

REPRESENTATIONS OF BRAIDED CATEGORIES

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HOUSTON, USA

• SLIDES •



REPRESENTATIONS

REPRESENTATIONS

↑ ALLOW US TO REALIZE STRUCTURES AS SYMMETRIES ...



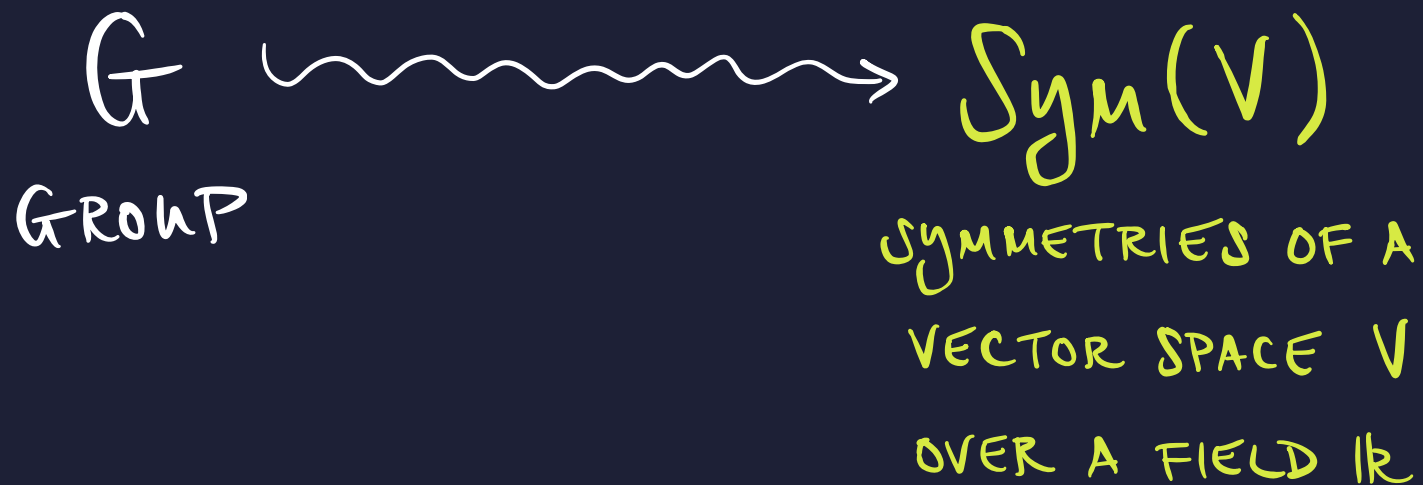
REPRESENTATIONS

↑ ALLOW US TO REALIZE STRUCTURES AS SYMMETRIES ...



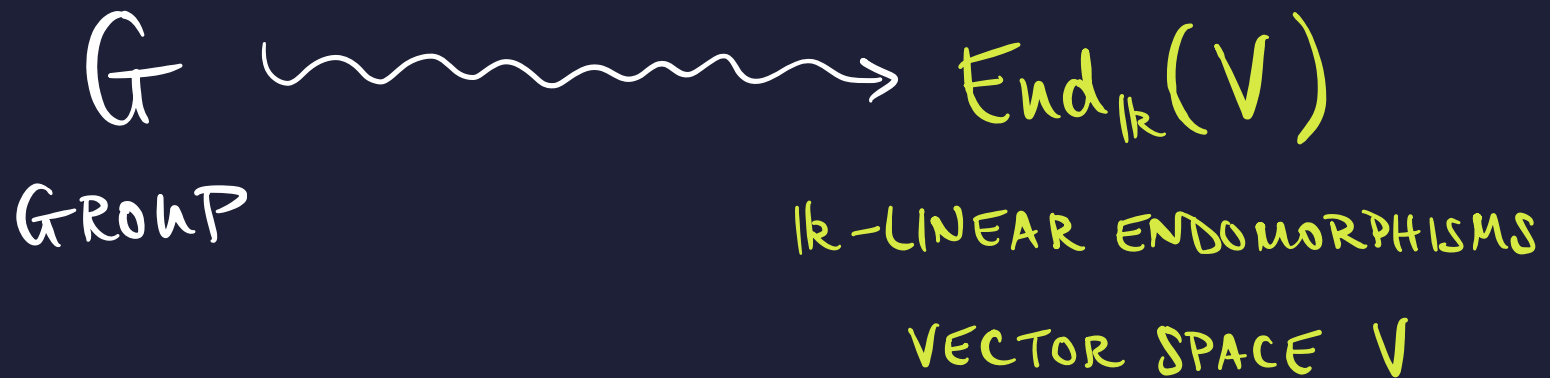
REPRESENTATIONS

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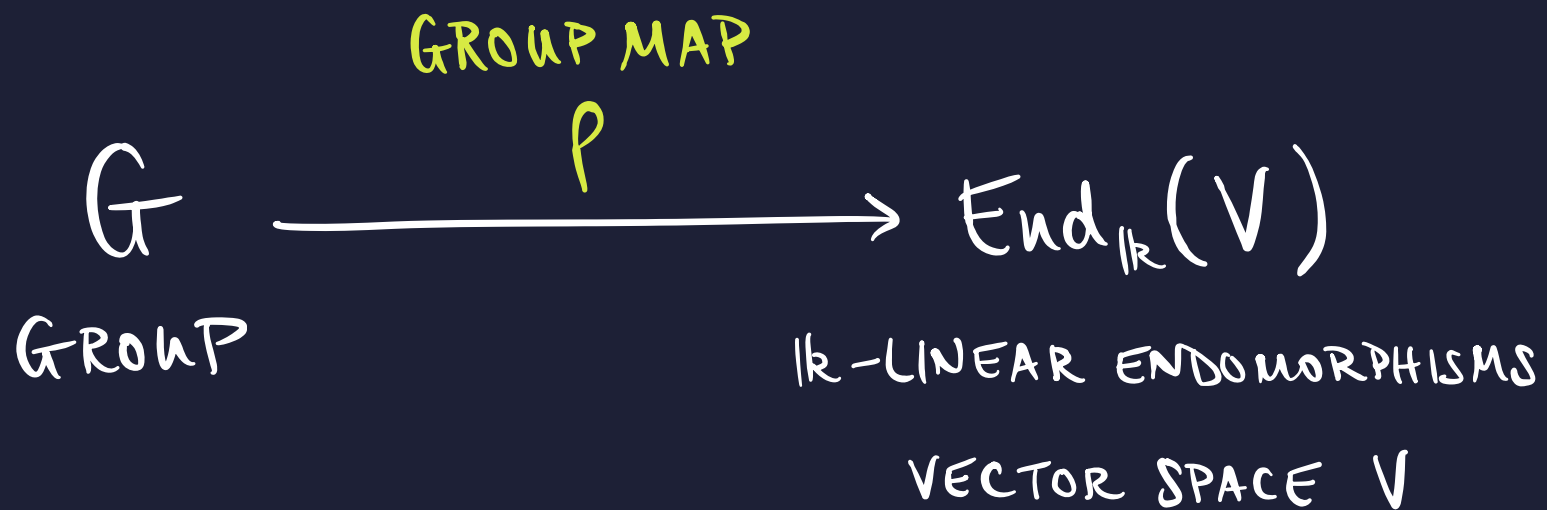
REPRESENTATIONS

↑ ALLOW US TO REALIZE STRUCTURES AS SYMMETRIES ...

$$\begin{array}{ccc} & \text{GROUP MAP} & \\ & \rho & \\ G & \longrightarrow & \text{End}_{\mathbb{K}}(V) \\ \text{GROUP} & & \mathbb{K}\text{-LINEAR ENDOMORPHISMS} \\ & & \text{VECTOR SPACE } V \end{array}$$

REPRESENTATIONS

↑ ALLOW US TO REALIZE STRUCTURES AS SYMMETRIES ...



NEEDS TO BE
A GROUP

REPRESENTATIONS

↑ ALLOW US TO REALIZE STRUCTURES AS SYMMETRIES ...

$$\begin{array}{ccc} & \text{GROUP MAP} & \\ & \rho & \\ G & \xrightarrow{\quad} & \text{Aut}_{\mathbb{R}}(V) =: \text{GL}(V) \\ \text{GROUP} & & \begin{array}{l} \mathbb{R}\text{-LINEAR AUTOMORPHISMS} \\ \text{VECTOR SPACE } V \end{array} \end{array}$$

REPRESENTATIONS

↑ ALLOW US TO REALIZE STRUCTURES AS SYMMETRIES ...

$$\begin{array}{ccc} & \text{GROUP MAP} & \\ & \rho & \\ G & \xrightarrow{\quad} & \text{Aut}_{\mathbb{R}}(V) =: \text{GL}(V) \\ \text{GROUP} & & \begin{array}{l} \mathbb{R}\text{-LINEAR AUTOMORPHISMS} \\ \text{VECTOR SPACE } V \end{array} \end{array}$$

REPRESENTATIONS

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$$\begin{array}{ccc} & \text{GROUP MAP} & \\ & \rho & \\ G & \xrightarrow{\quad} & \text{Aut}_{\mathbb{K}}(V) =: \text{GL}(V) \\ \text{GROUP} & & \begin{array}{l} \mathbb{K}\text{-LINEAR AUTOMORPHISMS} \\ \text{VECTOR SPACE } V \end{array} \end{array}$$

‡ AS MATRICES HERE...

$$G \xrightarrow{\rho} \text{GL}(\mathbb{K}^n) \cong \text{GL}_n(\mathbb{K})$$

REPRESENTATIONS

↑ ALLOW US TO REALIZE STRUCTURES AS SYMMETRIES ...

$$\begin{array}{ccc} & \text{GROUP MAP} & \\ & \rho & \\ G & \xrightarrow{\quad} & \text{Aut}_{\mathbb{K}}(V) =: \text{GL}(V) \\ \text{GROUP} & & \begin{array}{l} \mathbb{K}\text{-LINEAR AUTOMORPHISMS} \\ \text{VECTOR SPACE } V \end{array} \end{array}$$

‡ AS MATRICES HERE...

$$G \xrightarrow{\rho} \text{GL}(\mathbb{K}^n) \cong \text{GL}_n(\mathbb{K})$$

→ TRANSPORT GROUPS TO LINEAR ALGEBRA LAND

REPRESENTATIONS

↑ ALLOW US TO REALIZE STRUCTURES AS SYMMETRIES ...

$$\begin{array}{ccc} & \text{ALGEBRA MAP} & \\ & \rho & \\ A & \xrightarrow{\quad} & \text{End}_{\mathbb{K}}(V) \\ \mathbb{K}\text{-ALGEBRA} & & \mathbb{K}\text{-LINEAR ENDOMORPHISMS} \\ & & \text{VECTOR SPACE } V \end{array}$$

‡ AS MATRICES HERE...

$$A \xrightarrow{\rho} \text{End}_{\mathbb{K}}(\mathbb{K}^n) \cong \text{Mat}_n(\mathbb{K})$$

→ TRANSPORT $\mathbb{K}$ -ALGS TO LINEAR ALGEBRA LAND

REPRESENTATIONS

ABSTRACTLY...

$$\begin{array}{ccc} \mathcal{S} & \rightsquigarrow & \text{Sym}(U) \\ \text{STRUCTURE} & & \text{FOR SOME OBJECT } U \end{array}$$

REPRESENTATIONS

ABSTRACTLY...

STRUCTURE MAP

$$\mathcal{S} \xrightarrow{\rho} \text{Sym}(U)$$

STRUCTURE

SAME KIND OF STRUCTURE
FOR SOME OBJECT U

REPRESENTATIONS

ABSTRACTLY...

STRUCTURE MAP

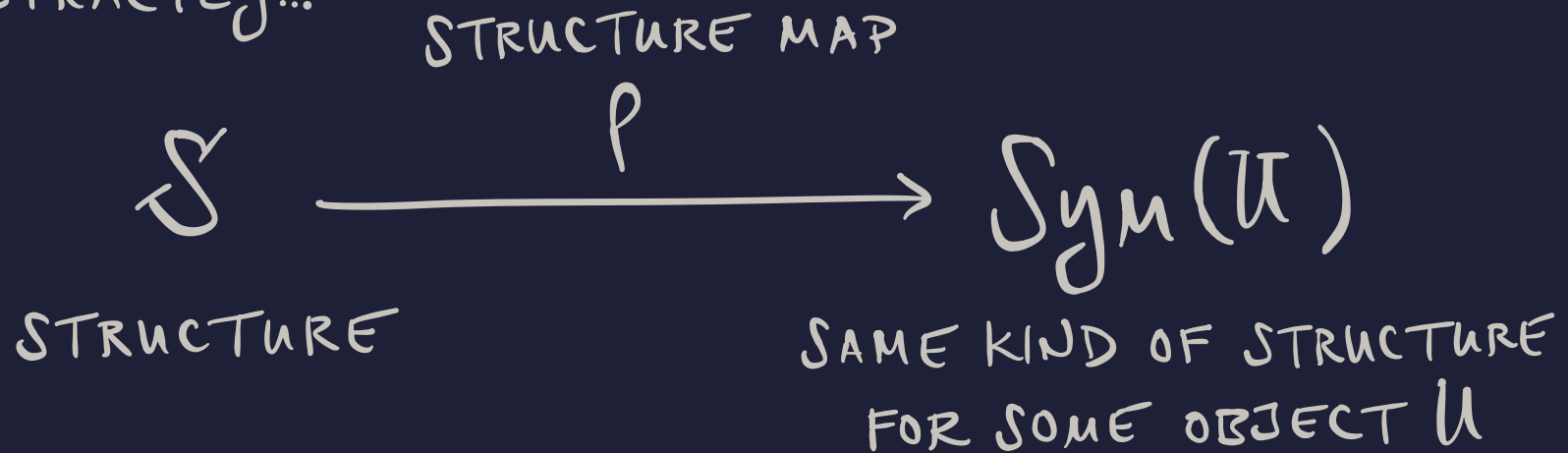
$$\mathcal{S} \xrightarrow{\rho} \text{Sym}(U)$$

STRUCTURE

SAME KIND OF STRUCTURE
FOR SOME OBJECT U

REPRESENTATIONS

ABSTRACTLY...



