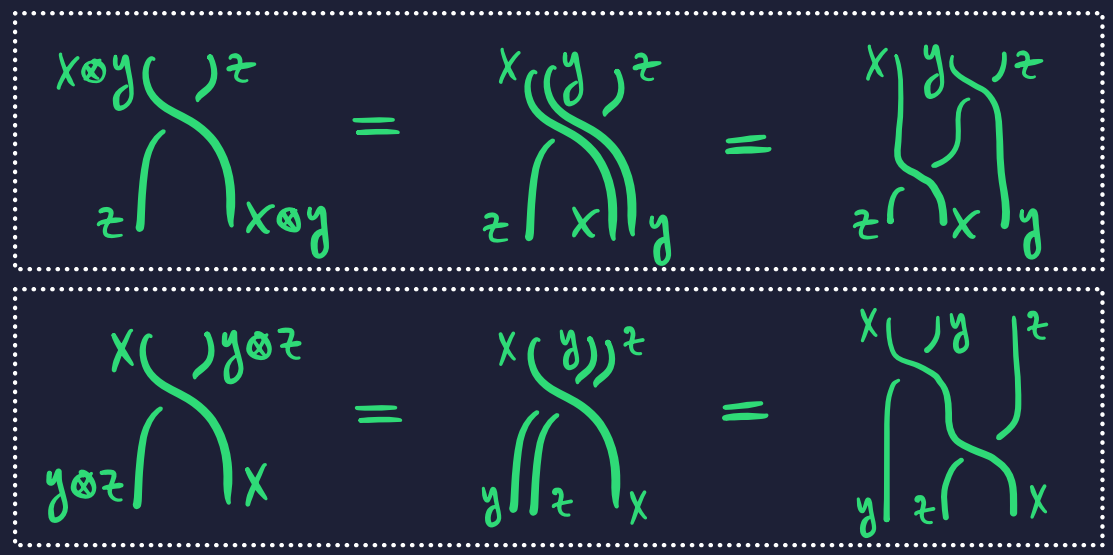

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$X, Y \in \mathcal{C}$

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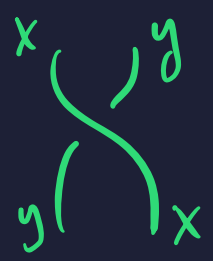

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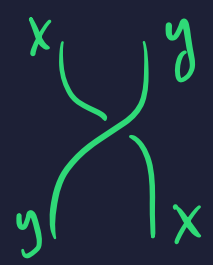

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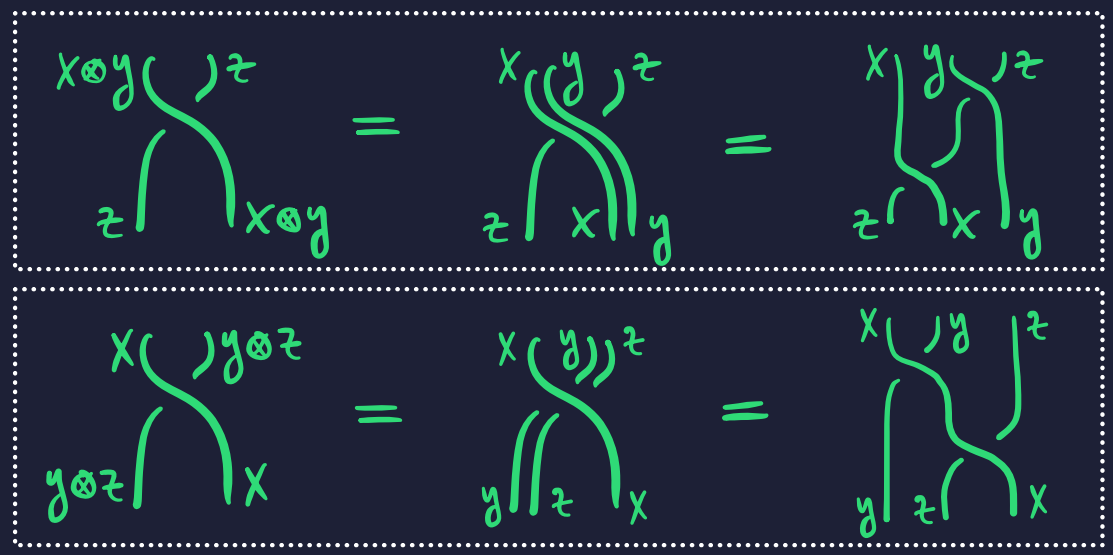

Ex. $(H\text{-Mod}_{\Bbbk}, \otimes_{\Bbbk}, \Bbbk, c)$

FOR: $H = (H, m, u, \Delta, \varepsilon, \qquad\qquad)$ BIALG.

REPRESENTATIONS OF BRAIDED CATEGORIES

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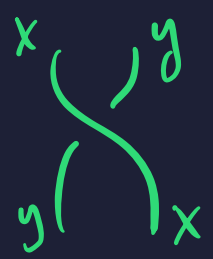
NAT'L $\cong$: BRAIDING

$X, Y \in \mathcal{C}$

$\boxed{c}$



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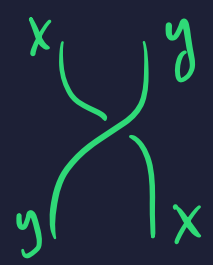

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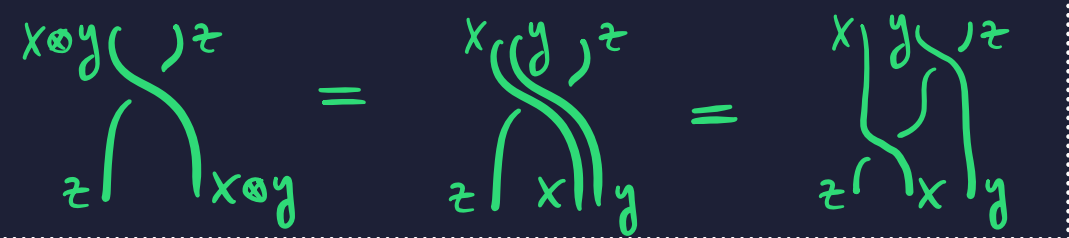
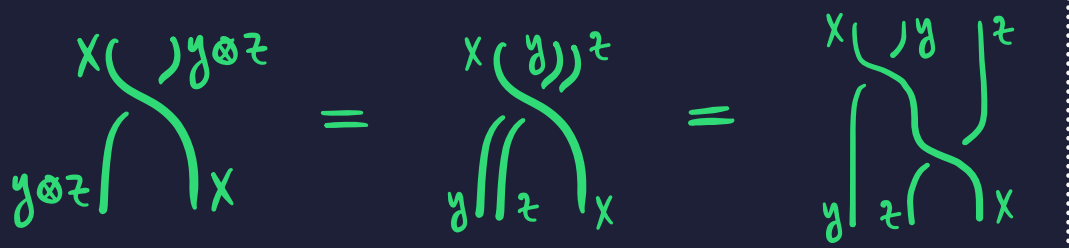




Ex. $\left(H\text{-Mod}_{\Bbbk},\ \otimes_{\Bbbk},\ \Bbbk,\ c\right)$

FOR: $H = (H, m, u, \Delta, \varepsilon, R \in H\otimes H)$ QUASITRI. BIALG.

R-MATRIX

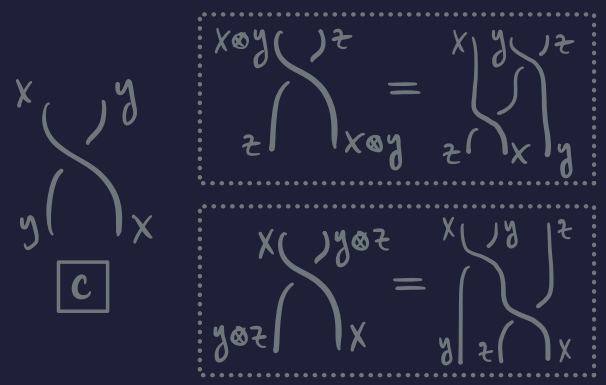
REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{S}$ = BRAIDED $\otimes$ CATEG.

$$(\mathcal{C}, \otimes, \mathbb{1}, c)$$

$$c = \{c_{X,Y} : X \otimes Y \xrightarrow{\sim} Y \otimes X\}$$

GRAPHICAL CALCULUS



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QUASITRI. BIALG.

$\mathcal{U}$ = BRAIDED $\mathcal{C}$-MOD. CATEG.

REPRESENTATIONS OF BRAIDED CATEGORIES

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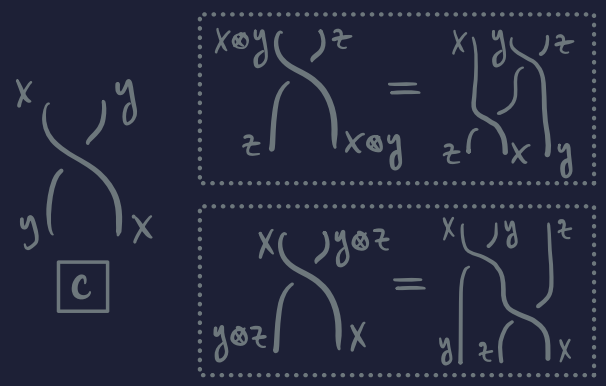

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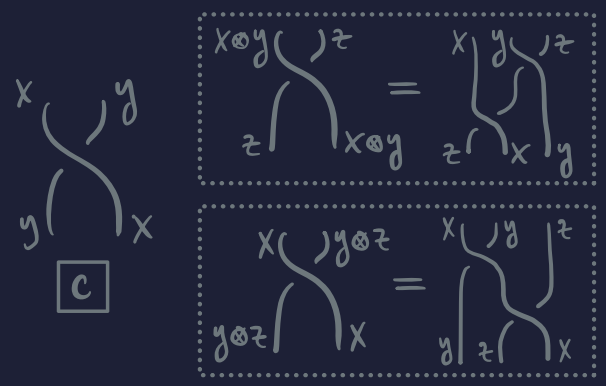

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NAT'L $\cong$: BRAIDING

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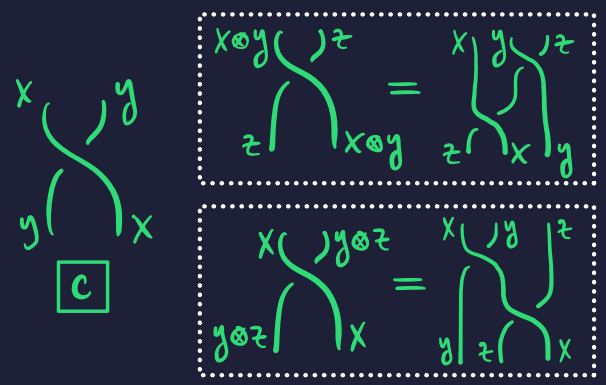

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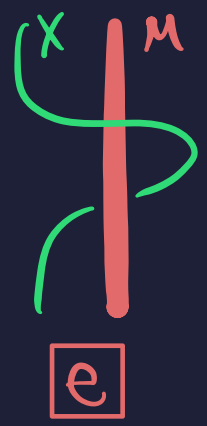

SUBJ. TO

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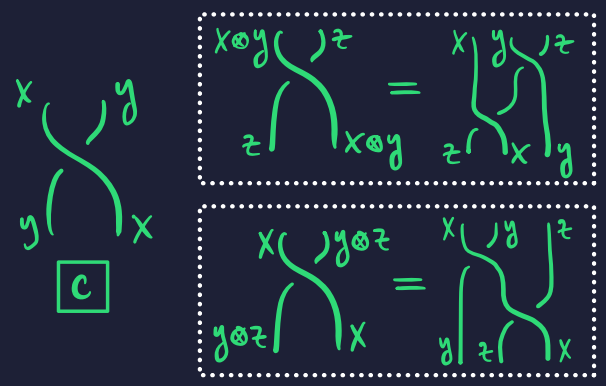

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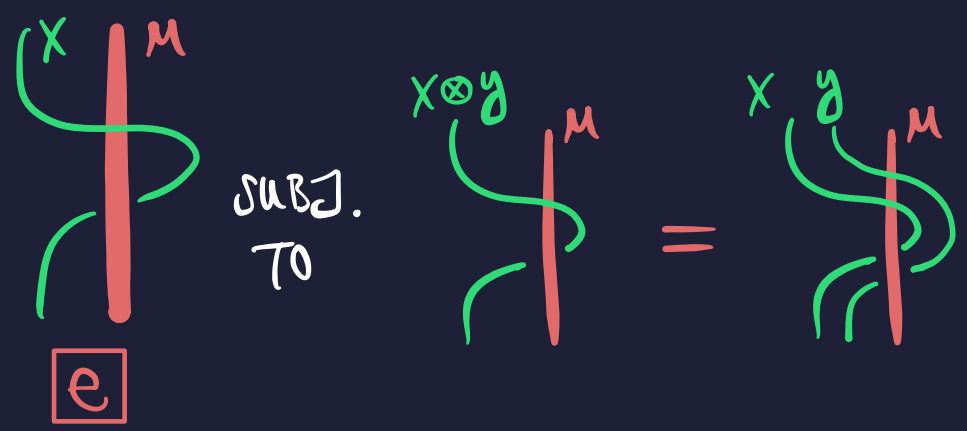

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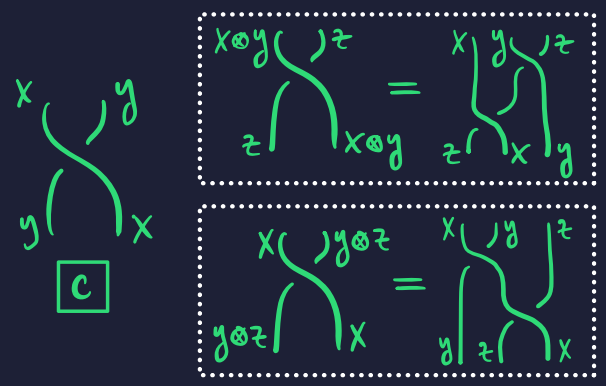

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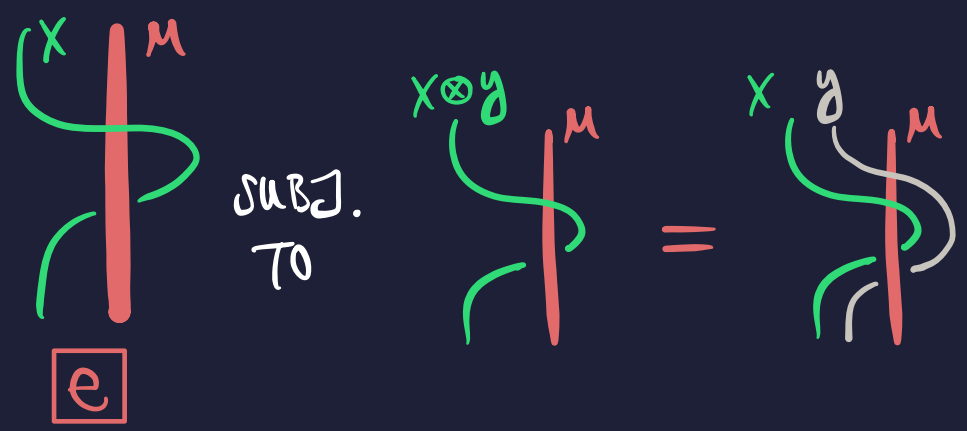

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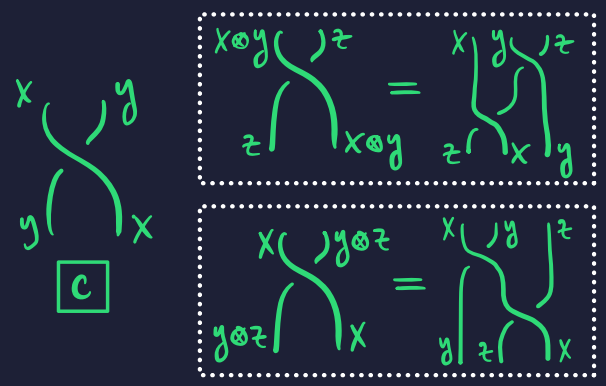

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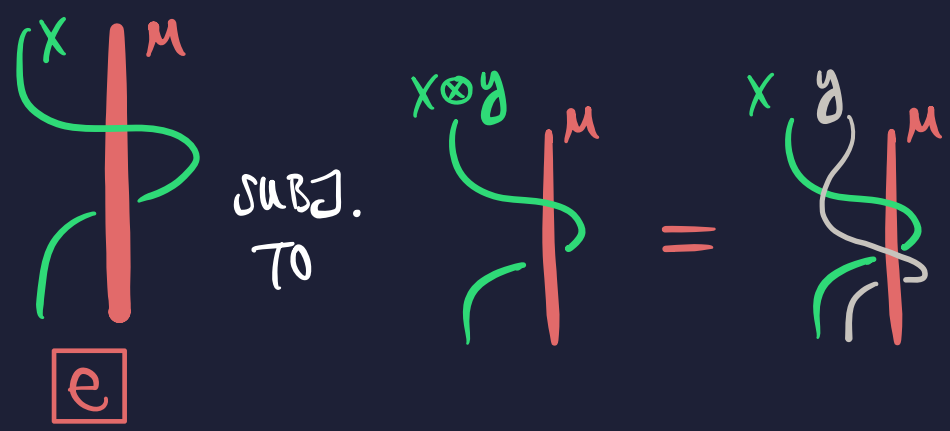

REPRESENTATIONS OF BRAIDED CATEGORIES

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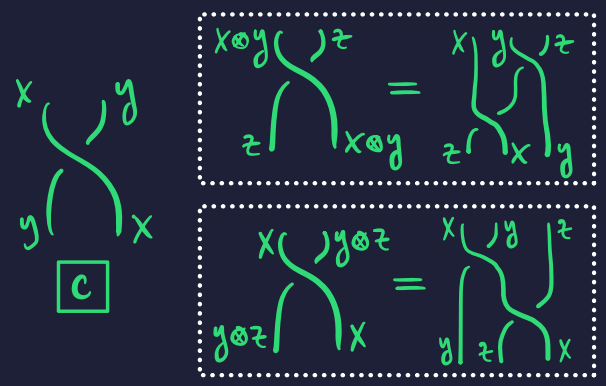

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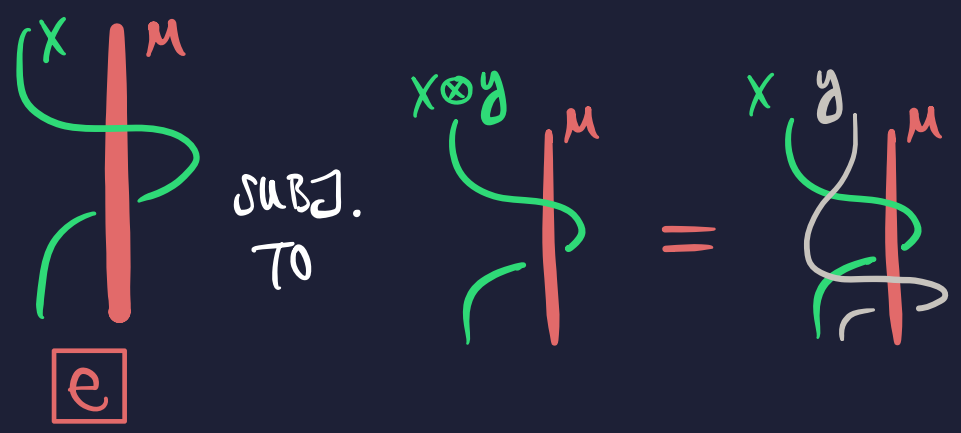

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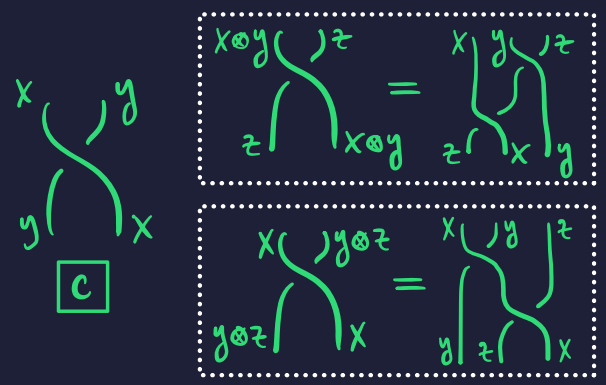

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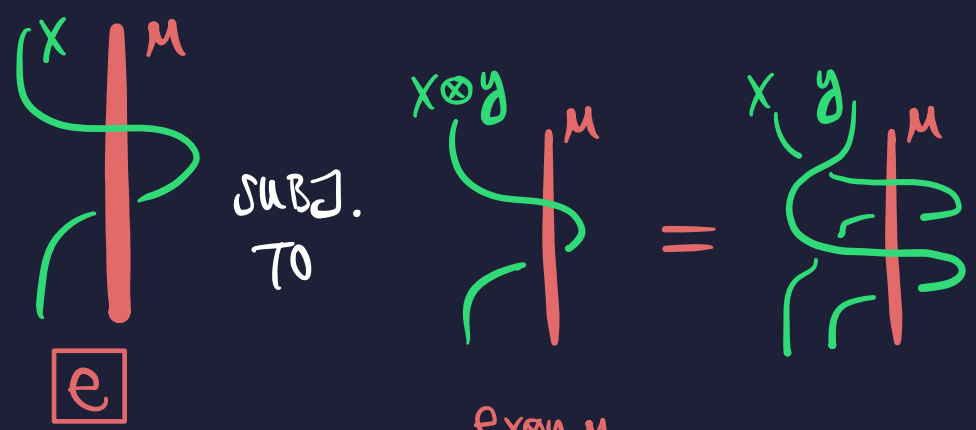

$$\begin{tikzcd}
X \otimes Y \triangleright M \arrow[rr, "e_{X\otimes Y, M}"] \arrow[d, "c^{-1}_{Y,X} \triangleright id"'] \arrow[dr, phantom, "\circlearrowright"] & & X \otimes Y \triangleright M \\
Y \otimes X \triangleright M \arrow[dr, "id \otimes e_{X,M}"'] & & X \otimes Y \triangleright M \arrow[u, "id \otimes e_{Y,M}"'] \\
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\end{tikzcd}$$