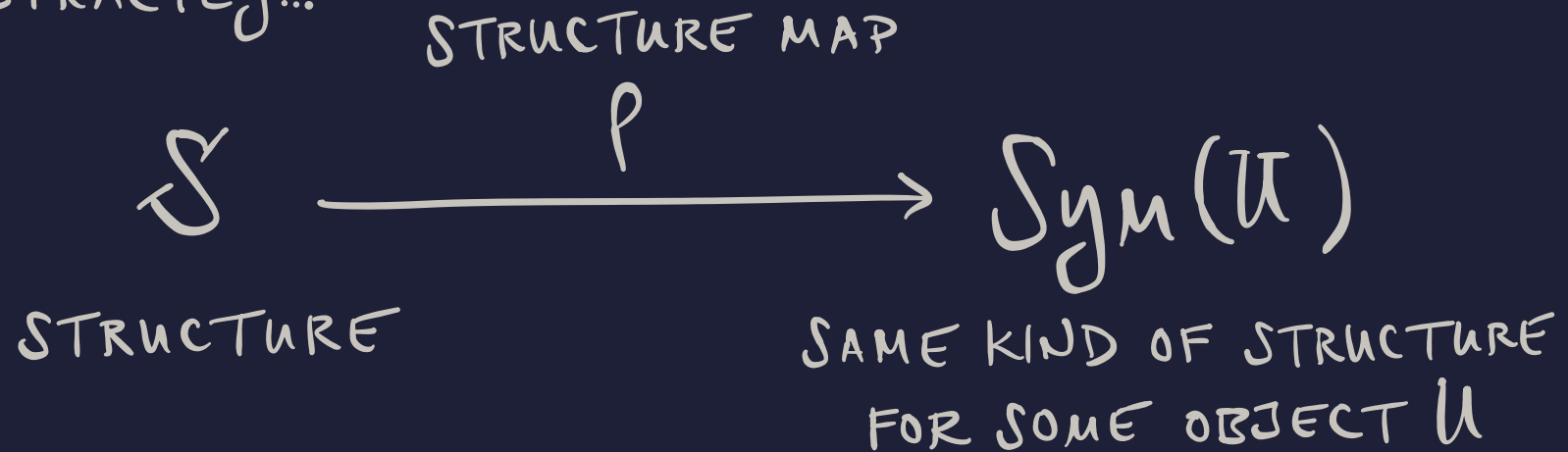
Ex.



ME

REPRESENTATIONS

ABSTRACTLY...



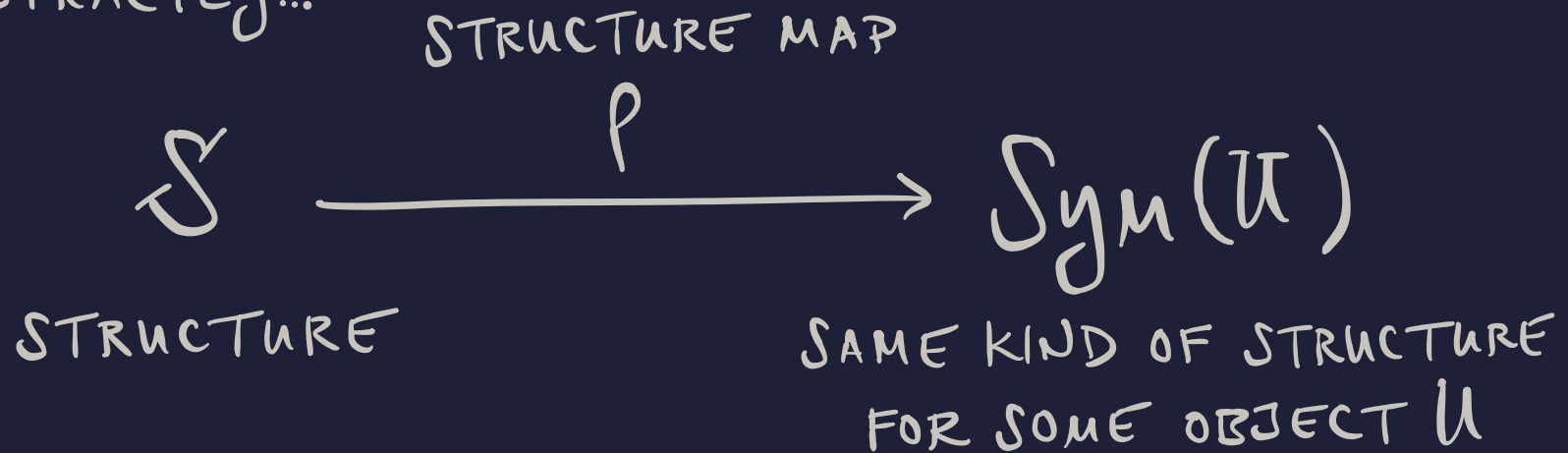
Ex.



ME

REPRESENTATIONS

ABSTRACTLY...



Ex.

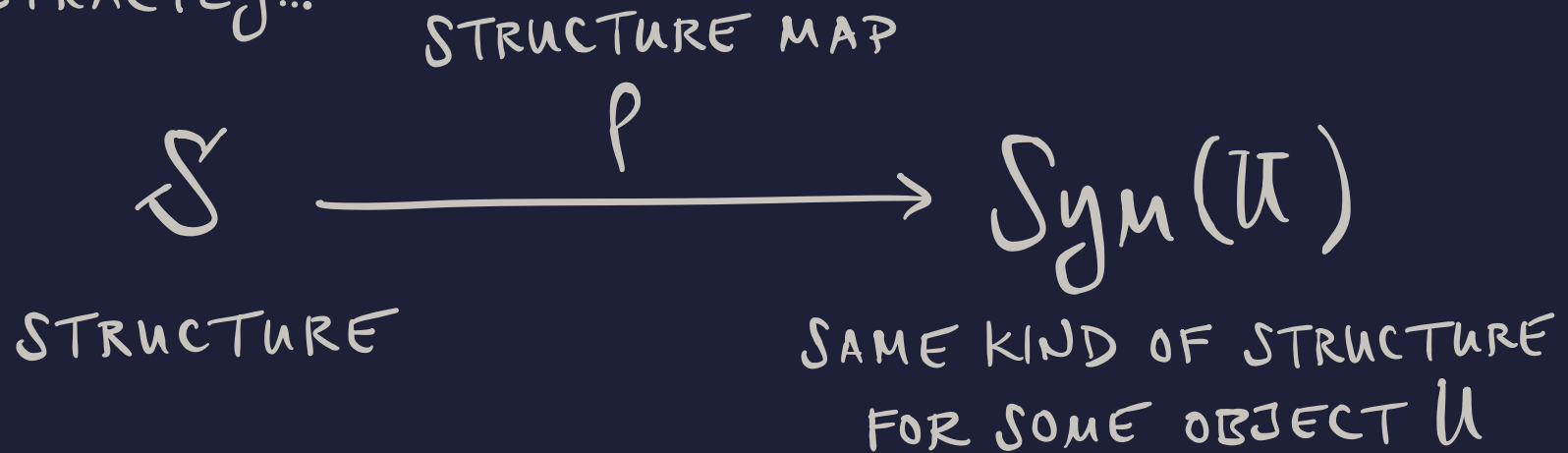


ME

✓ WORKS ... ALTHOUGH A BIT
BORING

REPRESENTATIONS

ABSTRACTLY...



Ex.

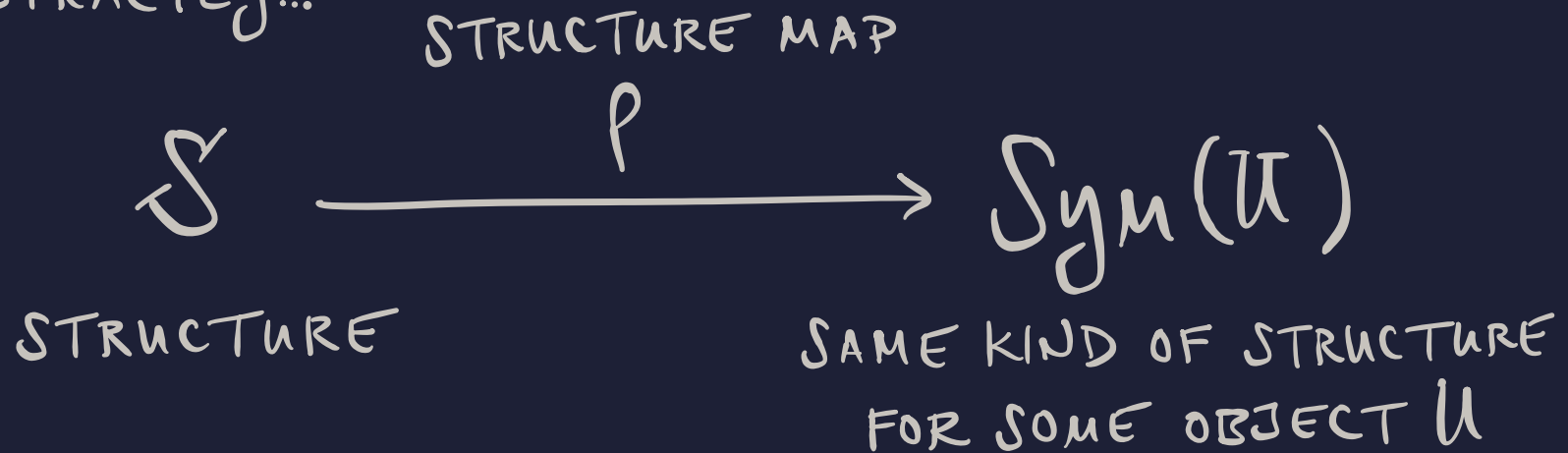


ME

✓ WORKS ... ALTHOUGH A BIT
BORING ... OR REGULAR

REPRESENTATIONS

ABSTRACTLY...



Ex.

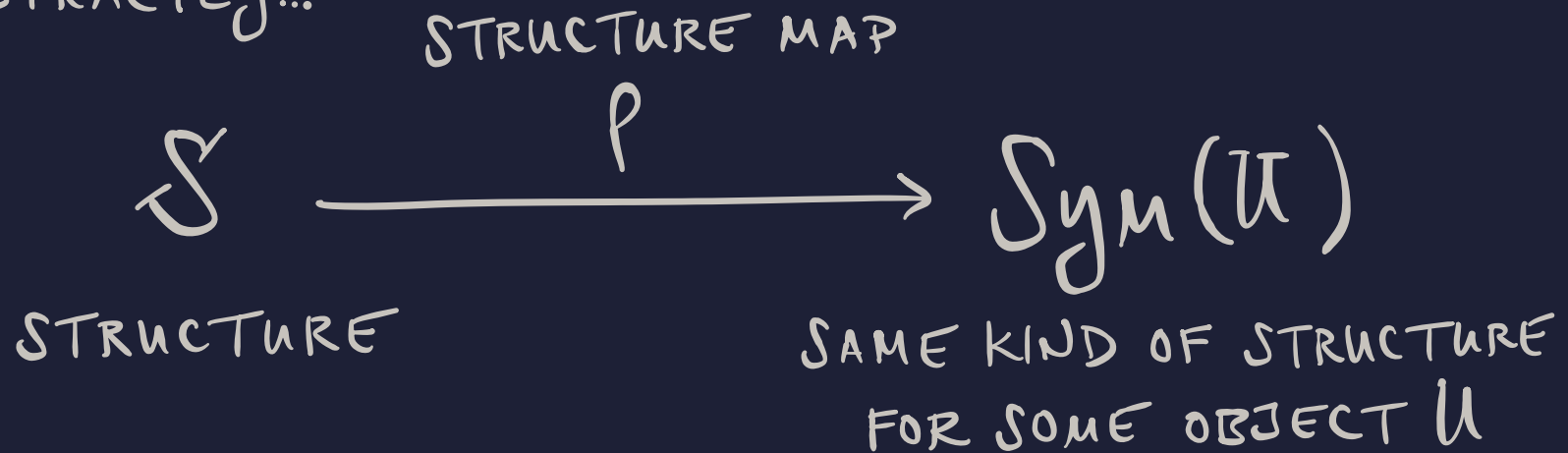


ME



REPRESENTATIONS

ABSTRACTLY...



Ex.



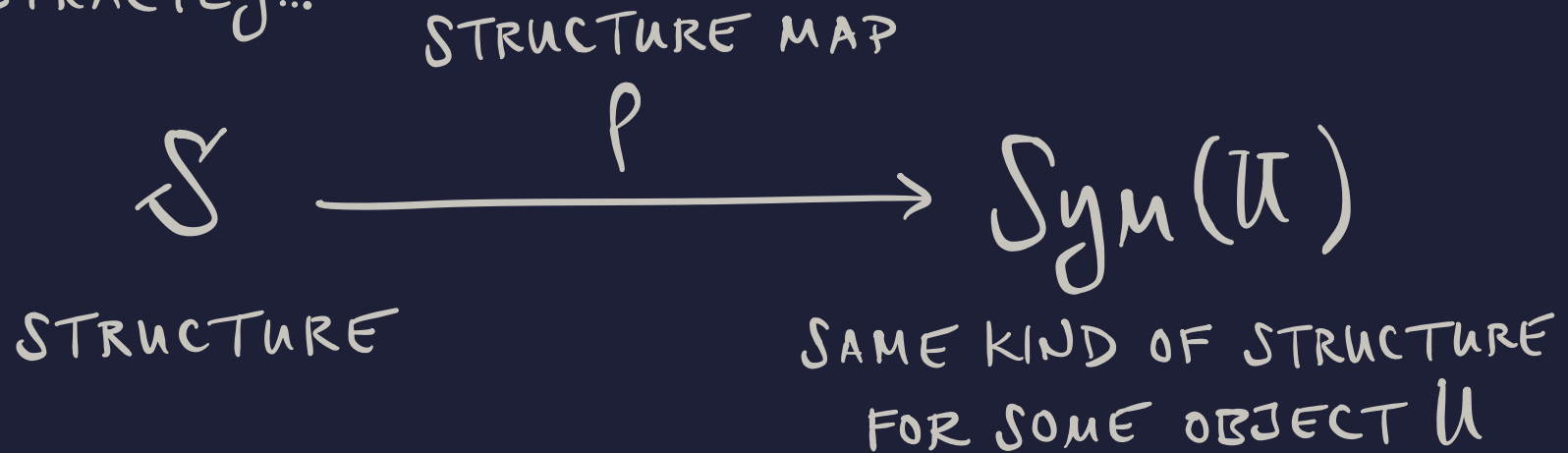
ME



WE ALL LIKE REP THEORY
SO THIS SHOULD WORK

REPRESENTATIONS

ABSTRACTLY...



Ex.