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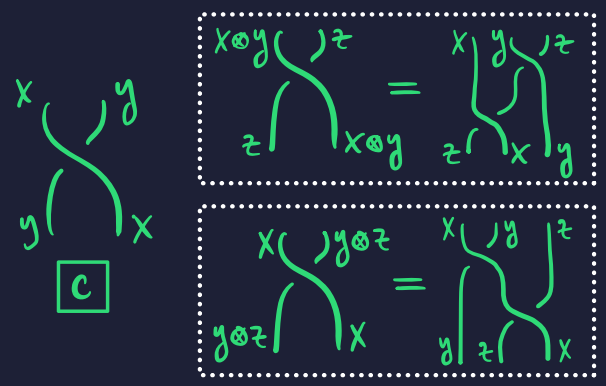

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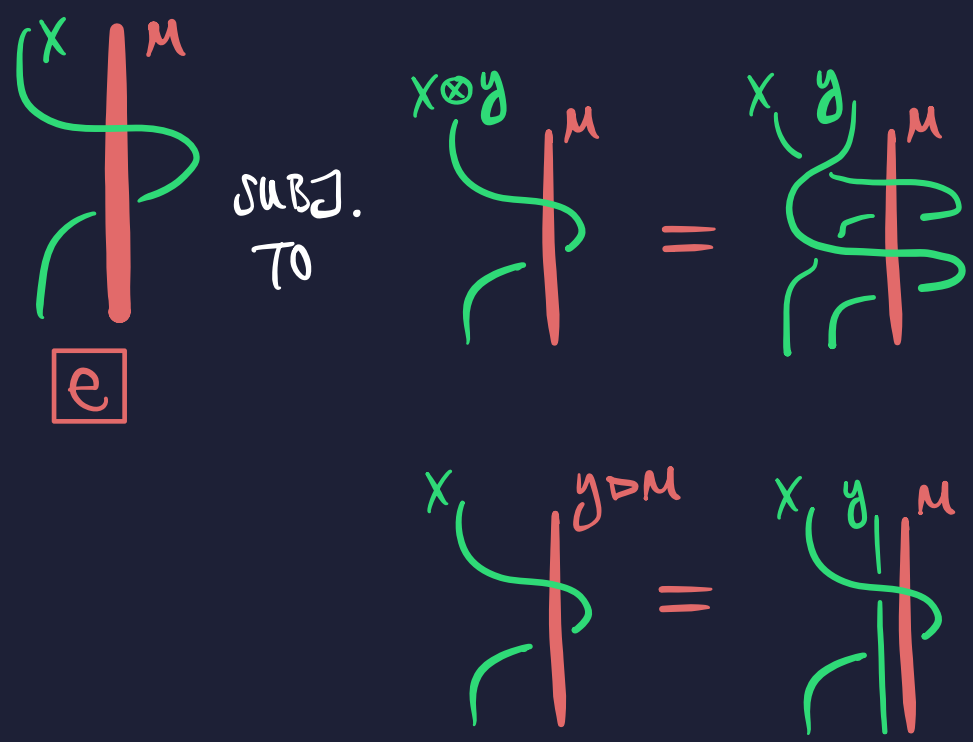

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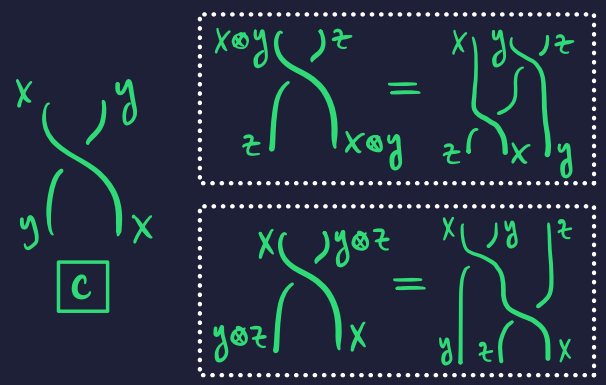

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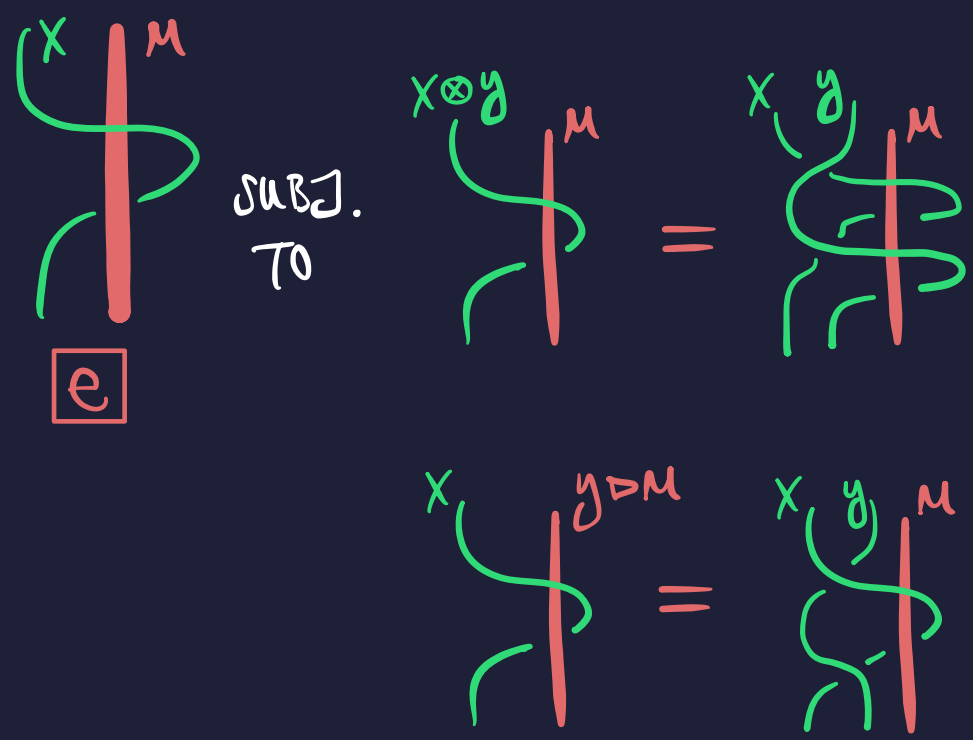

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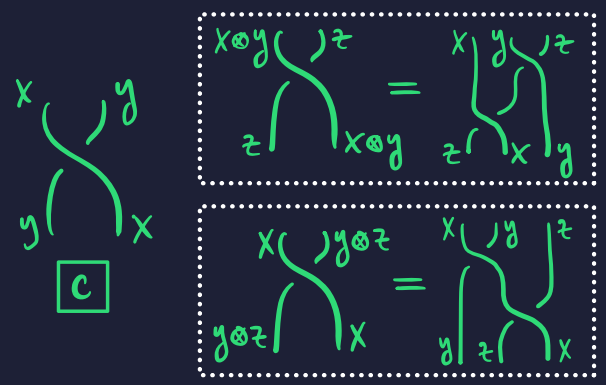

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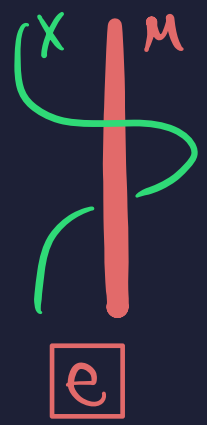

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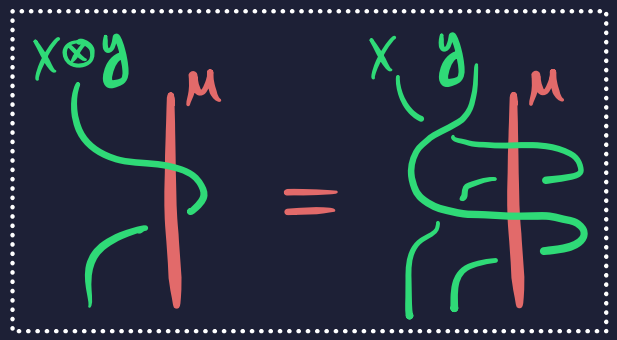

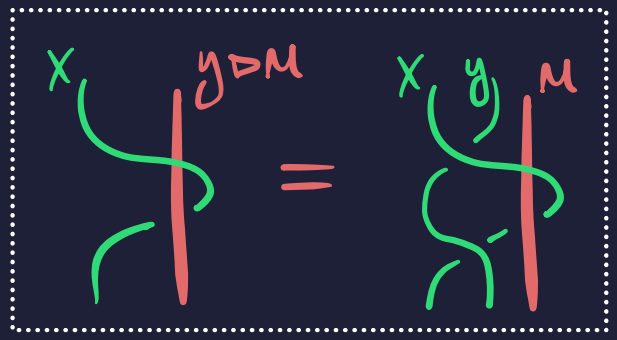

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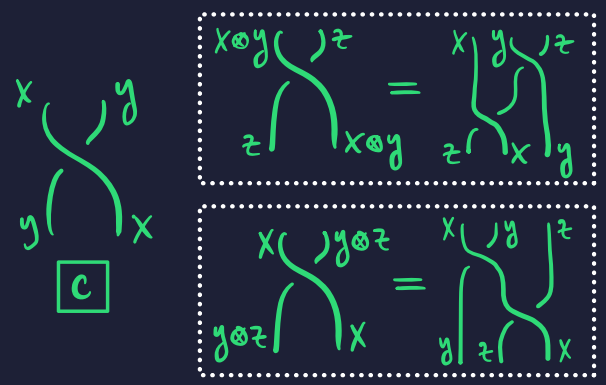

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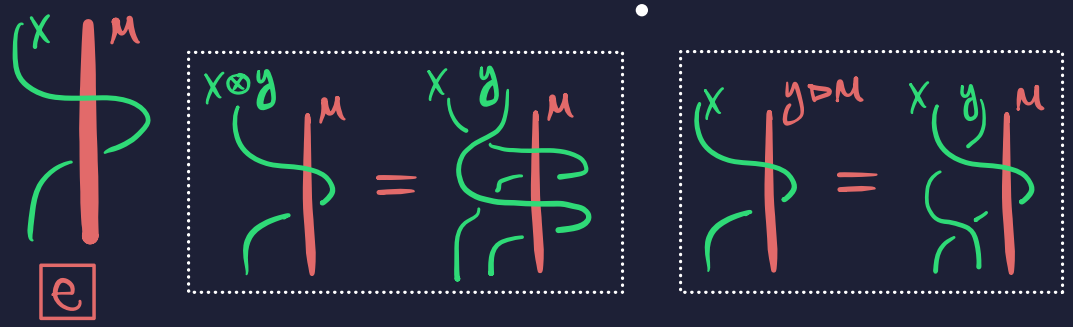

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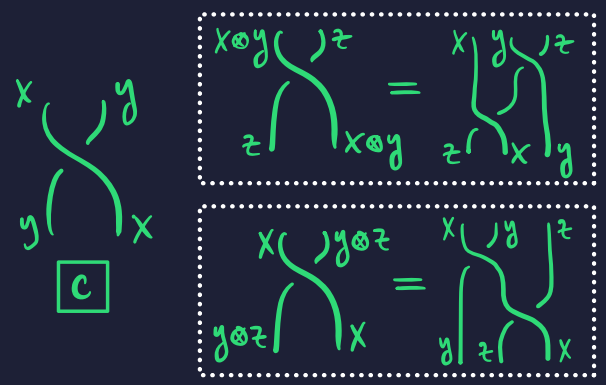

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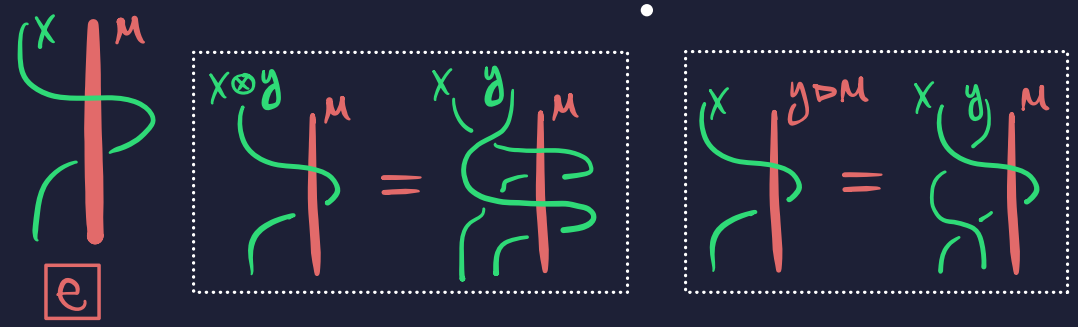

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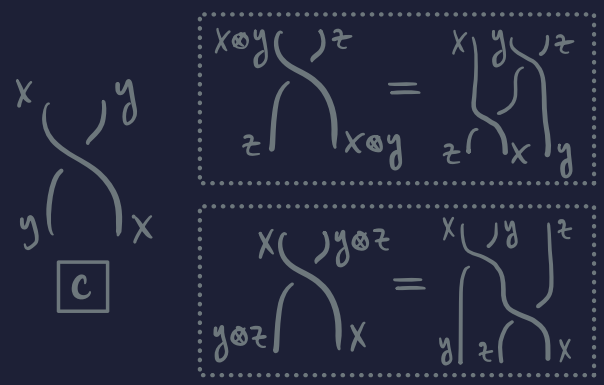

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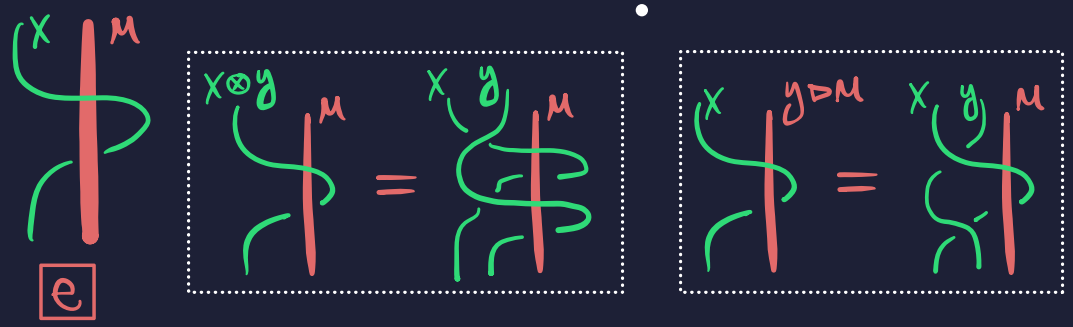

Ex. $(A\text{-Mod}_{\Bbbk}, \triangleright = \otimes_{\Bbbk}, e)$

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QUASITRI. LEFT H-COMODULE ALGEBRA

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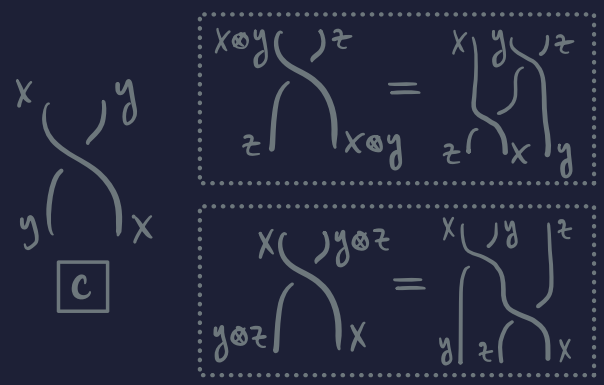

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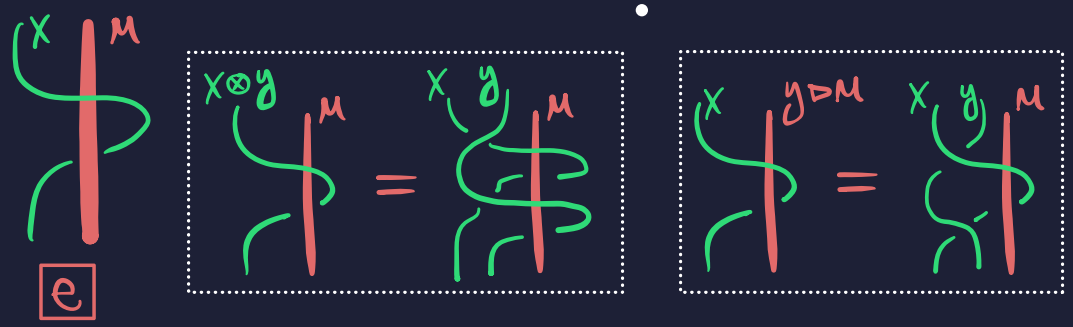

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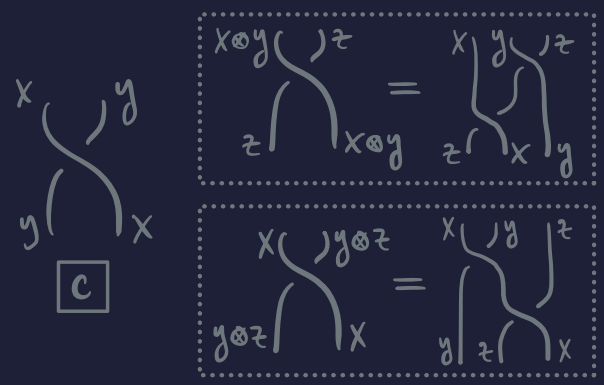

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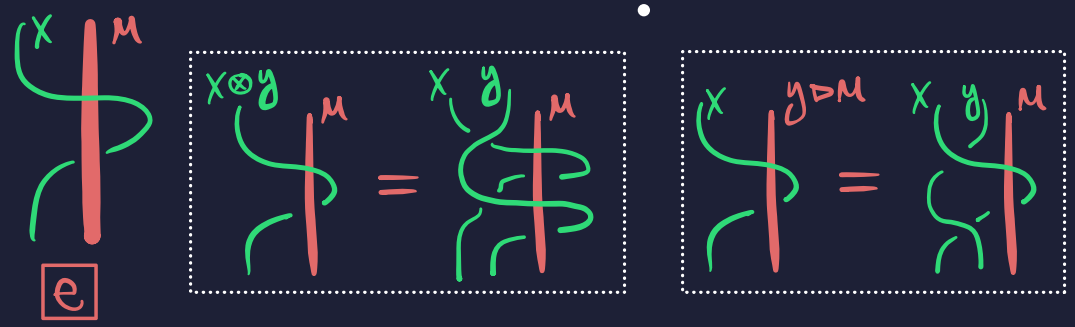

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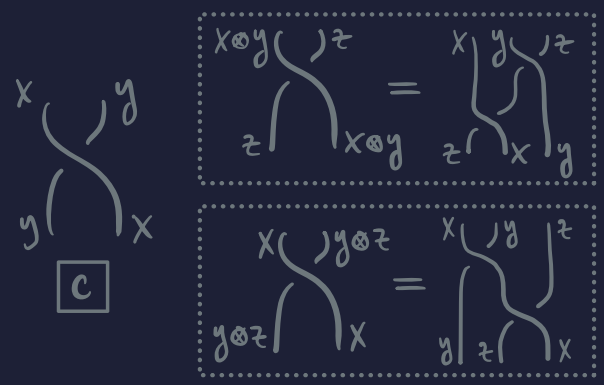

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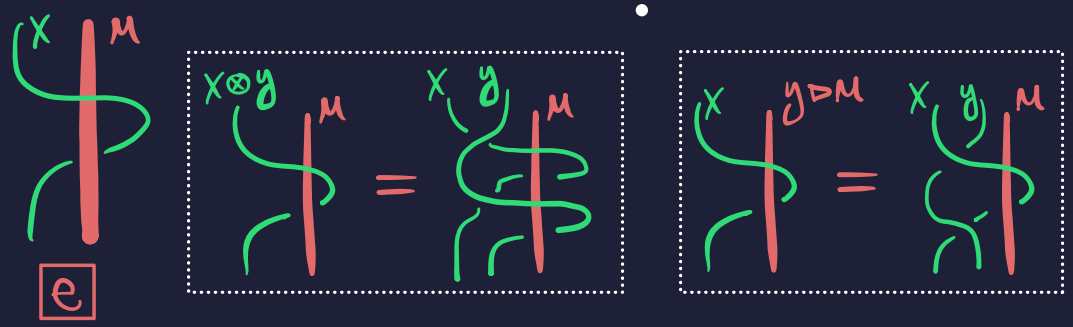

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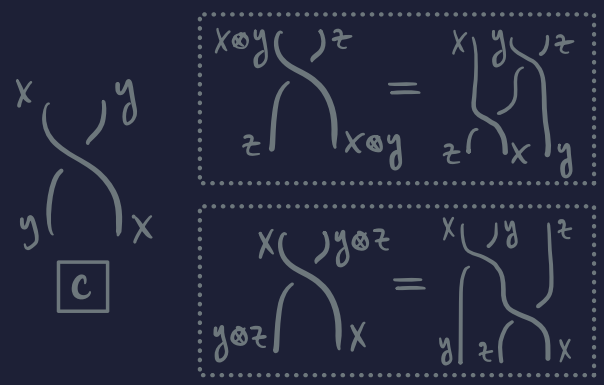