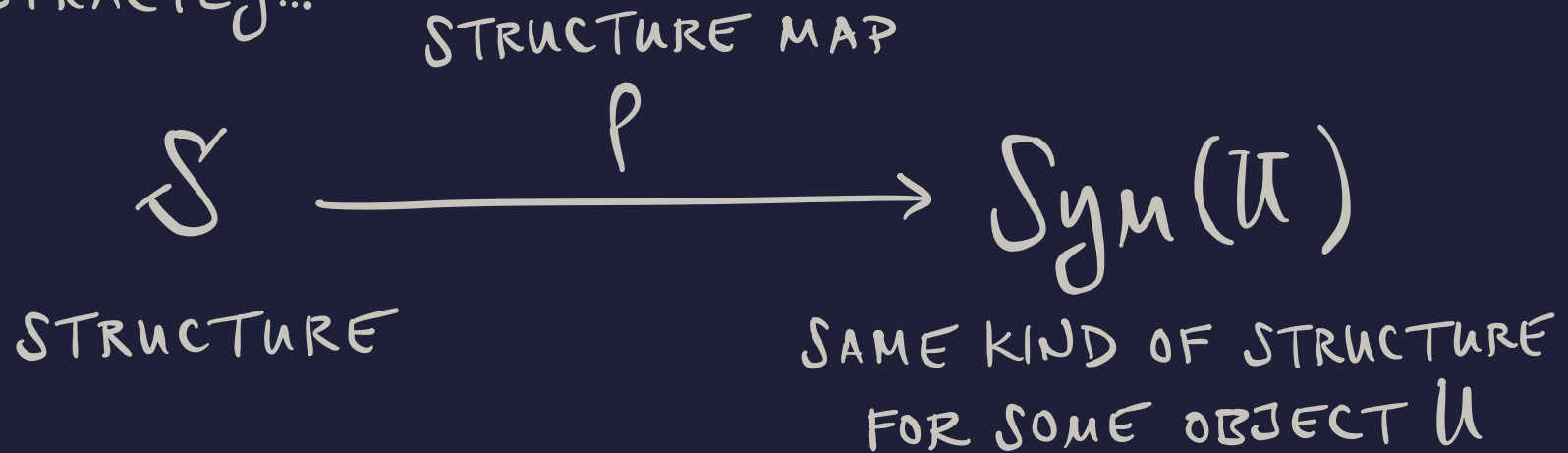


ME

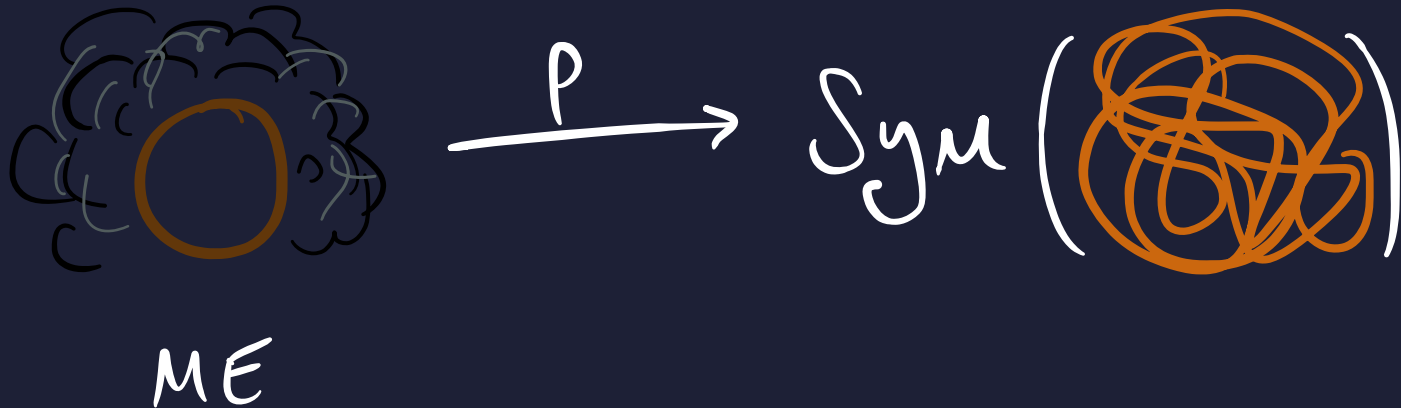


REPRESENTATIONS

ABSTRACTLY...

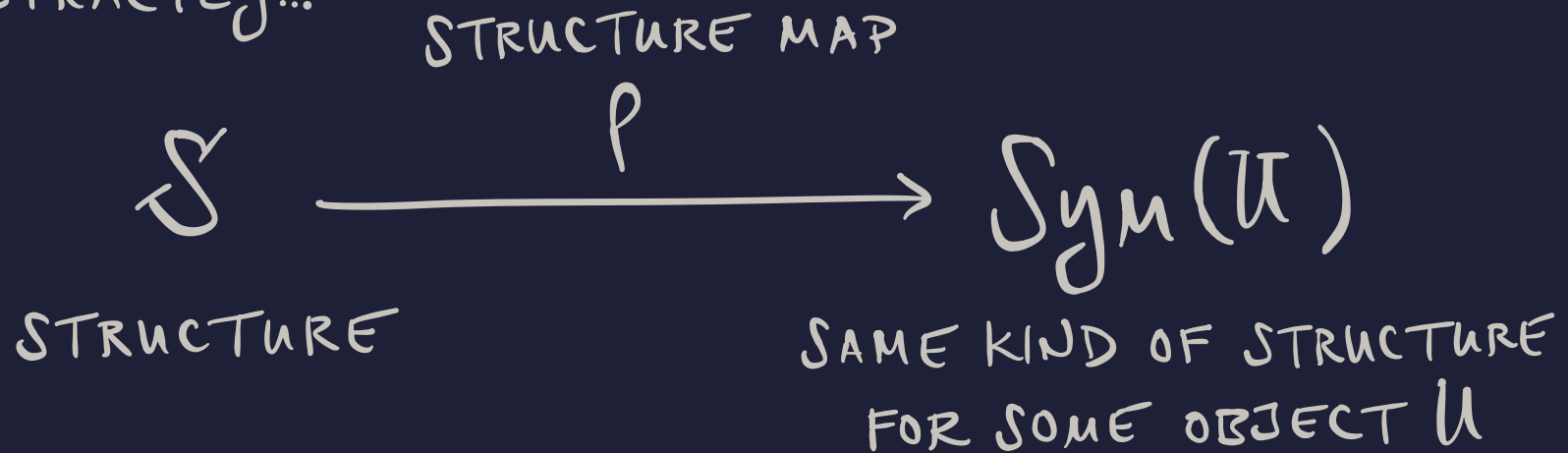


Ex.

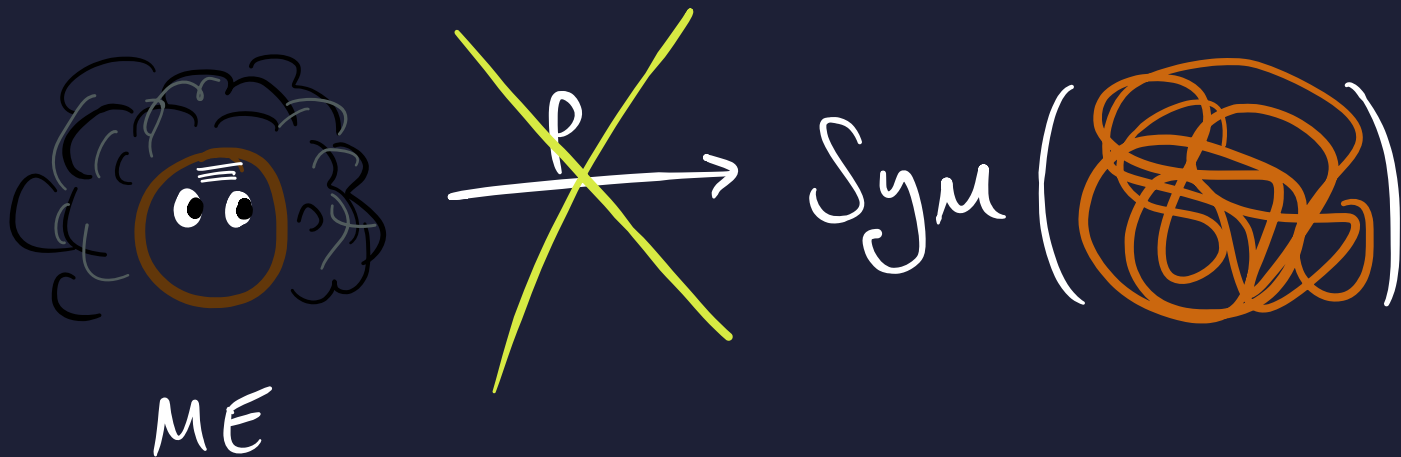


REPRESENTATIONS

ABSTRACTLY...



Ex.



REPRESENTATIONS



STRUCTURE MAP

$$\mathcal{S} \xrightarrow{\quad \rho \quad} \text{Sym}(U)$$

STRUCTURE

SAME KIND OF STRUCTURE
FOR SOME OBJECT U

REPRESENTATIONS

BRAIDED $\otimes$ CAT.

STRUCTURE MAP



$\mathcal{S}$

ρ

$\text{Sym}(\mathcal{U})$

STRUCTURE

SAME KIND OF STRUCTURE
FOR SOME OBJECT $\mathcal{U}$

REPRESENTATIONS

BRAIDED $\otimes$ CAT.

STRUCTURE MAP



$\mathcal{S}$

ρ

$\text{Sym}(\mathcal{U})$

STRUCTURE

SAME KIND OF STRUCTURE
FOR SOME OBJECT $\mathcal{U}$

SHIFT
FRAMEWORK



$$\mathcal{S} \times \mathcal{U} \xrightarrow{\Delta} \mathcal{U}$$

MODULES

REPRESENTATIONS

BRAIDED $\otimes$ CAT.

STRUCTURE MAP

$$\mathcal{S} \xrightarrow[\substack{s \mapsto p_s: \mathcal{U} \rightarrow \mathcal{U}}]{p} \text{Sym}(\mathcal{U})$$

STRUCTURE

SAME KIND OF STRUCTURE
FOR SOME OBJECT $\mathcal{U}$

SHIFT
FRAMEWORK

$$\mathcal{S} \times \mathcal{U} \xrightarrow{\triangleright} \mathcal{U}$$

$$(s, u) \mapsto p_s(u)$$

MODULES

REPRESENTATIONS

BRAIDED $\otimes$ CAT.

STRUCTURE MAP

$$\mathcal{S} \xrightarrow[\substack{s \mapsto p_s: \mathcal{U} \rightarrow \mathcal{U}}]{p} \text{Sym}(\mathcal{U})$$

STRUCTURE

SAME KIND OF STRUCTURE
FOR SOME OBJECT $\mathcal{U}$

SHIFT
FRAMEWORK

$$\mathcal{S} \times \mathcal{U} \xrightarrow{\triangleright} \mathcal{U}$$

$$(s, u) \mapsto p_s(u)$$

MODULES

↑
ALWAYS LEFT HERE

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{S} = \text{MONOIDAL CATEGORY}$

$$(\mathcal{C}, \otimes: \mathcal{C} \times \mathcal{C} \longrightarrow \mathcal{C}, \mathbb{1})$$

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{S} =$ MONOIDAL CATEGORY

$\left(\underset{\substack{\uparrow \\ \text{CATEGORY}}}{\mathcal{C}}, \otimes: \mathcal{C} \times \mathcal{C} \longrightarrow \mathcal{C}, \mathbb{1} \right)$

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{S} =$ MONOIDAL CATEGORY

$(\mathcal{C}, \otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, 1)$
CATEGORY $\uparrow$ BIFUNCTOR

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{S} =$ MONOIDAL CATEGORY

$(\mathcal{C}, \otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, \mathbb{1})$
CATEGORY $\uparrow$ BIFUNCTOR $\uparrow$ DISTINGUISHED OBJECT IN $\mathcal{C}$

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C}$ = MONOIDAL CATEGORY

$(\mathcal{C}, \otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, \mathbb{1})$
CATEGORY BIFUNCTOR DISTINGUISHED OBJECT IN $\mathcal{C}$

... THAT MIMICS A MONOID

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C}$ = MONOIDAL CATEGORY

$$\left(\underset{\substack{\uparrow \\ \text{CATEGORY}}}{\mathcal{C}}, \underset{\substack{\nwarrow \\ \text{BIFUNCTOR}}}{\otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}}, \underset{\substack{\uparrow \\ \text{DISTINGUISHED} \\ \text{OBJECT IN } \mathcal{C}}}{1} \right)$$

... THAT MIMICS A MONOID

$$\text{Ex. } \left(H\text{-Mod}_k, \otimes = \otimes_k, 1 = k \right)$$

FOR: $H = (H, m, u, \Delta, \varepsilon)$ BIALGEBRA

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C}$ = MONOIDAL CATEGORY

$$\left(\underset{\substack{\uparrow \\ \text{CATEGORY}}}{\mathcal{C}}, \underset{\substack{\uparrow \\ \text{BIFUNCTOR}}}{\otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}}, \underset{\substack{\uparrow \\ \text{DISTINGUISHED} \\ \text{OBJECT IN } \mathcal{C}}}{1} \right)$$

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$$\text{Ex. } \left(\text{H-Mod}_{\mathbb{k}}, \otimes = \otimes_{\mathbb{k}}, 1 = \mathbb{k} \right)$$

$$\text{FOR: } H = \left(\overbrace{H, m, u}^{\text{ALGEBRA}}, \Delta, \varepsilon \right) \text{ BIALGEBRA}$$

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C}$ = MONOIDAL CATEGORY

$$\left(\underset{\substack{\uparrow \\ \text{CATEGORY}}}{\mathcal{C}}, \underset{\substack{\uparrow \\ \text{BIFUNCTOR}}}{\otimes}: \mathcal{C} \times \mathcal{C} \longrightarrow \mathcal{C}, \underset{\substack{\uparrow \\ \text{DISTINGUISHED} \\ \text{OBJECT IN } \mathcal{C}}}{1} \right)$$

... THAT MIMICS A MONOID

$$\text{Ex. } \left(H\text{-Mod}_k, \underset{\substack{\uparrow \\ \text{COMULTIPIN}}}{\otimes} = \otimes_k, 1 = k \right)$$

FOR: $H = (H, m, u, \Delta, \varepsilon)$ BIALGEBRA

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C}$ = MONOIDAL CATEGORY

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$$\text{Ex. } \left(H\text{-Mod}_{\mathbb{K}}, \otimes = \otimes_{\mathbb{K}}, 1 = \mathbb{K} \right)$$

$$\text{FOR: } H = (H, m, u, \overset{\text{COUNIT}}{\Delta}, \varepsilon) \text{ BIALGEBRA}$$

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C}$ = MONOIDAL CATEGORY

$$\left(\underset{\substack{\uparrow \\ \text{CATEGORY}}}{\mathcal{C}}, \underset{\substack{\uparrow \\ \text{BIFUNCTOR}}}{\otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}}, \underset{\substack{\uparrow \\ \text{DISTINGUISHED} \\ \text{OBJECT IN } \mathcal{C}}}{1} \right)$$

... THAT MIMICS A MONOID

$$\text{Ex. } \left(H\text{-Mod}_k, \otimes = \otimes_k, 1 = k \right)$$

FOR: $H = (H, m, u, \Delta, \varepsilon)$ BIALGEBRA

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C} = \text{RIGID MONOIDAL CATEGORY}$

$\left(\underset{\substack{\uparrow \\ \text{CATEGORY}}}{\mathcal{C}}, \underset{\substack{\nwarrow \\ \text{BIFUNCTOR}}}{\otimes}: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, \underset{\substack{\uparrow \\ \text{DISTINGUISHED OBJECT IN } \mathcal{C}}}{1\mathbb{L}} \right)$

... WITH DUAL OBJECTS

$(X^*, \text{ev}_X^L: X^* \otimes X \rightarrow 1\mathbb{L}, \text{coev}_X^L: 1\mathbb{L} \rightarrow X \otimes X^*)$

$(*X, \text{ev}_X^R: X \otimes *X \rightarrow 1\mathbb{L}, \text{coev}_X^R: 1\mathbb{L} \rightarrow *X \otimes X)$

Ex. $(H\text{-Mod}_{1k}, \otimes = \otimes_{1k}, 1\mathbb{L} = 1k)$

FOR: $H = (H, m, u, \Delta, \varepsilon)$ BIALGEBRA

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C} = \overset{\text{RIGID}}{\text{MONOIDAL CATEGORY}}$

$\left(\underset{\substack{\uparrow \\ \text{CATEGORY}}}{\mathcal{C}}, \underset{\substack{\uparrow \\ \text{BIFUNCTOR}}}{\otimes}: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, \underset{\substack{\uparrow \\ \text{DISTINGUISHED OBJECT IN } \mathcal{C}}}{1\mathbb{L}} \right)$

... WITH DUAL OBJECTS

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$(*X, \text{ev}_X^R: X \otimes *X \rightarrow 1\mathbb{L}, \text{coev}_X^R: 1\mathbb{L} \rightarrow *X \otimes X)$

Ex. $\left(H\text{-Mod}_{\mathbb{K}}, \otimes = \otimes_{\mathbb{K}}, 1\mathbb{L} = \mathbb{K} \right)$
+ DUALS

FOR: $H = (\underset{\substack{\uparrow \\ \text{HOPF}}}{H}, m, u, \Delta, \underset{\substack{\uparrow \\ \text{BIALGEBRA}}}{\varepsilon})$

REPRESENTATIONS OF BRAIDED CATEGORIES

• STRICT CASE •

$\mathcal{C} = \text{MONOIDAL CATEGORY}$

$$(\mathcal{C}, \otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, \mathbb{1})$$

GRAPHICAL
CALCULUS

$$\text{Ex. } (\mathcal{H}\text{-Mod}_{\mathbb{k}}, \otimes = \otimes_{\mathbb{k}}, \mathbb{1} = \mathbb{k})$$

FOR: $\mathcal{H} = (\mathcal{H}, m, u, \Delta, \varepsilon)$ BIALGEBRA

REPRESENTATIONS OF BRAIDED CATEGORIES

• STRICT CASE •

$\mathcal{C}$ = MONOIDAL CATEGORY

$$(\mathcal{C}, \otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, \mathbb{1})$$

GRAPHICAL
CALCULUS

x

$x \in \mathcal{C}$

$$\text{Ex. } (\mathcal{H}\text{-Mod}_{\mathbb{k}}, \otimes = \otimes_{\mathbb{k}}, \mathbb{1} = \mathbb{k})$$

FOR: $\mathcal{H} = (\mathcal{H}, m, u, \Delta, \varepsilon)$ BIALGEBRA

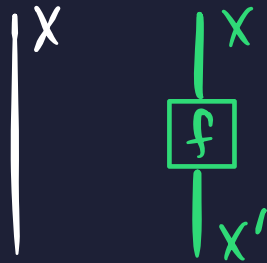
REPRESENTATIONS OF BRAIDED CATEGORIES

• STRICT CASE •

$\mathcal{C}$ = MONOIDAL CATEGORY

$$(\mathcal{C}, \otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, \mathbb{1})$$

GRAPHICAL
CALCULUS
(READ DOWN)



$$X \in \mathcal{C}$$

$$f: X \rightarrow X' \in \mathcal{C}$$

Ex. $(H\text{-Mod}_{\mathbb{k}}, \otimes = \otimes_{\mathbb{k}}, \mathbb{1} = \mathbb{k})$

FOR: $H = (H, m, u, \Delta, \varepsilon)$ BIALGEBRA

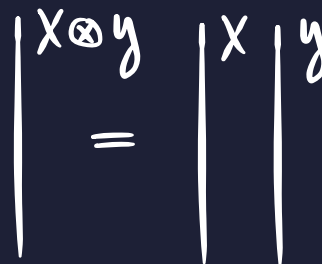
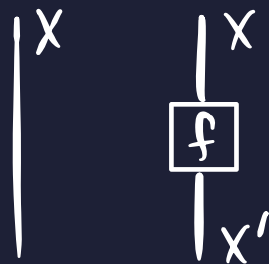
REPRESENTATIONS OF BRAIDED CATEGORIES

• STRICT CASE •

$\mathcal{C}$ = MONOIDAL CATEGORY

$$(\mathcal{C}, \otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, \mathbb{1})$$

GRAPHICAL
CALCULUS
(READ DOWN)



$X, Y \in \mathcal{C}$
 $f: X \rightarrow X' \in \mathcal{C}$

Ex. $(H\text{-Mod}_{\mathbb{k}}, \otimes = \otimes_{\mathbb{k}}, \mathbb{1} = \mathbb{k})$

FOR: $H = (H, m, u, \Delta, \varepsilon)$ BIALGEBRA

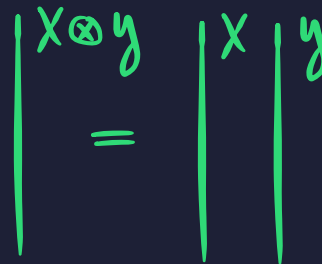
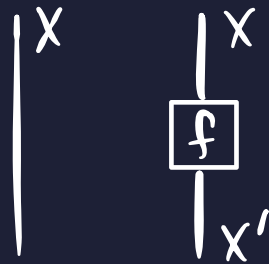
REPRESENTATIONS OF BRAIDED CATEGORIES

• STRICT CASE •

$\mathcal{C}$ = MONOIDAL CATEGORY

$$(\mathcal{C}, \otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, \mathbb{1})$$

GRAPHICAL
CALCULUS
(READ DOWN)



$X, Y \in \mathcal{C}$
 $f: X \rightarrow X' \in \mathcal{C}$

Ex. $(H\text{-Mod}_k, \otimes = \otimes_k, \mathbb{1} = k)$

FOR: $H = (H, m, u, \Delta, \varepsilon)$ BIALGEBRA

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C}$ = MONOIDAL CATEG.

$$(\mathcal{C}, \otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, \mathbb{1})$$

GRAPHICAL CALCULUS

$$\begin{array}{c} |^X \\ | \\ \boxed{f} \\ | \\ |^{X'} \end{array} \quad \begin{array}{c} |^{\mathbb{1}} \\ | \\ | \\ | \end{array} \quad \begin{array}{c} |^{X \otimes y} \\ | \\ = \\ |^X |^y \end{array}$$

$$X, y \in \mathcal{C} \quad f: X \rightarrow X' \in \mathcal{C}$$

Ex.

$$(\mathcal{H}\text{-Mod}_{\mathbb{k}}, \otimes_{\mathbb{k}}, \mathbb{k})$$

$$\mathcal{H} = (\mathcal{H}, m, u, \Delta, \varepsilon)$$

BIALGEBRA

$\mathcal{U}$ = $\mathcal{C}$ -MODULE CATEGORY

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C}$ = MONOIDAL CATER.

$$(\mathcal{C}, \otimes : \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, \mathbb{1})$$

GRAPHICAL CALCULUS

$$\begin{array}{c} |^X \\ | \\ \boxed{f} \\ | \\ |^{X'} \end{array} \quad \begin{array}{c} |^{\mathbb{1}} \\ | \\ | \end{array} \quad \begin{array}{c} |^{X \otimes y} \\ | \\ = \\ |^X |^y \end{array}$$

$$X, y \in \mathcal{C} \quad f: X \rightarrow X' \in \mathcal{C}$$

Ex.

$$(\mathcal{H}\text{-Mod}_{\mathbb{k}}, \otimes_{\mathbb{k}}, \mathbb{k})$$

$$\mathcal{H} = (\mathcal{H}, m, u, \Delta, \varepsilon)$$

BIALGEBRA

$\mathcal{U}$ = $\mathcal{C}$ -MODULE CATEGORY

$$(\mathcal{U}, \triangleright : \mathcal{C} \times \mathcal{U} \rightarrow \mathcal{U})$$

$\uparrow$ CATEGORY $\uparrow$ BIFUNCTOR

... LIKE A MODULE OVER A MONOID

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C}$ = MONOIDAL CATER.

$$(\mathcal{C}, \otimes : \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, \mathbb{1})$$

GRAPHICAL CALCULUS

$$\begin{array}{c} |^X \\ |^X \\ \boxed{f} \\ |_{X'} \end{array} \quad \begin{array}{c} |^{\mathbb{1}} \\ \vdots \end{array} \quad \begin{array}{c} |^{X \otimes y} \\ = \\ |^X |^y \end{array}$$

$$X, y \in \mathcal{C} \quad f: X \rightarrow X' \in \mathcal{C}$$

Ex.

$$(\mathcal{H}\text{-Mod}_{\mathbb{k}}, \otimes_{\mathbb{k}}, \mathbb{k})$$

$$\mathcal{H} = (\mathcal{H}, m, u, \Delta, \varepsilon)$$

BIALGEBRA

$\mathcal{U}$ = $\mathcal{C}$ -MODULE CATEGORY

$$(\mathcal{U}, \triangleright : \mathcal{C} \times \mathcal{U} \rightarrow \mathcal{U})$$

$\uparrow$ CATEGORY $\uparrow$ BIFUNCTOR

... LIKE A MODULE OVER A MONOID

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C}$ = MONOIDAL CATER.

$$(\mathcal{C}, \otimes : \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, \mathbb{1})$$

GRAPHICAL CALCULUS

$$\begin{array}{c} |^X \\ | \\ \boxed{f} \\ | \\ |^{X'} \end{array} \quad \begin{array}{c} |^{\mathbb{1}} \\ | \\ | \end{array} \quad \begin{array}{c} |^{X \otimes Y} \\ | \\ = \\ |^X |^Y \end{array}$$

$$X, Y \in \mathcal{C} \quad f: X \rightarrow X' \in \mathcal{C}$$

Ex.

$$(\mathcal{H}\text{-Mod}_{\mathbb{K}}, \otimes_{\mathbb{K}}, \mathbb{K})$$

$$\mathcal{H} = (\mathcal{H}, m, u, \Delta, \varepsilon)$$

BIALGEBRA

$\mathcal{U}$ = $\mathcal{C}$ -MODULE CATEGORY

$$(\mathcal{U}, \triangleright : \mathcal{C} \times \mathcal{U} \rightarrow \mathcal{U})$$

$\uparrow$ CATEGORY $\uparrow$ BIFUNCTOR

... LIKE A MODULE OVER A MONOID

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C}$ = MONOIDAL CATEG.

$$(\mathcal{C}, \otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, \mathbb{1})$$

GRAPHICAL CALCULUS

$$\begin{array}{c} |^X \\ | \\ \boxed{f} \\ | \\ |^{X'} \end{array} \quad \begin{array}{c} |^{\mathbb{1}} \\ | \\ | \end{array} \quad \begin{array}{c} |^{X \otimes Y} \\ | \\ = \\ |^X |^Y \end{array}$$

$$X, Y \in \mathcal{C} \quad f: X \rightarrow X' \in \mathcal{C}$$

Ex.

$$(\mathcal{H}\text{-Mod}_{\mathbb{k}}, \otimes_{\mathbb{k}}, \mathbb{k})$$

$$\mathcal{H} = (\mathcal{H}, m, u, \Delta, \varepsilon)$$

BIALGEBRA

$\mathcal{U} = \mathcal{C}\text{-MODULE CATEGORY}$

$$\begin{array}{c} \mathcal{U} \\ \uparrow \\ \text{CATEGORY} \end{array}, \quad \begin{array}{c} \triangleright: \mathcal{C} \times \mathcal{U} \rightarrow \mathcal{U} \\ \uparrow \\ \text{BIFUNCTOR} \end{array}$$

... LIKE A MODULE OVER A MONOID

$$\text{Ex. } (\mathcal{A}\text{-Mod}_{\mathbb{k}}, \triangleright = \otimes_{\mathbb{k}})$$

$$\text{FOR: } \mathcal{A} = (\mathcal{A}, m, u, \delta: \mathcal{A} \rightarrow \mathcal{H} \otimes \mathcal{A})$$

LEFT $\mathcal{H}$ -COMODULE ALGEBRA

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C}$ = MONOIDAL CATEG.

$$(\mathcal{C}, \otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, \mathbb{1})$$

GRAPHICAL CALCULUS

$$\begin{array}{c} |^X \\ | \\ \boxed{f} \\ | \\ |^{X'} \end{array} \quad \begin{array}{c} |^{\mathbb{1}} \\ | \\ | \end{array} \quad \begin{array}{c} |^{X \otimes Y} \\ | \\ = \\ |^X |^Y \end{array}$$

$$X, Y \in \mathcal{C} \quad f: X \rightarrow X' \in \mathcal{C}$$

Ex.

$$(\mathcal{H}\text{-Mod}_{\mathbb{k}}, \otimes_{\mathbb{k}}, \mathbb{k})$$

$$\mathcal{H} = (\mathcal{H}, m, u, \Delta, \varepsilon)$$

BIALGEBRA

$\mathcal{U} = \mathcal{C}\text{-MODULE CATEGORY}$

$$\left(\underset{\substack{\uparrow \\ \text{CATEGORY}}}{\mathcal{U}}, \underset{\substack{\uparrow \\ \text{BIFUNCTOR}}}{\triangleright: \mathcal{C} \times \mathcal{U} \rightarrow \mathcal{U}} \right)$$

... LIKE A MODULE OVER A MONOID

$$\text{Ex. } (\mathcal{A}\text{-Mod}_{\mathbb{k}}, \triangleright = \otimes_{\mathbb{k}})$$

$$\text{FOR: } \mathcal{A} = (\underbrace{\mathcal{A}, m, u}_{\text{ALGEBRA}}, \delta: \mathcal{A} \rightarrow \mathcal{H} \otimes \mathcal{A})$$

LEFT $\mathcal{H}$ -COMODULE ALGEBRA

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C}$ = MONOIDAL CATEG.

$$(\mathcal{C}, \otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, \mathbb{1})$$

GRAPHICAL CALCULUS

$$\begin{array}{c} |^X \\ |^X \\ \boxed{f} \\ |_{X'} \end{array} \quad \begin{array}{c} |^{\mathbb{1}} \\ \vdots \end{array} \quad \begin{array}{c} |^{X \otimes y} \\ = \\ |^X |^y \end{array}$$

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Ex.