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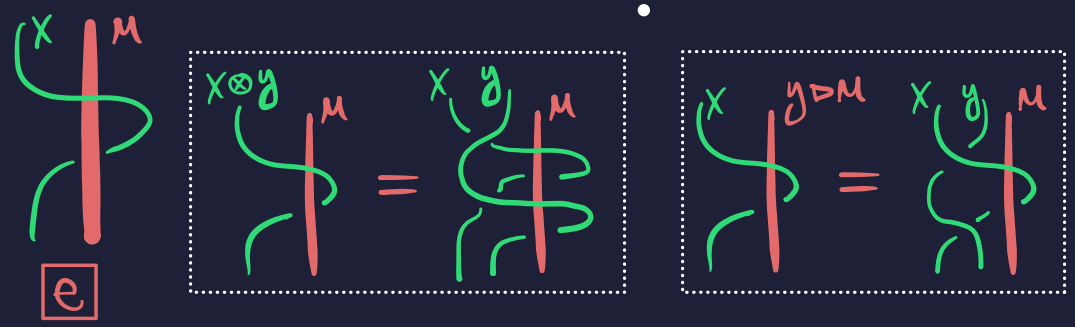

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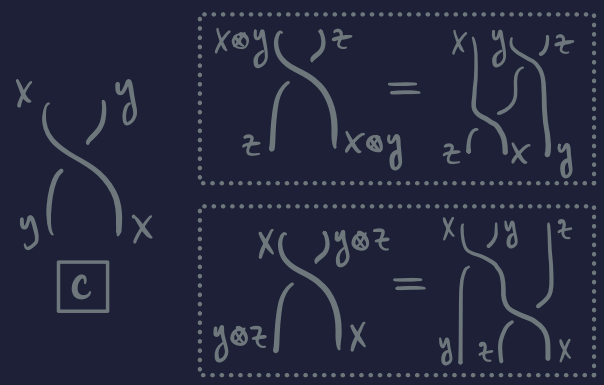

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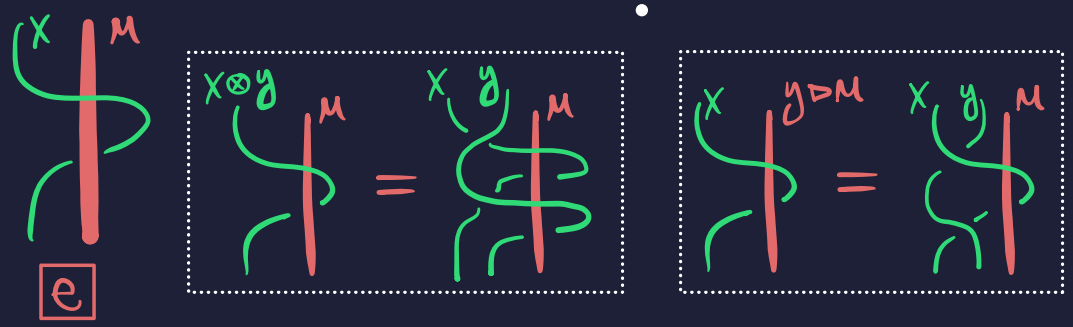

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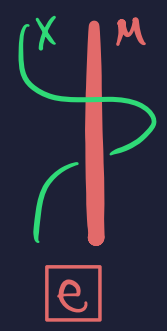

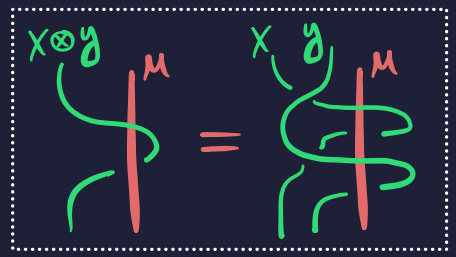

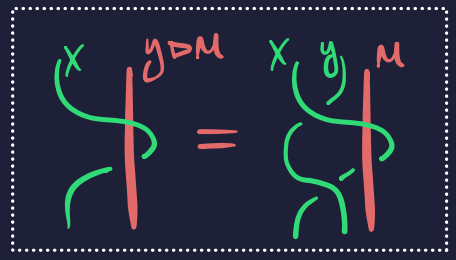

Ex. $(A\text{-Mod}_{\Bbbk}, \triangleright = \otimes_{\Bbbk}, e)$

FOR: $A = (A, m, u, \delta, \overset{\text{K-MATRIX}}{K \in H \otimes A})$

QUASITRI. LEFT H-COMODULE ALG

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C} := (\mathcal{C}, \otimes, \mathbb{1}, c)$

EXAMPLE: REGULAR $\mathcal{C}$-MOD. CAT.

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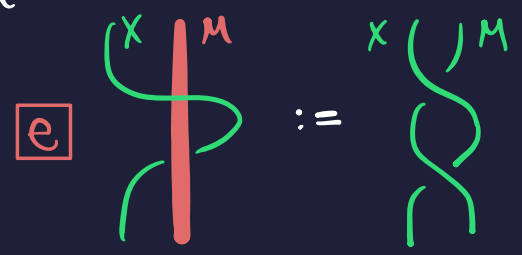

$X, M \in \mathcal{C}$

BRAIDED $\mathcal{C}$-MOD. CATEG.

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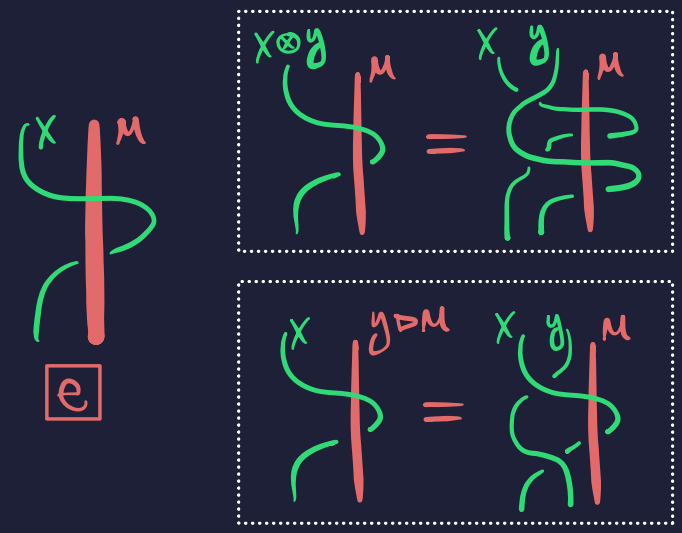

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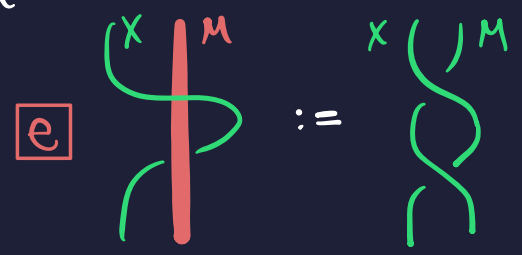

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- VERIFY FOR ENTERTAINMENT.

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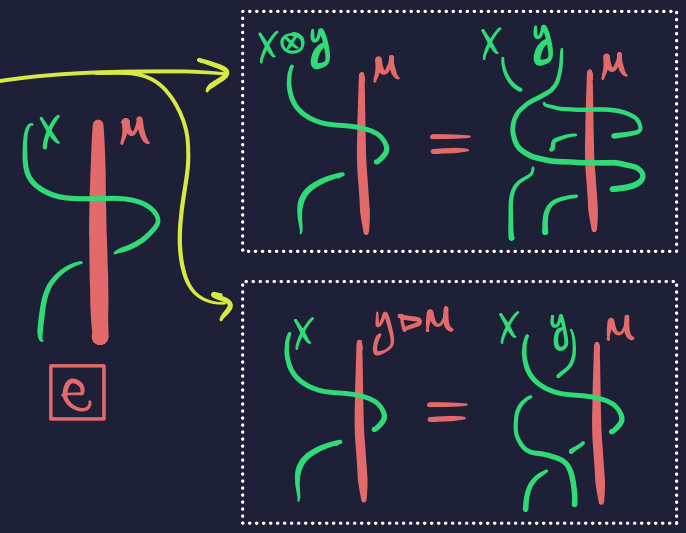

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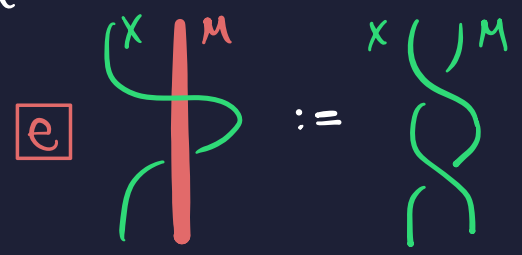

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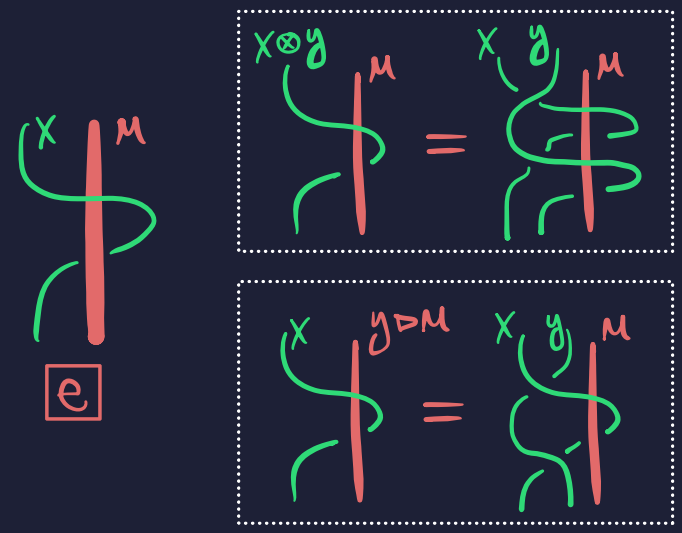

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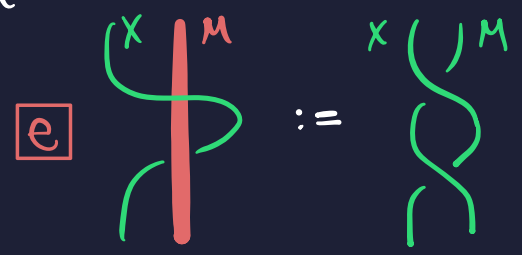

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YIELD SOLNS TO THE QUANTUM REFLECTION EQ. $\longrightarrow$

(LIKE R-MATRICES $\rightsquigarrow$ SOLNS TO YBE)

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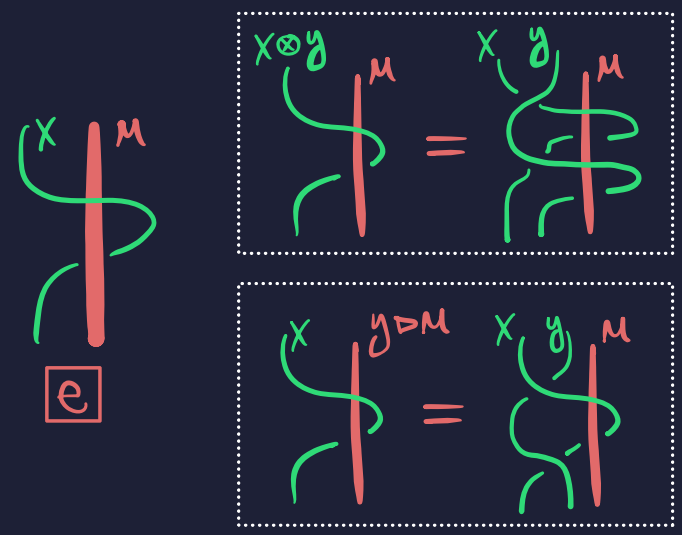