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BIALGEBRA

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$$\text{FOR: } \mathcal{A} = (\mathcal{A}, m, u, \overset{\text{COACTION}}{\delta}: \mathcal{A} \rightarrow \mathcal{H} \otimes \mathcal{A})$$

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REPRESENTATIONS OF BRAIDED CATEGORIES

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$\mathcal{U} = \mathcal{C}\text{-MODULE CATEGORY}$

$$(\mathcal{U}, \triangleright: \mathcal{C} \times \mathcal{U} \rightarrow \mathcal{U})$$

GRAPHICAL CALCULUS



$$X, Y \in \mathcal{C} \quad f: X \rightarrow X' \in \mathcal{C}$$

Ex.

$$(\mathcal{H}\text{-Mod}_{\mathbb{K}}, \otimes_{\mathbb{K}}, \mathbb{K})$$

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BIALGEBRA

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LEFT $\mathcal{H}$ -COMODULE ALGEBRA

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$$(\mathcal{U}, \triangleright: \mathcal{C} \times \mathcal{U} \rightarrow \mathcal{U})$$

GRAPHICAL CALCULUS



$$X, Y \in \mathcal{C} \quad f: X \rightarrow X' \in \mathcal{C}$$

M

$$M \in \mathcal{U}$$

Ex.

$$(\mathcal{H}\text{-Mod}_{\mathbb{k}}, \otimes_{\mathbb{k}}, \mathbb{k})$$

$$\mathcal{H} = (\mathcal{H}, m, u, \Delta, \varepsilon)$$

BIALGEBRA

$$\text{Ex. } (\mathcal{A}\text{-Mod}_{\mathbb{k}}, \triangleright = \otimes_{\mathbb{k}})$$

$$\text{FOR: } \mathcal{A} = (\mathcal{A}, m, u, \delta: \mathcal{A} \rightarrow \mathcal{H} \otimes \mathcal{A})$$

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GRAPHICAL CALCULUS



$$X, Y \in \mathcal{C} \quad f: X \rightarrow X' \in \mathcal{C}$$

Ex.

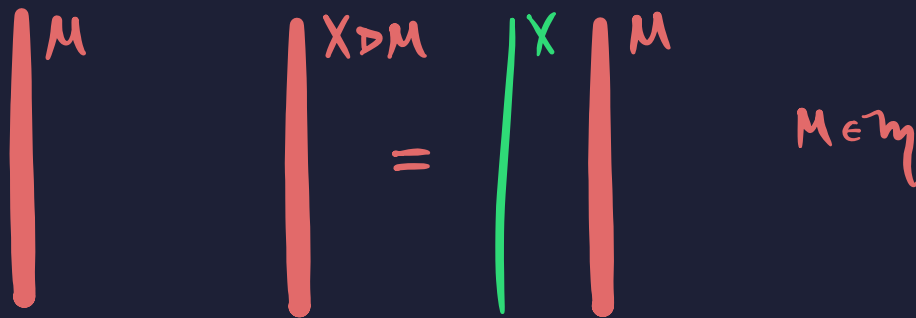
$$(\mathcal{H}\text{-Mod}_{\mathbb{k}}, \otimes_{\mathbb{k}}, \mathbb{k})$$

$$\mathcal{H} = (\mathcal{H}, m, u, \Delta, \varepsilon)$$

BIALGEBRA

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$$(\mathcal{U}, \triangleright: \mathcal{C} \times \mathcal{U} \rightarrow \mathcal{U})$$



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LEFT $\mathcal{H}$ -COMODULE ALGEBRA

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{S}$ = BRAIDED MONOIDAL CATEGORY
 $(\mathcal{C}, \otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, \mathbb{1}, c = \{c_{x,y}: x \otimes y \xrightarrow{\sim} y \otimes x\})$
 $\uparrow_{\text{NAT'L } \cong} : \text{BRAIDING}$

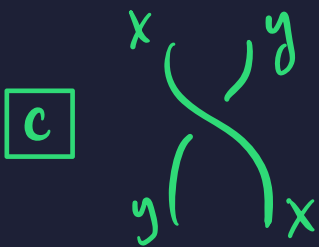
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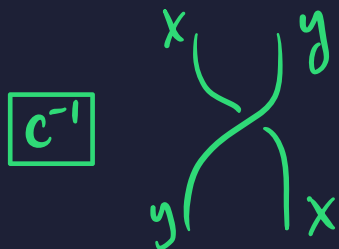
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 $\uparrow_{\text{NAT'L } \cong} : \text{BRAIDING}$

$x, y \in \mathcal{C}$



SUBJECT
TO

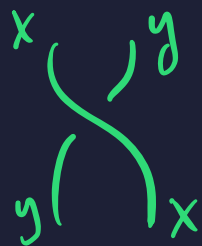


REPRESENTATIONS OF BRAIDED CATEGORIES

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 $(\mathcal{C}, \otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, \mathbb{1}, c = \{c_{x,y}: x \otimes y \xrightarrow{\sim} y \otimes x\})$
 $\uparrow_{\text{NAT'L } \cong} : \text{BRAIDING}$

$x, y \in \mathcal{C}$

$\boxed{c}$



SUBJECT
TO

$\boxed{c^{-1}}$



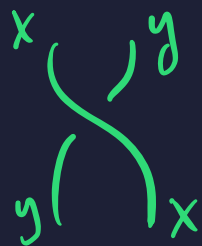
$$\begin{array}{c} x \otimes y \quad z \\ \diagdown \quad \diagup \\ \quad \quad x \otimes y \end{array} = \begin{array}{c} x \quad y \quad z \\ \diagdown \quad \diagup \\ \quad \quad x \parallel y \end{array}$$

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C}$ = BRAIDED MONOIDAL CATEGORY
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 $\uparrow_{\text{NAT'L } \cong} : \text{BRAIDING}$

$x, y \in \mathcal{C}$

$\boxed{c}$



SUBJECT
TO

$\boxed{c^{-1}}$



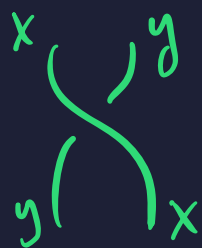
$$\begin{array}{c} x \otimes y \quad z \\ \diagdown \quad \diagup \\ \quad \quad \quad \\ \diagup \quad \diagdown \\ z \quad x \otimes y \end{array} = \begin{array}{c} x \quad y \quad z \\ \diagdown \quad \diagup \quad \diagdown \\ \quad \quad \quad \\ \diagup \quad \diagdown \quad \diagup \\ z \quad x \quad y \end{array} = \begin{array}{c} x \quad y \quad z \\ \diagdown \quad \diagup \quad \diagdown \\ \quad \quad \quad \\ \diagup \quad \diagdown \quad \diagup \\ z \quad x \quad y \end{array}$$

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C}$ = BRAIDED MONOIDAL CATEGORY
 $(\mathcal{C}, \otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}, \mathbb{1}, c = \{c_{x,y}: X \otimes y \xrightarrow{\sim} y \otimes x\})$
 $\uparrow_{\text{NAT'L } \cong} : \text{BRAIDING}$

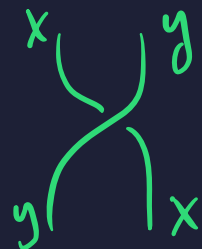
$x, y \in \mathcal{C}$

$\boxed{c}$



SUBJECT
TO

$\boxed{c^{-1}}$



$$\begin{array}{c} x \otimes y \quad z \\ \diagdown \quad \diagup \\ \quad \quad \quad \\ \diagup \quad \diagdown \\ z \quad x \otimes y \end{array} = \begin{array}{c} x \quad y \quad z \\ \diagdown \quad \diagup \quad \diagdown \\ \quad \quad \quad \\ \diagup \quad \diagdown \quad \diagup \\ z \quad x \quad y \end{array} = \begin{array}{c} x \quad y \quad z \\ \diagdown \quad \diagup \quad \diagdown \\ \quad \quad \quad \\ \diagup \quad \diagdown \quad \diagup \\ z \quad x \quad y \end{array}$$

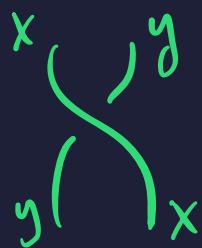
$$\begin{array}{ccc} x \otimes y \otimes z & \xrightarrow{c_{x \otimes y, z}} & z \otimes x \otimes y \\ & \searrow \text{id} \otimes c_{y, z} & \nearrow c_{x, z} \otimes \text{id} \\ & x \otimes z \otimes y & \end{array}$$

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C}$ = BRAIDED MONOIDAL CATEGORY
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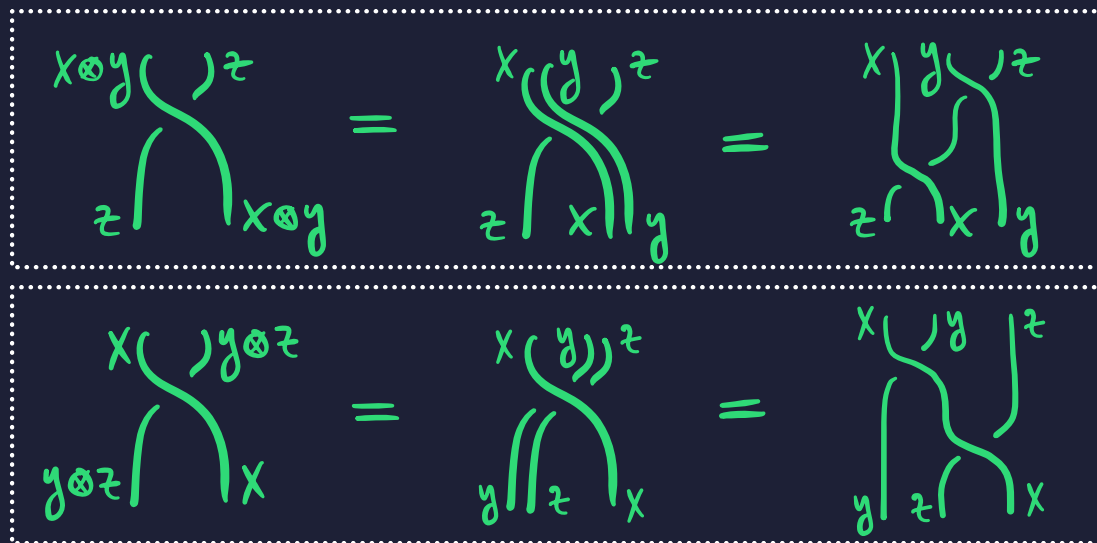
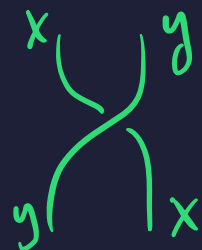
$x, y \in \mathcal{C}$

$\boxed{c}$



SUBJECT
TO

$\boxed{c^{-1}}$

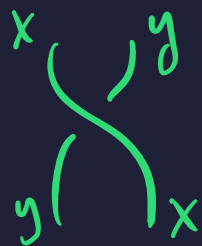


REPRESENTATIONS OF BRAIDED CATEGORIES

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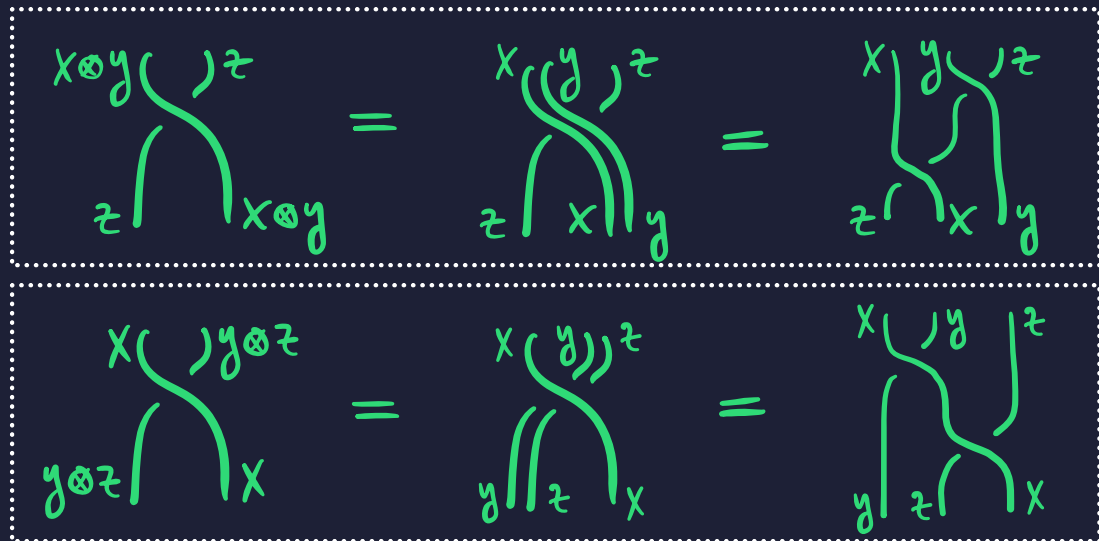
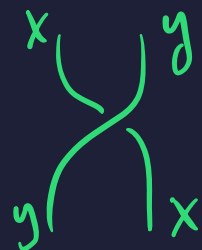
$x, y \in \mathcal{C}$

$\boxed{c}$



SUBJECT
TO

$\boxed{c^{-1}}$



Ex. $(H\text{-Mod}_k, \otimes_k, k, c)$

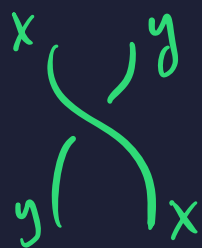
FOR: $H = (H, m, u, \Delta, \varepsilon, \quad) \quad \text{BIALG.}$

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{C}$ = BRAIDED MONOIDAL CATEGORY
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 $\uparrow_{\text{NAT'L } \cong} : \text{BRAIDING}$

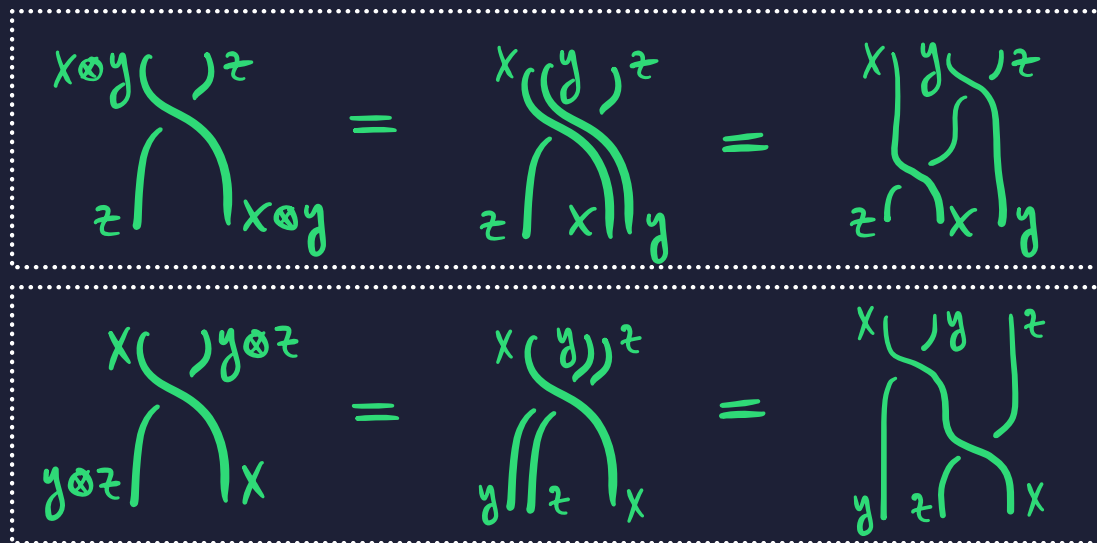
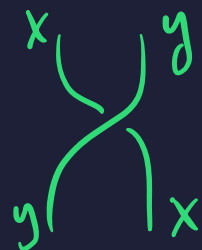
$x, y \in \mathcal{C}$

$\boxed{c}$



SUBJECT
TO

$\boxed{c^{-1}}$



Ex. $(H\text{-Mod}_k, \otimes_k, k, c)$

FOR: $H = (H, m, u, \Delta, \varepsilon, \overset{\text{R-MATRIX}}{R} \in H \otimes H)$ QUASITRI. BIALG.

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{S} = \text{BRAIDED } \otimes \text{ CATEG.}$

$$(\mathcal{C}, \otimes, \mathbb{1}, c)$$

$$c = \{c_{x,y} : x \otimes y \xrightarrow{\sim} y \otimes x\}$$

GRAPHICAL CALCULUS

$$\begin{array}{c} x \quad y \\ \diagdown \quad \diagup \\ y \quad x \\ \boxed{c} \end{array} \quad \begin{array}{c} \boxed{x \otimes y \quad z} \\ \diagdown \quad \diagup \\ z \quad x \otimes y \\ \boxed{x \otimes y \quad z} \end{array} = \begin{array}{c} x \quad y \quad z \\ \diagdown \quad \diagup \\ z \quad x \quad y \end{array}$$

$$\begin{array}{c} x \quad y \quad z \\ \diagdown \quad \diagup \\ y \quad z \quad x \end{array} = \begin{array}{c} x \quad y \quad z \\ \diagdown \quad \diagup \\ y \quad z \quad x \end{array}$$

Ex.

$$(H\text{-Mod}_{\mathbb{k}}, \otimes_{\mathbb{k}}, \mathbb{k}, c)$$

$$H = (H, m, u, \Delta, \varepsilon, \overset{\text{R-MATRIX}}{R} \in H \otimes H)$$

QUASITRI. BIALG.

$\mathcal{U} = \text{BRAIDED } \mathcal{C}\text{-MOD. CATEG.}$

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{S}$ = BRAIDED $\otimes$ CATEG.

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GRAPHICAL CALCULUS

Ex.

$$(H\text{-Mod}_{|k}, \otimes_{|k}, |k, c)$$

$$H = (H, m, u, \Delta, \varepsilon, \overset{\text{R-MATRIX}}{R} \in H \otimes H)$$

QUASITRI. BIALG.

$\mathcal{U}$ = BRAIDED $\mathcal{C}$ -MOD. CATEG.

$$(\mathcal{M}, \triangleright : \mathcal{C} \times \mathcal{M} \rightarrow \mathcal{M}, e)$$

$\mathcal{C}$ -MODULE CATEG.

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{S} = \text{BRAIDED } \otimes \text{ CATEG.}$

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GRAPHICAL CALCULUS

Ex.

$$(\mathcal{H}\text{-Mod}_{|k}, \otimes_{|k}, |k, c)$$

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QUASITRI. BIALG.

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$$(\mathcal{M}, \triangleright : \mathcal{C} \times \mathcal{M} \rightarrow \mathcal{M}, e)$$

$$e := \{e_{x,M} : x \triangleright M \xrightarrow{\sim} x \triangleright M\}$$

NAT'L $\cong$: BRAIDING

REPRESENTATIONS OF BRAIDED CATEGORIES

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GRAPHICAL CALCULUS

Ex.

$$(\mathcal{H}\text{-Mod}_{|k}, \otimes_{|k}, |k, c)$$

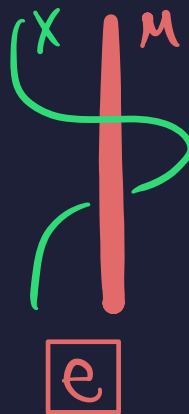
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QUASITRI. BIALG.

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SUBJ.
TO

REPRESENTATIONS OF BRAIDED CATEGORIES

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GRAPHICAL CALCULUS

Ex.

$$(\mathcal{H}\text{-Mod}_{\mathbb{K}}, \otimes_{\mathbb{K}}, \mathbb{1}_{\mathbb{K}}, c)$$

$$H = (H, m, u, \Delta, \varepsilon, \overset{\text{R-MATRIX}}{R} \in H \otimes H)$$

QUASITRI. BIALG.

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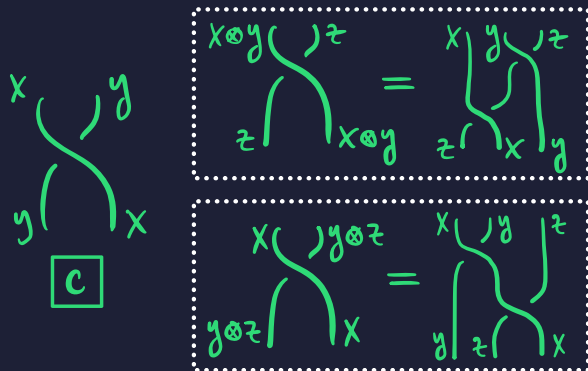
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$$(\mathcal{C}, \otimes, \mathbb{1}, c)$$

$$c = \{c_{x,y} : x \otimes y \xrightarrow{\sim} y \otimes x\}$$

GRAPHICAL CALCULUS



Ex.

$$(H\text{-Mod}_{|k|}, \otimes_{|k|}, |k|, c)$$

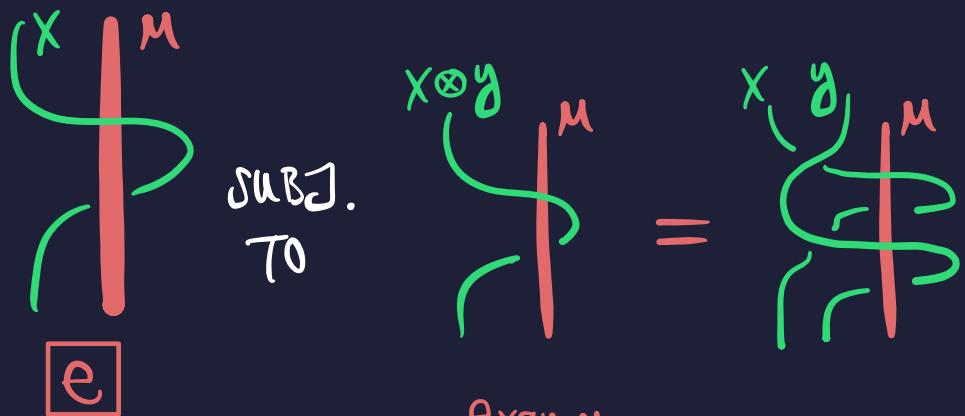
$$H = (H, m, u, \Delta, \varepsilon, R\text{-MATRIX } R \in H \otimes H)$$

QUASITRI. BIALG.

$\mathcal{U}$ = BRAIDED $\mathcal{C}$ -MOD. CATEG.

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$$\begin{array}{ccc} & x \otimes y \triangleright M & \xrightarrow{e_{x \otimes y, M}} x \otimes y \triangleright M \\ \swarrow \scriptstyle c_{y,x}^{-1} \triangleright \text{id} & & \nwarrow \scriptstyle \text{id} \otimes e_{y,M} \\ y \otimes x \triangleright M & & x \otimes y \triangleright M \\ \searrow \scriptstyle \text{id} \otimes e_{x,M} & & \nearrow \scriptstyle c_{x,y} \triangleright \text{id} \\ & y \otimes x \triangleright M & \end{array}$$

REPRESENTATIONS OF BRAIDED CATEGORIES

$\mathcal{S}$ = BRAIDED $\otimes$ CATEG.

$$(\mathcal{C}, \otimes, \mathbb{1}, c)$$

$$c = \{c_{x,y} : x \otimes y \xrightarrow{\sim} y \otimes x\}$$

GRAPHICAL CALCULUS

Ex.

$$(H\text{-Mod}_{\mathbb{k}}, \otimes_{\mathbb{k}}, \mathbb{k}, c)$$

$$H = (H, m, u, \Delta, \varepsilon, \overset{\text{R-MATRIX}}{R} \in H \otimes H)$$

QUASITRI. BIALG.

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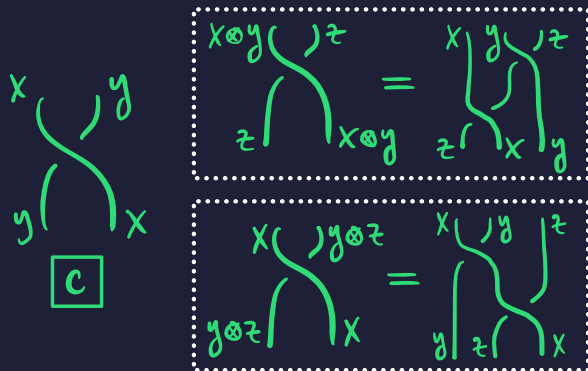
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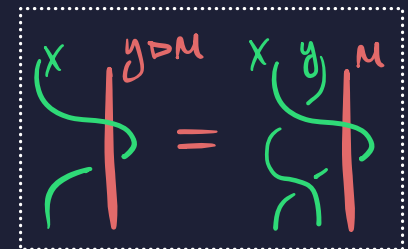
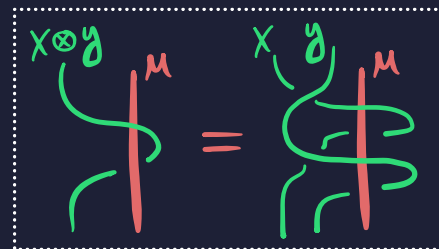
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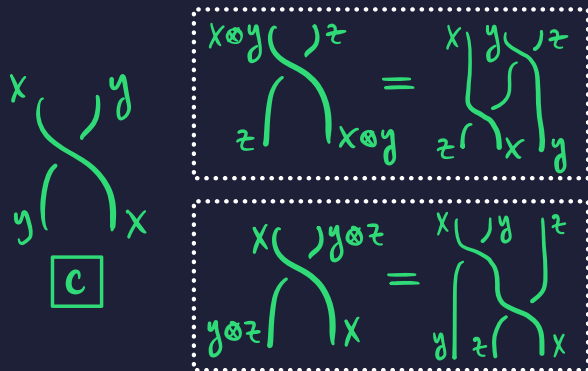
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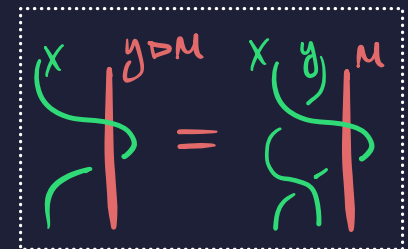
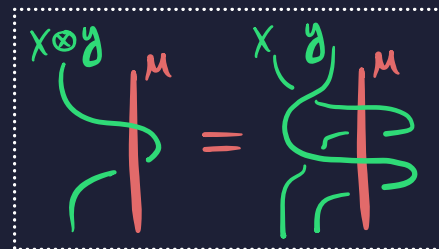
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QUASITRI. LEFT H-COMODULE ALGEBRA

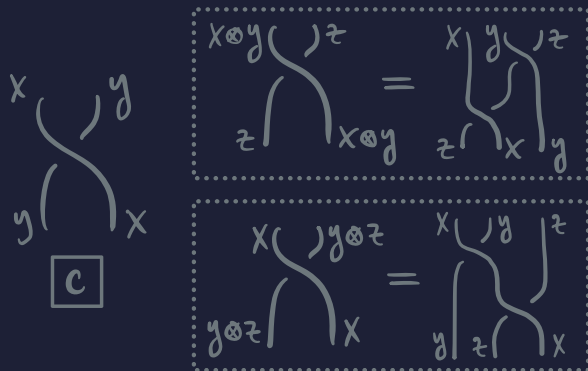
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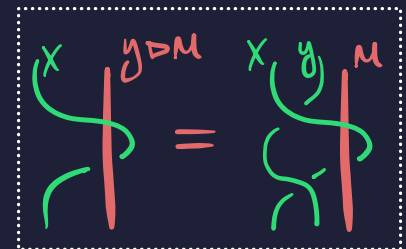
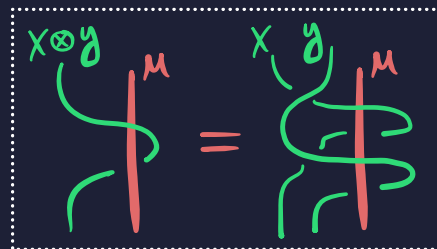
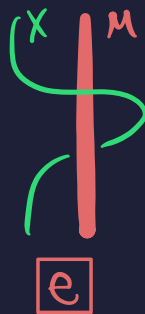
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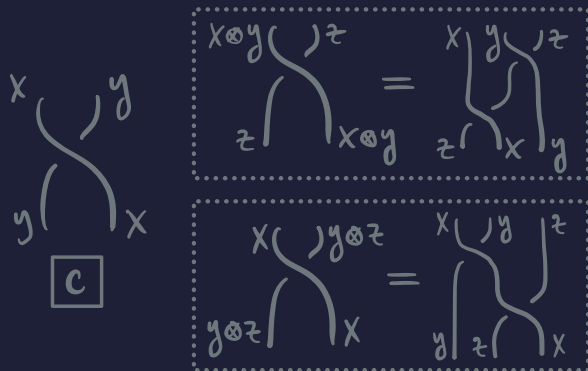
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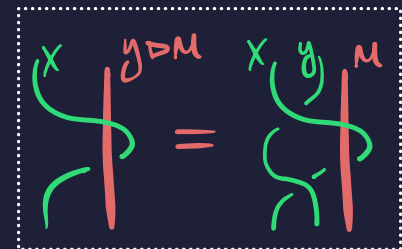
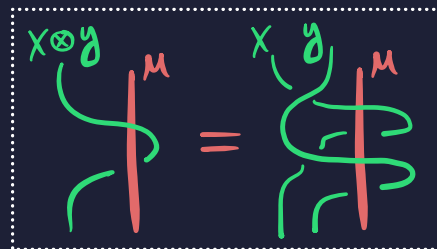
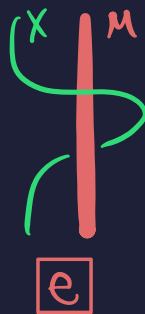
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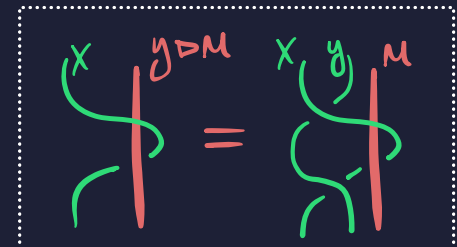
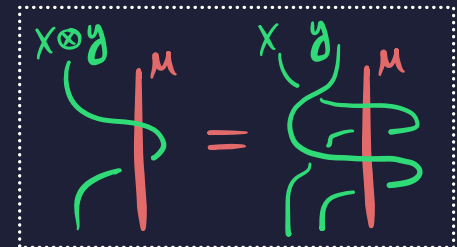
REPRESENTATIONS OF BRAIDED CATEGORIES

$$\mathcal{C} := (\mathcal{C}, \otimes, \mathbb{1}, c)$$

BRAIDED $\mathcal{C}$ -MOD. CATEG.