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YIELD REP'NS OF $\longrightarrow$

ARTIN BRAID GROUPS OF TYPE B

(BR $\otimes$ CATS YIELD REP'NS OF ... TYPE A)

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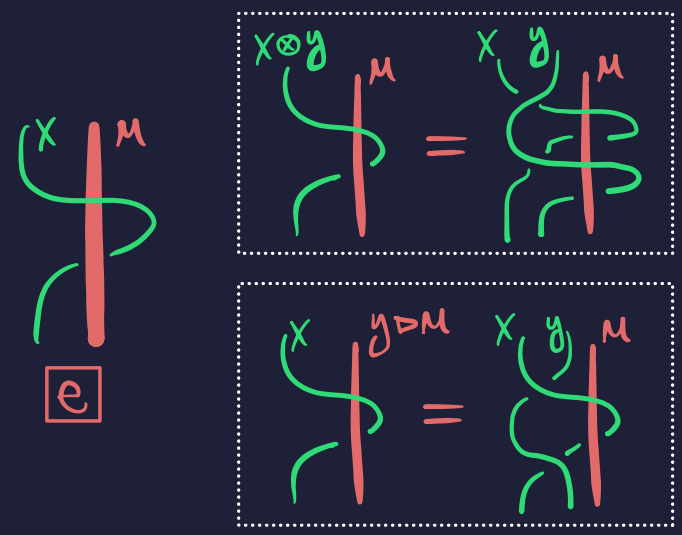

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[DAVYDOV–NIKSHYCH]

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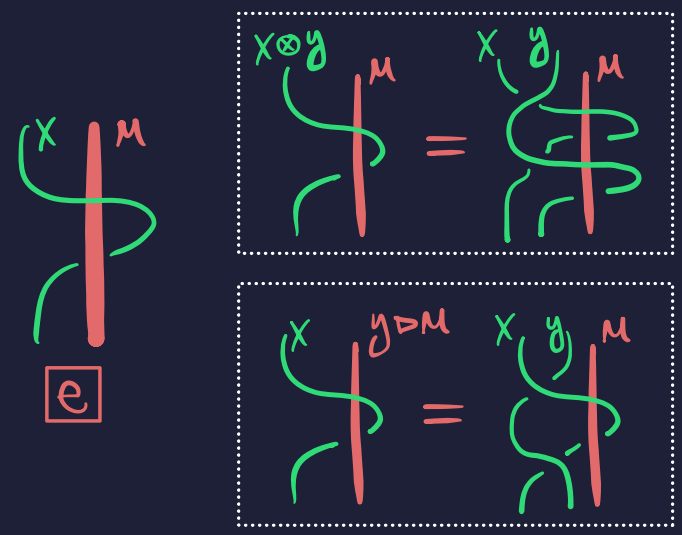

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2-CAT. OF BRAIDED $\mathcal{C}$-MOD. CATS

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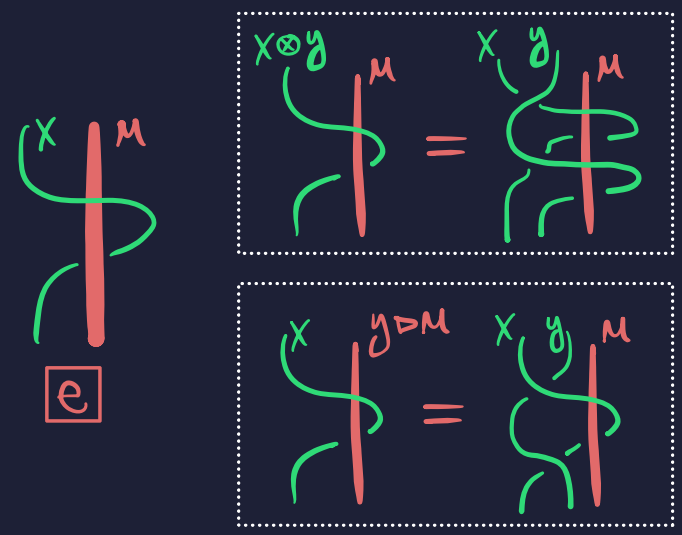

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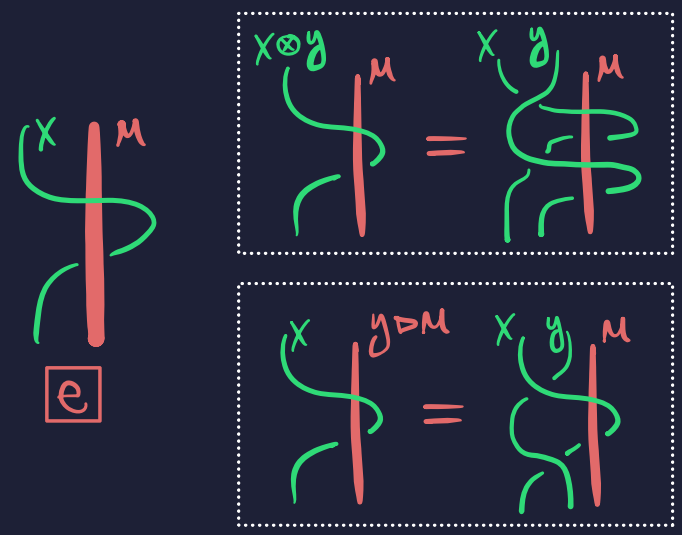

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SEE INTRO TO LAUGWITZ–W–YAKIMOV 2307.14764 FOR REFS & MORE!

KEY PEOPLE:

APPEL BALAGOVIĆ BAO
BROCHIER KOLB
LETZTER VLAAR WANG
...

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

↑ CONSTRUCT BRAIDED $\mathcal{C}$-MODULE CATEGORIES ...

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

RECALL: DRINFELD CENTERS $\mathcal{Z}(\mathcal{C})$ OF $\mathcal{C} := (\mathcal{C}, \otimes, \mathbb{1})$

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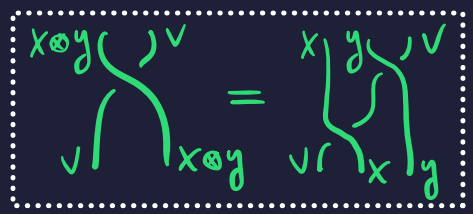
OBJECTS: $(V \in \mathcal{C},\ c^V)$

$c^V := \{ c^V_X : X \otimes V \xrightarrow{\sim} V \otimes X \}$

"HALF-BRAIDING"

WHERE $c^V_{X \otimes Y}$ SATISFIES



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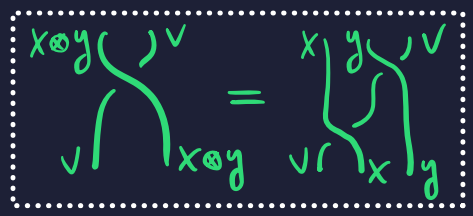

MORPHISMS: $(V, c^V) \longrightarrow (W, c^W)$

$= V \longrightarrow W \in \mathcal{C}$ COMP. W/ c^V, c^W

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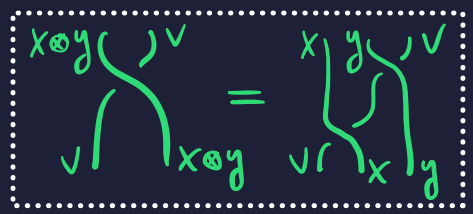

MONOIDAL PRODUCT:

$(V, c^V) \otimes (W, c^W)$
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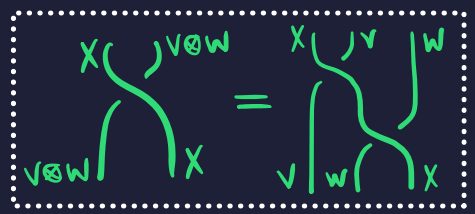
WHERE $c^{V\otimes W}_X$ SATISFIES



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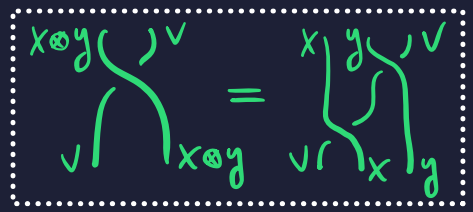

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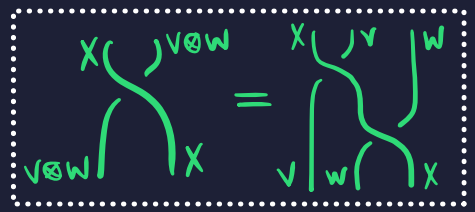

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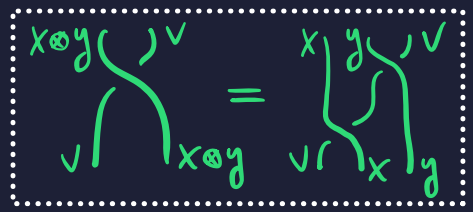

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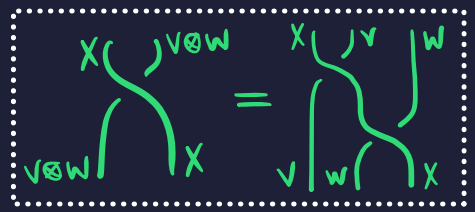

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• A KEY CONSTR'N OF BRAIDED $\otimes$ CATS •

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

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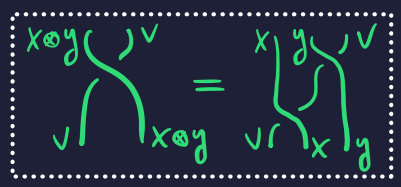

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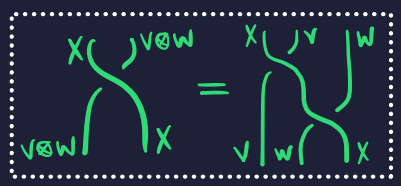

$c_{(V, c^V), (W, c^W)} := c^V_W$

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

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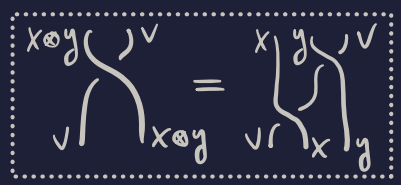

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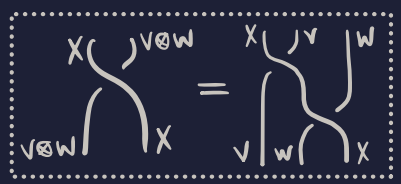

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RECALL FOR H = FIN DIM HOPF ALG.:

DRINFELD CENTER OF H-Mod	H-YETTER DRINFELD MODULES	DRINFELD DOUBLE OF H

$$\mathcal{Z}(H\text{-Mod}) \overset{BR\otimes}{\simeq} {}^H_H\mathcal{YD} \overset{BR\otimes}{\simeq} D(H)\text{-Mod}$$

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

DRIN. CTR $\mathcal{Z}(\mathcal{C})$ OF $(\mathcal{C}, \otimes, \mathbb{1})$

<u>OBJ</u>: $(V \in \mathcal{C}, c^V)$

$\{ c^V_X : X \otimes V \xrightarrow{\sim} V \otimes X \}$

"HALF-BRAIDING"

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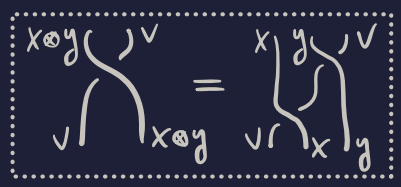

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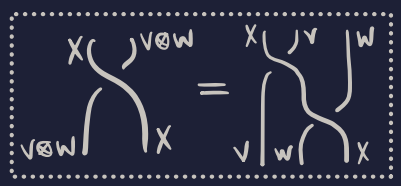

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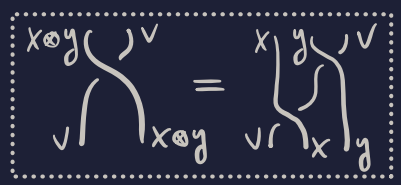

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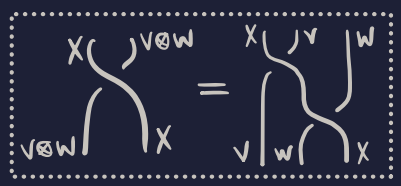

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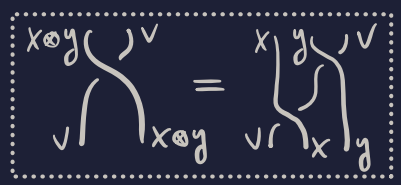

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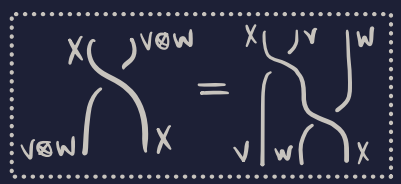

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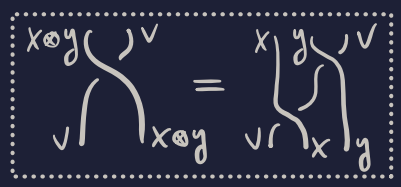

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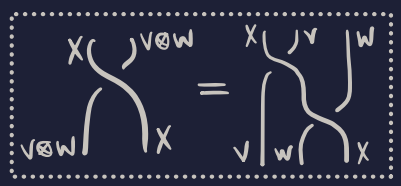

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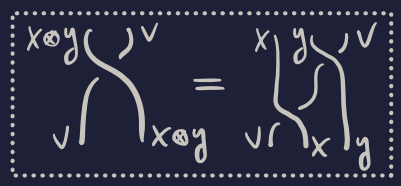

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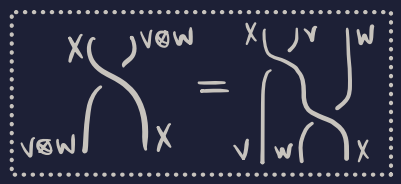

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$\in H$-Mod

$\in H$-CoMod

$(H^*)^{op} \otimes H$

SUBJ TO RELS

+ COMPATIBILITY

QUASITRIANG HOPF ALG

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

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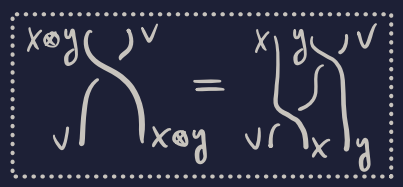

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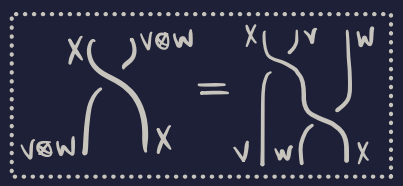

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SII AS COALGS

$(M, \bullet, \delta)$

$\in H$-Mod

$\in H$-Comod

$(H^*)^{op} \otimes H$

SUBJ TO RELS

+ COMPATIBILITY

QUASITRIANG HOPF ALG

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

→ DEVELOP THIS FOR $\mathcal{C}$-MOD. CATS

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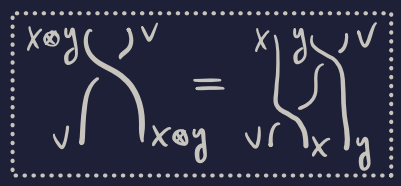

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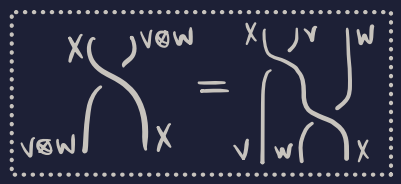

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SUBJ TO RELS

+ COMPATIBILITY

QUASITRIANG HOPF ALG

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

TAKE $\begin{cases} (\mathcal{C}, \otimes, \mathbb{1}, c) & \text{BRAIDED MONOIDAL CATEGORY} \\ (\mathcal{M}, \triangleright) & \text{(NOT NEC. BRAIDED) } \mathcal{C}\text{-MODULE CATEGORY} \end{cases}$

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WILL BUILD A

BRAIDED $\mathcal{C}$-MODULE CATEGORY

"REFLECTIVE CENTER

$\mathcal{E}_{\mathcal{C}}(\mathcal{M})$ OF $\mathcal{M}$"

AKIN TO BUILDING $\mathcal{Z}(\mathcal{C})$ FOR $\mathcal{C}$

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OBJECTS: $(M \in \mathcal{M}, e^M)$

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"REFLECTION"

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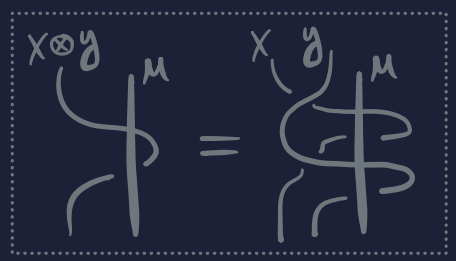

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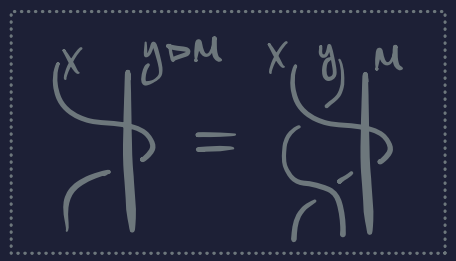

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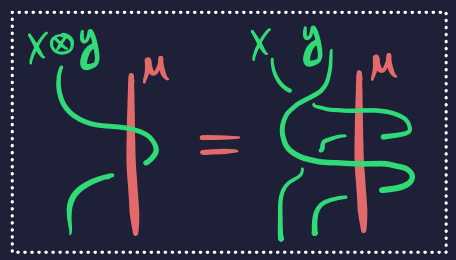

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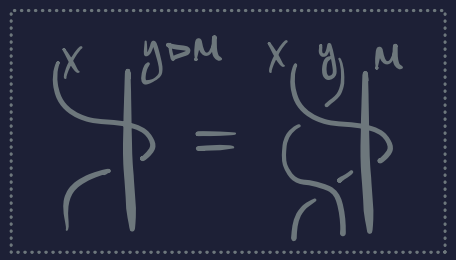

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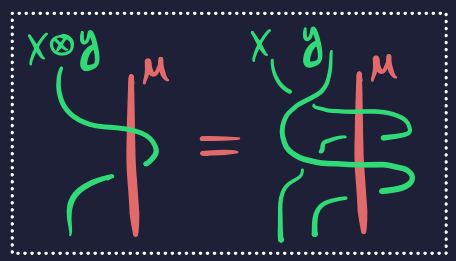

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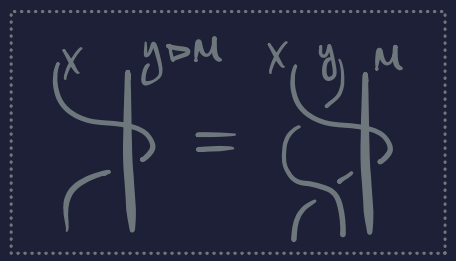

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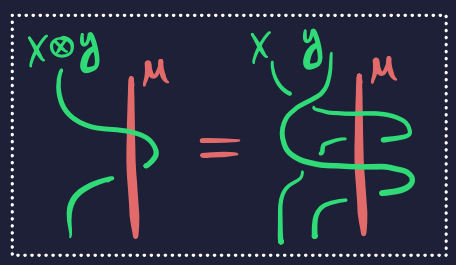

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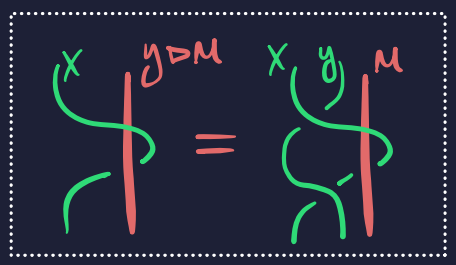

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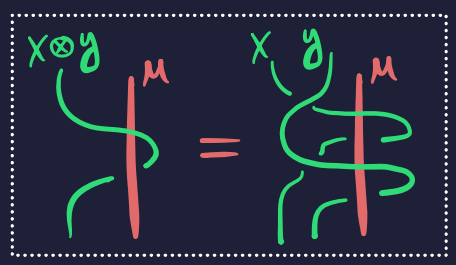

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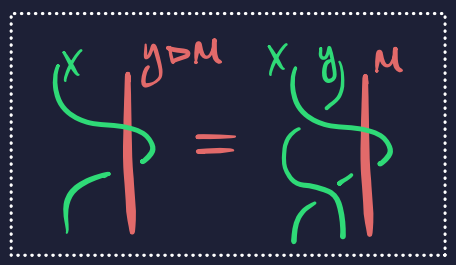

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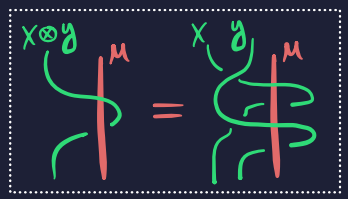

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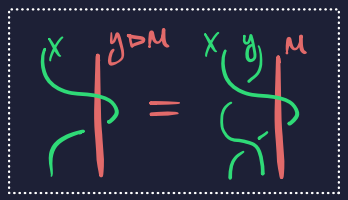

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$\mathcal{C} = (\mathcal{C}, \otimes, \mathbb{1}, c)$

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REFLECTIVE CTR $\mathcal{E}_r(\mathcal{M})$

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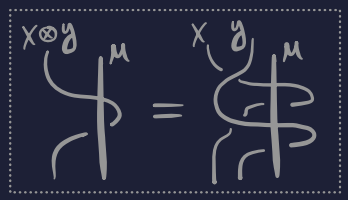