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Ex. $(A\text{-Mod}_{\mathbb{k}}, \triangleright = \otimes_{\mathbb{k}}, e)$

For: $A = (A, m, u, \delta, K \in H \otimes A)$ ^{K-MATRIX}

QUASITRI. LEFT H -COMODULE ALG

REPRESENTATIONS OF BRAIDED CATEGORIES

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EXAMPLE: REGULAR $\mathcal{C}$ -MOD. CAT.

$$\mathcal{M} := \mathcal{C}, \quad \triangleright := \otimes, \quad e := c^2$$

$$\boxed{e} \quad \begin{array}{c} x \\ \text{---} \\ m \end{array} := \begin{array}{c} x \\ \text{---} \\ m \end{array} \quad x, m \in \mathcal{C}$$

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$$\boxed{e} \quad \begin{array}{c} x \\ \text{---} \\ y \triangleright m \end{array} = \begin{array}{c} x \quad y \\ \text{---} \\ m \end{array}$$

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- VERIFY FOR ENTERTAINMENT.

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QUASITRI. LEFT H -COMODULE ALG

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WHY CARE?

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YIELD SOLINS TO THE
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(LIKE R-MATRICES $\leadsto$ SOLINS TO YBE)

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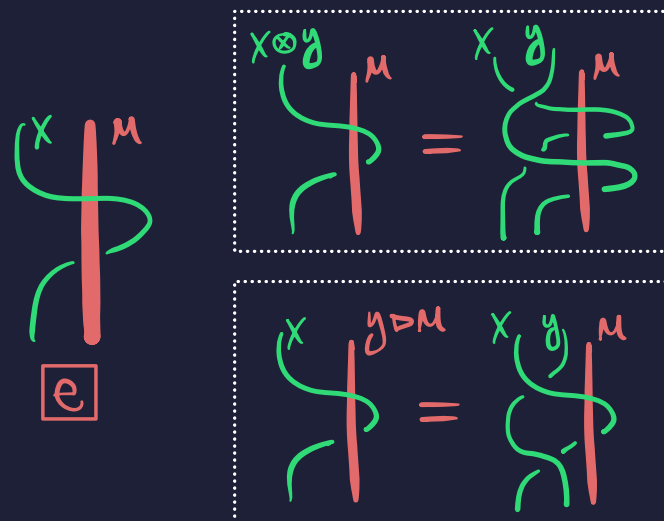
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[DAVYDOV - NIKSHYCH]

$\exists$ 2-EQUIVALENCE:

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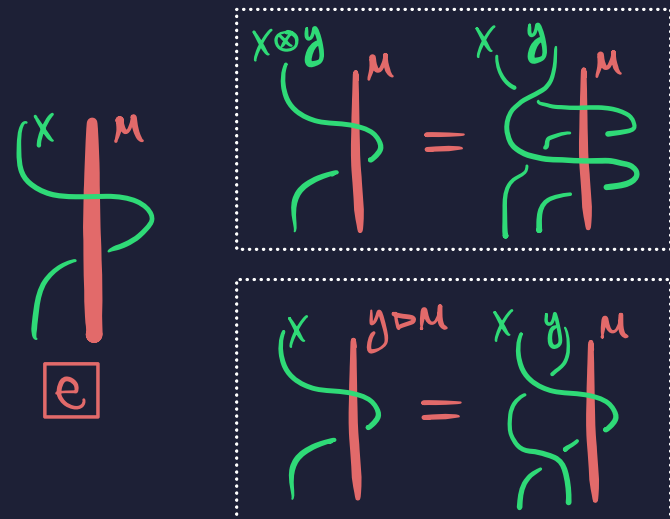
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$\uparrow$
2-CAT. OF BRAIDED $\mathcal{C}$ -MOD. CATS

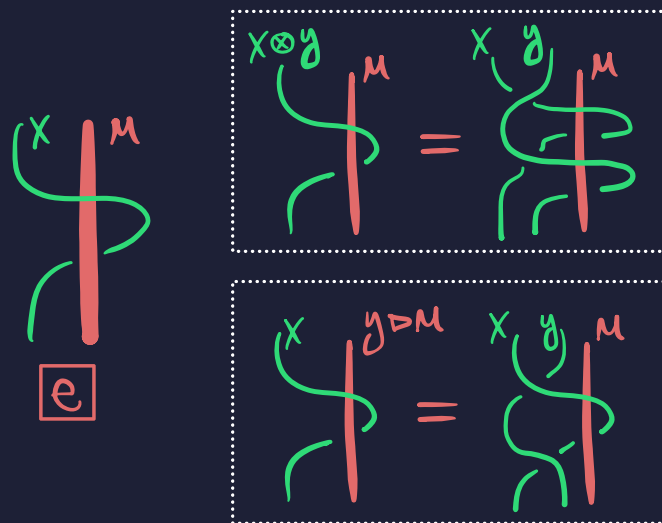
YIELD REPRS OF $\longrightarrow$
ARTIN BRAID GROUPS OF TYPE B
(BR $\otimes$ CATS YIELD REPRS OF ... TYPE A)

YIELD SOLNS TO THE
QUANTUM REFLECTION EQ. $\longrightarrow$
(LIKE R-MATRICES $\leadsto$ SOLNS TO YBE)

BRAIDED $\mathcal{C}$ -MOD. CATEG.

$$(\mathcal{M}, \triangleright : \mathcal{C} \times \mathcal{M} \longrightarrow \mathcal{M}, e)$$

$$e := \{e_{x,M} : x \triangleright M \xrightarrow{\sim} x \triangleright M\}$$



$$\text{Ex. } (\mathcal{A}\text{-Mod}_{\mathbb{K}}, \triangleright = \otimes_{\mathbb{K}}, e)$$

$$\text{FOR: } \mathcal{A} = (\mathcal{A}, m, u, \delta, \overset{\text{K-MATRIX}}{K} \in H \otimes \mathcal{A})$$

QUASITRI. LEFT H -COMODULE ALG

REPRESENTATIONS OF BRAIDED CATEGORIES

$$\mathcal{C} := (\mathcal{C}, \otimes, \mathbb{1}, c)$$

WHY CARE?

[DAVYDOV - NIKSHYCH]

$\exists$ 2-EQUIVALENCE:

$$\text{Mod}_{\text{Br}}(\mathcal{C}) \simeq \mathcal{Z}(\text{Mod}(\mathcal{C}))$$

↑
CENTER OF $\otimes$ 2-CAT

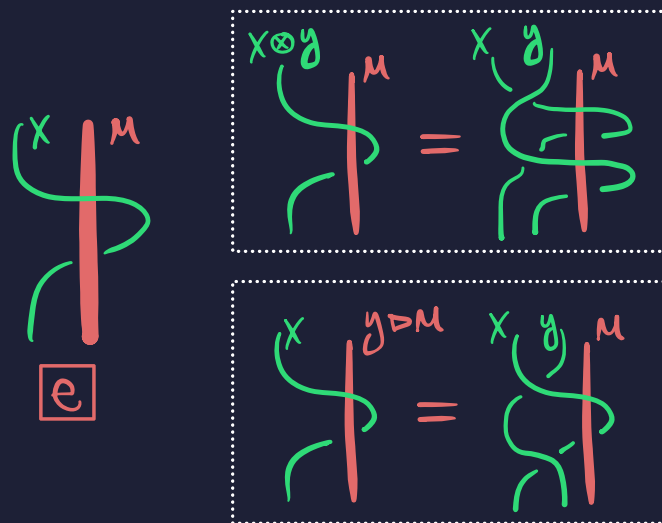
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[DAVYDOV-NIKSHYCH]

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YIELD REPS OF
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$$(\mathcal{M}, \triangleright: \mathcal{C} \times \mathcal{M} \rightarrow \mathcal{M}, e)$$

SEE INTRO TO
LAUGWITZ-W-YAKIMOV
2307.14764
FOR REFS & MORE!

KEY PEOPLE:

APPEL BALAGOVIC BAO
BROCHIER KOLB
LETZTER VLAAR WANG
...

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

↑ CONSTRUCT BRAIDED $\mathcal{C}$ -MODULE CATEGORIES ...

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

RECALL: DRINFELD CENTERS $\mathcal{Z}(\mathcal{C})$ OF $\mathcal{C} := (\mathcal{C}, \otimes, \mathbb{1})$

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

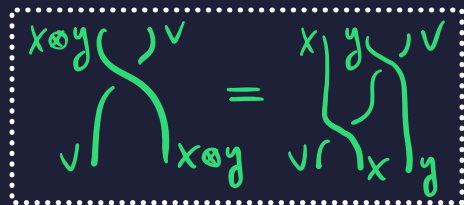
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OBJECTS: $(V \in \mathcal{C}, c^V)$

$$c^V := \{ c_X^V : X \otimes V \xrightarrow{\sim} V \otimes X \}$$

"HALF-BRAIDING"

WHERE $c_{X \otimes Y}^V$ SATISFIES



$$\begin{array}{c} X \otimes Y \\ \downarrow \\ \downarrow \end{array} \begin{array}{c} \downarrow \\ \downarrow \end{array} \begin{array}{c} V \\ \downarrow \end{array} = \begin{array}{c} X \otimes Y \\ \downarrow \\ \downarrow \end{array} \begin{array}{c} \downarrow \\ \downarrow \end{array} \begin{array}{c} V \\ \downarrow \end{array}$$

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

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MORPHISMS: $(V, c^V) \longrightarrow (W, c^W)$

$= V \longrightarrow W \in \mathcal{C}$ COMP. W/ c^V, c^W

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

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WHERE $c_{X \otimes Y}^V$ SATISFIES

$$\begin{array}{c} X \otimes Y \quad V \\ \text{braiding} \end{array} = \begin{array}{c} X \quad Y \quad V \\ \text{braiding} \end{array}$$

MONOIDAL PRODUCT:

$$(V, c^V) \otimes (W, c^W) := (V \otimes W, c^{V \otimes W})$$

WHERE $c_X^{V \otimes W}$ SATISFIES

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BRAIDING:

$$(V \otimes W, c^{V \otimes W}) \xrightarrow{c_W^V} (W \otimes V, c^{W \otimes V})$$

$$\parallel \quad \parallel$$

$$(V, c^V) \otimes (W, c^W) \quad (W, c^W) \otimes (V, c^V)$$

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

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$$(V, c^V) \otimes (W, c^W) \qquad (W, c^W) \otimes (V, c^V)$$

• A KEY CONSTR'N OF BRAIDED $\otimes$ CATS •

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

DRIN. CTR $\mathcal{Z}(\mathcal{C})$ OF $(\mathcal{C}, \otimes, \mathbb{1})$

OBJ: $(V \in \mathcal{C}, c^V)$

$$\{c_X^V: X \otimes V \xrightarrow{\sim} V \otimes X\}$$

"HALF-BRAIDING"

WHERE $c_{X \otimes Y}^V$ SAT

$$\begin{array}{c} X \otimes Y \\ \text{ } \end{array} \begin{array}{c} V \\ \text{ } \end{array} = \begin{array}{c} X \text{ } Y \\ \text{ } \end{array} \begin{array}{c} V \\ \text{ } \end{array}$$

$$(V, c^V) \otimes (W, c^W) := (V \otimes W, c^{V \otimes W})$$

WHERE $c_X^{V \otimes W}$ SAT

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$$c_{(V, c^V), (W, c^W)} := c_W^V$$

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

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 $:= (V \otimes W, c^{V \otimes W})$

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$$\begin{array}{c} X \quad V \otimes W \\ \downarrow \quad \downarrow \\ \downarrow \quad \downarrow \\ V \otimes W \end{array} = \begin{array}{c} X \quad V \quad W \\ \downarrow \quad \downarrow \quad \downarrow \\ \downarrow \quad \downarrow \quad \downarrow \\ V \quad W \quad X \end{array}$$

$c_{(V, c^V), (W, c^W)} := c_W^V$

RECALL FOR $H = \text{FIN DIM HOPF ALG.}$:

DRINFELD
CENTER
OF $H\text{-Mod}$

H -YETTER
DRINFELD
MODULES

DRINFELD
DOUBLE
OF H

$$\mathcal{Z}(H\text{-Mod}) \stackrel{\text{BR} \otimes}{\simeq} \begin{array}{c} H \\ \text{Y} \\ H \end{array} \mathcal{D} \stackrel{\text{BR} \otimes}{\simeq} D(H)\text{-Mod}$$

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

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DRINFELD
CENTER
OF $H\text{-Mod}$

$H\text{-GETTER}$
 DRINFELD
 MODULES

DRINFELD
DOUBLE
OF H

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$(M, \bullet, \partial)$

$\in H\text{-mod}$

$\in H\text{-comod}$

+ COMPATIBILITY

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

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+ COMPATIBILITY

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

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$$(M, \bullet, \partial)$$

$\rightarrow \in H\text{-mod}$

$\rightarrow \in H\text{-comod}$

+ COMPATIBILITY

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

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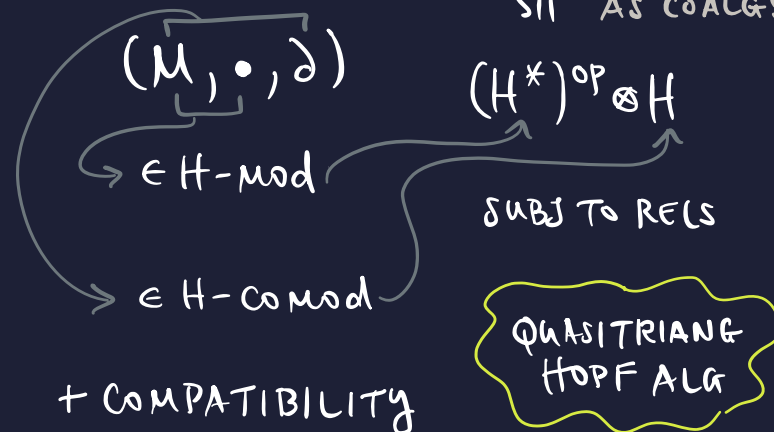
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RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