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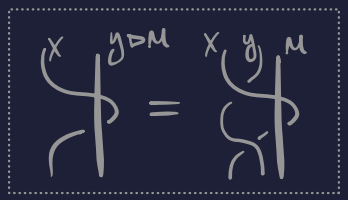

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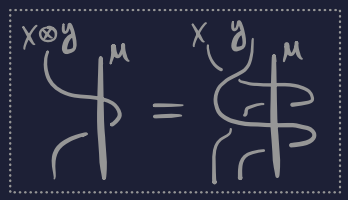

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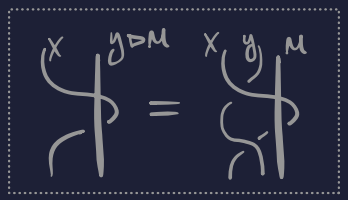

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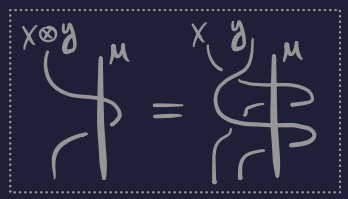

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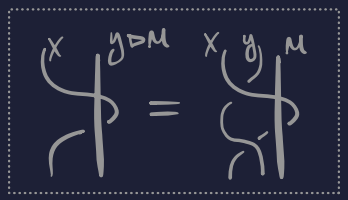

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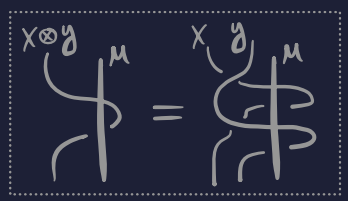

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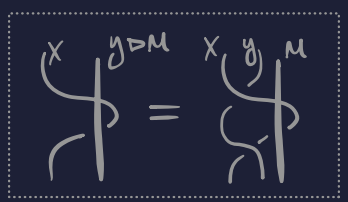

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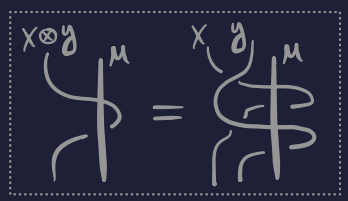

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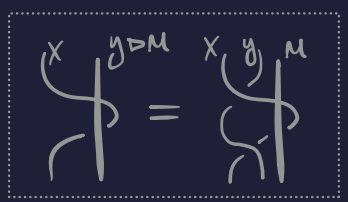

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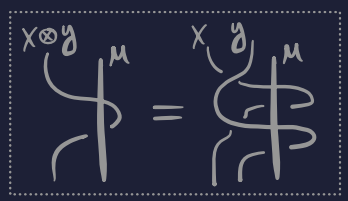

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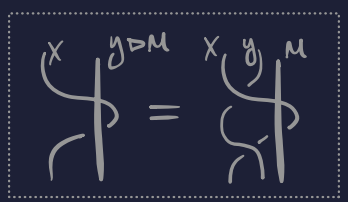

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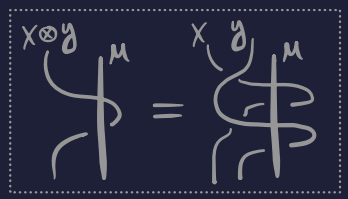

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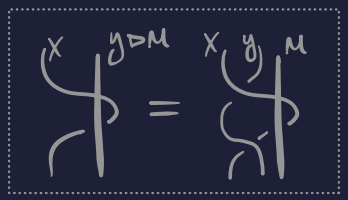

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RESULTS W/ YADAV [2411.18453]

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$$\left\{ X \in \mathcal{C} : \begin{tikzcd} X \arrow[r, bend left] \arrow[r, bend right] & Y \end{tikzcd} = \begin{tikzcd} X \arrow[r, equal] & Y \end{tikzcd} \ \forall Y \in \mathcal{C} \right\}$$

OBJS

$\simeq \mathrm{Vec}_{\mathbb{k}}$

RESULTS W/ YADAV [2411.18453]

$\mathcal{C}$ BR. FIN. $\otimes$ CAT. $\quad \mathcal{M} \in \mathcal{C}$-BrMod

SHIMIZU'S THEOREM: THE FOLLOWING ARE EQUIV. FOR A BRAIDED FINITE TENSOR CATEGORY $\mathcal{C} = (\mathcal{C}, \otimes, \mathbb{1}, c)$:

$\mathcal{C}$ IS NONDEGENERATE:

$$C_{\mathcal{C}} := \int^{X \in \mathcal{C}} X^* \otimes X \in \mathrm{HopfAlg}(\mathcal{C})$$

HAS A NONDEG HOPF PAIRING

$$\omega_{\mathcal{C}} : C_{\mathcal{C}} \otimes C_{\mathcal{C}} \to \mathbb{1}$$

$\Longleftrightarrow$

$\mathcal{C}$ IS WEAKLY FACTOR'BLE:

$$\mathrm{Hom}_{\mathcal{C}}(\mathbb{1}, C_{\mathcal{C}}) \to \mathrm{Hom}_{\mathcal{C}}(C_{\mathcal{C}}, \mathbb{1})$$
$$f \longmapsto [\omega_{\mathcal{C}} \circ (f \otimes \mathrm{id})]$$

IS BIJECTIVE

$\Downarrow$

$\mathcal{C}$ IS FACTORIZABLE:

EMBEDDINGS

$$\mathcal{C} := (\mathcal{C}, c) \longrightarrow \mathcal{Z}(\mathcal{C})$$
$$\overline{\mathcal{C}} := (\mathcal{C}, c^{-1}) \longrightarrow \mathcal{Z}(\mathcal{C})$$

YIELD $\mathcal{C} \boxtimes \overline{\mathcal{C}} \simeq \mathcal{Z}(\mathcal{C})$

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CONSIDER THE TERMS BELOW...

$\mathcal{M}$ IS NONDEGENERATE:

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(IN A CERTAIN WAY)

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CONSIDER THE TERMS BELOW.

WE OBTAIN THE IMPLICATIONS...

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RESULTS W/ YADAV [2411.18453]

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CONSIDER THE TERMS BELOW.

FOR YOU!

WE OBTAIN THE IMPLICATIONS & HAVE THIS QUESTION

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$\overset{?}{\Longleftarrow}$

$?$

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RESULTS W/ YADAV [2411.18453]

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HOPF CASE:

(H,R) = FIN. DIM. QUASITRI. HOPF ALG.

(B,K) = FIN. DIM. QUASITRI. H-COMOD. ALG

$\exists$ NOTION OF (B,K) BEING FACTORIZABLE

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  $\iff$ H IS A FACTORIZABLE HOPF ALG.

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- $(R_H(A), K_{REF})$ IS A FACTOR'BLE H-COMOD ALG.
  WHEN $R_H(A)$ IS H-SIMPLE

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  ~~WHEN $R_H(A)$ IS H-SIMPLE~~ QUESTION

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THM: B-Mod IS A NONDEG (H-Mod)-MOD CAT
$\iff$ B IS A FACTORIZABLE H-COMOD ALG.

REPRESENTATIONS OF BRAIDED CATEGORIES

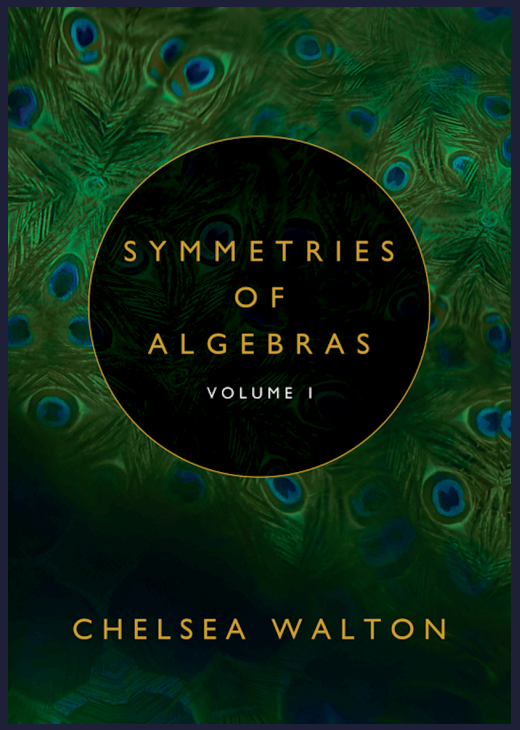
⋮

MORE TO COME

STAY TUNED :)

REPRESENTATIONS OF BRAIDED CATEGORIES

COMMERCIAL BREAK



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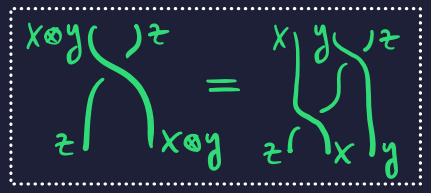
I HOPE THAT YOU LIKE IT!



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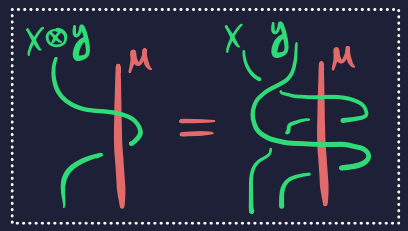
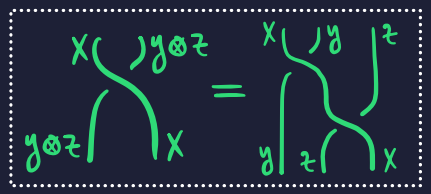
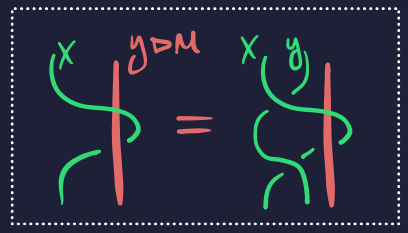
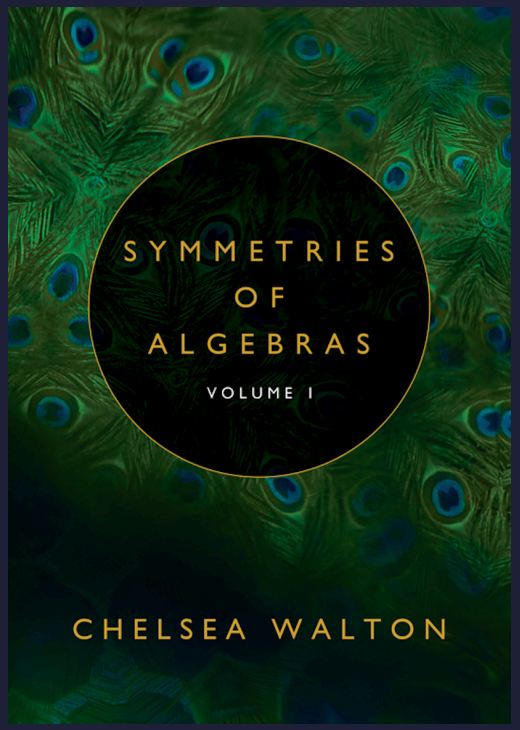






Thanks for listening!



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