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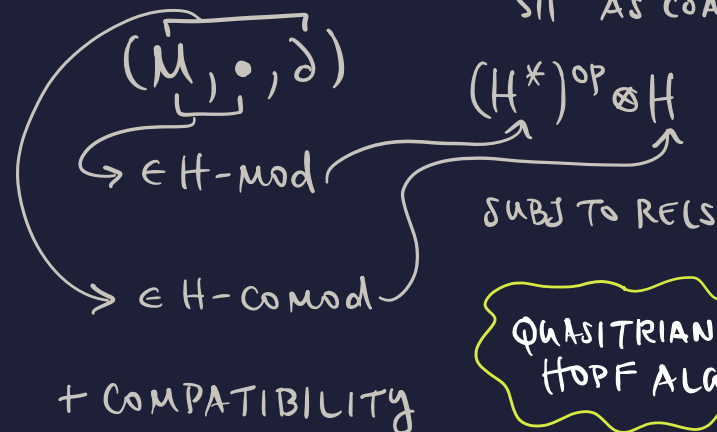
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H -YETTER
DRINFELD
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SII AS COALGS



RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

→ DEVELOP THIS
FOR $\mathcal{C}$ -MOD. CATS

DRINF. CTR $\mathcal{Z}(\mathcal{C})$ OF $(\mathcal{C}, \otimes, \mathbb{1})$

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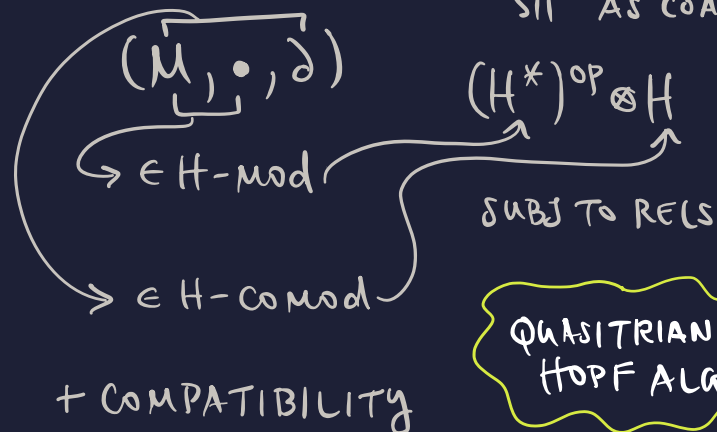
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CENTER
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H -GETTER
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DOUBLE
OF H

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SII AS COALGS



RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

TAKE $\begin{cases} (\mathcal{C}, \otimes, \mathbb{1}, c) \text{ BRAIDED MONOIDAL CATEGORY} \\ (\mathcal{M}, \triangleright) \text{ (NOT NEC. BRAIDED) } \mathcal{C}\text{-MODULE CATEGORY} \end{cases}$

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

TAKE $\begin{cases} (\mathcal{C}, \otimes, \mathbb{1}, c) \text{ BRAIDED MONOIDAL CATEGORY} \\ (\mathcal{M}, \triangleright) \text{ (NOT NEC. BRAIDED) } \mathcal{C}\text{-MODULE CATEGORY} \end{cases}$

WILL BUILD A
BRAIDED $\mathcal{C}$ -MODULE CATEGORY

"REFLECTIVE CENTER
 $\mathcal{Z}_{\mathcal{C}}(\mathcal{M})$ OF $\mathcal{M}$ "

AKIN TO BUILDING $\mathcal{Z}(\mathcal{C})$ FOR $\mathcal{C}$.

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

$$\mathcal{C} = (\mathcal{C}, \otimes, \mathbb{1}, c)$$

$$\mathcal{M} = (\mathcal{M}, \triangleright) \in \mathcal{C}\text{-mod}$$

REFLECTIVE CENTER $\mathcal{Z}_{\mathcal{C}}(\mathcal{M}) \in \mathcal{C}\text{-BrMod}$:

OBJECTS: $(M \in \mathcal{M}, e^M)$

$$e^M := \{ e_x^M : X \triangleright M \xrightarrow{\sim} X \triangleright M \}$$

"REFLECTION"

WHERE $e_{x \otimes y}^M$ SATISFIES

ACTION:

$$y \triangleright (M, e^M)$$

$$:= (y \triangleright M, e^{y \triangleright M})$$

WHERE $e_x^{y \triangleright M}$ SATISFIES

MORPHISMS: $(M, e^M) \rightarrow (N, e^N)$
 $= M \rightarrow N \in \mathcal{M}$ comp. w/ e^M, e^N

BRAIDING:

$$\begin{array}{ccc} (y \triangleright M, e^{y \triangleright M}) & \xrightarrow{e_y^M} & (y \triangleright M, e^{y \triangleright M}) \\ \parallel & & \parallel \\ y \triangleright (M, e^M) & & y \triangleright (M, e^M) \end{array}$$

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

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OBJECTS: $(M \in \mathcal{M}, e^M)$

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"REFLECTION"

WHERE $e_{X \otimes Y}^M$ SATISFIES

ACTION:

$$Y \triangleright (M, e^M)$$

$$:= (Y \triangleright M, e^{Y \triangleright M})$$

WHERE $e_x^{Y \triangleright M}$ SATISFIES

MORPHISMS: $(M, e^M) \rightarrow (N, e^N)$
 $= M \rightarrow N \in \mathcal{M}$ comp. w/ e^M, e^N

BRAIDING:

$$\begin{array}{ccc} (Y \triangleright M, e^{Y \triangleright M}) & \xrightarrow{e_Y^M} & (Y \triangleright M, e^{Y \triangleright M}) \\ \parallel & & \parallel \\ Y \triangleright (M, e^M) & & Y \triangleright (M, e^M) \end{array}$$

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

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RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

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REFLECTIVE CTR $\mathcal{E}_{\mathcal{C}}(\mathcal{M})$

OBJ: $(M \in \mathcal{M}, e^M)$

$$\{e_x^M: X \triangleright M \xrightarrow{\sim} X \triangleright M\}$$

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$$e_{y, (M, e^M)} := e_y^M$$

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$$e_{y, (M, e^M)} := e_y^M$$

TAKE $H = \text{FIN DIM QUASITRI. HOPF ALG.}$

$A = \text{LEFT } H\text{-COMODULE ALG.}$

GET AS BRAIDED $(\mathcal{C} = H\text{-mod})\text{-mod}$ CATS:

$$\mathcal{E}_{H\text{-mod}}(A\text{-Mod}) \simeq \hat{H}_A^H \text{ DH}(H) \simeq R_H(A)\text{-Mod}$$

HOPF-DOL MODS REFLECTIVE ALG

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

$$\mathcal{C} = (\mathcal{C}, \otimes, \mathbb{1}, c)$$

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$$\{e_x^M: X \triangleright M \xrightarrow{\sim} X \triangleright M\}$$

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$$e_{y, (M, e^M)} := e_y^M$$

TAKE $H = \text{FIN DIM QUASITRI. HOPF ALG.}$

$A = \text{LEFT } H\text{-COMODULE ALG.}$

GET AS BRAIDED $(\mathcal{C} = H\text{-mod})\text{-MOD CATS:}$

$$\mathcal{E}_{H\text{-mod}}(A\text{-Mod}) \simeq \hat{H}_A^H \text{ DH}(H) \simeq R_H(A)\text{-Mod}$$

HOPF-DOL MODS REFLECTIVE ALG

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

$$\mathcal{C} = (\mathcal{C}, \otimes, \mathbb{1}, c)$$

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REFLECTIVE CTR $\mathcal{E}_{\mathcal{C}}(\mathcal{M})$

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$$\{e_x^M: X \triangleright M \xrightarrow{\sim} X \triangleright M\}$$

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TAKE $H = \text{FIN DIM QUASITRI. HOPF ALG.}$

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GET AS BRAIDED $(\mathcal{C} = H\text{-mod})\text{-mod}$ CATS:

$$\mathcal{E}_{H\text{-mod}}(A\text{-Mod}) \simeq \hat{\hat{H}}_A^H \text{DH}(H) \simeq R_H(A)\text{-Mod}$$

HOPF-DOL MODS REFLECTIVE ALG

$$\hat{H} = H\text{-MOD. COALG.}$$

$$= H \text{ AS VS}$$

RELATED TO MAJID'S

"TRANSMUTED
HOPF ALGS"

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

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REFLECTIVE CTR $\mathcal{E}_{\mathcal{C}}(\mathcal{M})$

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$$\{e_x^M: X \triangleright M \xrightarrow{\sim} X \triangleright M\}$$

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$$y \triangleright (M, e^M)$$

$$:= (y \triangleright M, e^{y \triangleright M})$$

WHERE $e_x^{y \triangleright M}$ SAT

$$e_{y, (M, e^M)} := e_y^M$$

TAKE $H = \text{FIN DIM QUASITRI. HOPF ALG.}$

$A = \text{LEFT } H\text{-COMODULE ALG.}$

GET AS BRAIDED $(\mathcal{C} = H\text{-mod})\text{-mod}$ CATS:

$$\mathcal{E}_{H\text{-mod}}(A\text{-Mod}) \simeq \hat{\hat{H}}_A^H \text{ DH}(H) \simeq R_H(A)\text{-Mod}$$

HOPF-DOH MODS REFLECTIVE ALG

$$\hat{H} = H\text{-MOD. COALG.}$$

$$= H \text{ AS VS}$$

RELATED TO MAJID'S
"TRANSMUTED
HOPF ALGS"

$$(M, *, \varphi)$$

$$\in A\text{-mod}$$

$$\in \hat{H}\text{-comod}$$

+ COMP.

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

$$\mathcal{C} = (\mathcal{C}, \otimes, \mathbb{1}, c)$$

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TAKE $H = \text{FIN DIM QUASITRI. HOPF ALG.}$

$A = \text{LEFT } H\text{-COMODULE ALG.}$

GET AS BRAIDED $(\mathcal{C} = H\text{-mod})\text{-mod}$ CATS:

$$\mathcal{E}_{H\text{-mod}}(A\text{-Mod}) \simeq_{\substack{\hat{H} \\ A \text{ DUAL MODS}}} \text{DH}(H) \simeq_{\text{REFLECTIVE ALG}} R_H(A)\text{-Mod}$$

$$\hat{H} = H\text{-MOD. COALG.}$$

$$= H \text{ AS VS}$$

RELATED TO MAJID'S
"TRANSMUTED
HOPF ALGS"

$$(M, *, \varphi)$$

$\in A\text{-mod}$

$\in \hat{H}\text{-comod}$

+ COMP.

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

$$\mathcal{C} = (\mathcal{C}, \otimes, \mathbb{1}, c)$$

$$\eta = (\eta, \triangleright) \in \mathcal{C}\text{-mod}$$

REFLECTIVE CTR $\mathcal{E}_c(\eta)$

OBJ: $(M \in \tau, e^M)$

$$\{e_x^M: X \supset M \xrightarrow{\sim} X \supset M\}$$

"REFLECTION"

WHERE $e_{x \otimes y}^M \text{ SAT}$

A diagrammatic equation on a black background with a dashed white border. On the left, two strands labeled x and y are crossed, with the label $x \otimes y$ above the crossing. This is followed by an equals sign. On the right, the two strands x and y are shown separately, each with its own crossing, and the label M is placed to the right of the second crossing.

$$y \triangleright (M, e^M)$$
$$:= (y^{\triangleright M}, e^{y^{\triangleright M}})$$

WHERE e_x^{yDM} SAT

$$x \circ y \supset M = x \circ (y \circ M)$$

$$e_{y, (M, e^M)} := e_y^M$$

TAKE $H = \text{FIN DIM QUASITRI. HOPF ALG.}$

A = LEFT H-COMODULE ALG.

GET AS BRAIDED $(\mathcal{C} = H\text{-mod})\text{-mod}$ CATS:

$$\mathcal{E}_{H\text{-mod}}(A\text{-Mod}) \simeq \hat{H}_A \text{ DH}(H) \simeq R_H(A)\text{-Mod}$$

HOPF-DO1 MODS REFLECTIVE ALG

$$\begin{aligned}\hat{H} &= H - \text{MOD. COALG.} \\ &= H \text{ AS VS}\end{aligned}$$

RELATED TO MAJID'S
"TRANSMUTED
HOPF ALGS"

$(M, *, \varphi)$

$\rightarrow \in A\text{-mod}$

$\Rightarrow \in \hat{H}\text{-comod}$

+ COMP.

$$= A \otimes_H (\hat{H}^*)^{\text{op}}$$

CROSSED PROD. ALG.

$$= A \otimes H^* \text{ as vs.}$$

QUASITRIANGULAR H-COMOD. ALG.

RESULTS W/ LANGWITZ & YAKIMOV [2307.14764]

$$\mathcal{C} = (\mathcal{C}, \otimes, \mathbb{1}, c)$$

$$\eta = (\eta, \triangleright) \in \mathcal{C}\text{-mod}$$

REFLECTIVE CTR $\mathcal{E}_c(m)$

OBJ: $(M \in \tau, e^M)$

$$\{e_x^M: X \supset M \xrightarrow{\sim} X \supset M\}$$

"REFLECTION"

WHERE $e_{x \otimes y}^M$ SAT

A diagrammatic equation on a black background with a dashed white border. On the left, two strands labeled x and y are crossed, with the label $x \otimes y$ above the crossing. This is followed by an equals sign. On the right, the two strands x and y are shown separately, each passing through a box labeled M .

$$y \triangleright (M, e^M)$$
$$:= (y^{\triangleright M}, e^{y^{\triangleright M}})$$

WHERE e_x^{yDM} SAT

$$x \circ y \supset M = x \circ (y \circ M)$$

$$e_{y, (M, e^M)} := e_y^M$$

TAKE $H = \text{FIN DIM QUASITRI. HOPF ALG.}$

A = LEFT H-COMODULE ALG.

GET AS BRAIDED $(\mathcal{C} = H\text{-mod})\text{-mod}$ CATS:

$$\mathcal{E}_{H\text{-Mod}}(A\text{-Mod}) \simeq \hat{H} \underset{\text{HOPF-DO1 MODS}}{A} \text{DH}(H) \simeq \underset{\text{REFLECTIVE ALG}}{R_H(A)\text{-Mod}}$$

$$\begin{aligned}\hat{H} &= H - \text{MOD. COALG.} \\ &= H \text{ AS VS}\end{aligned}$$

RELATED TO MAJID'S
"TRANSMUTED
HOPF ALGS"

$(M, *, \varphi)$
 $\rightarrow \in A\text{-mod}$
 $\rightarrow \in \hat{H}\text{-comod}$

$= A \rtimes_H (\hat{H}^*)^{\text{op}}$
 CROSSED PROD. ALG.
 $= A \otimes H^*$ AS VS.

+ COMP.

QUASITRIANGULAR
H-COMOD. ALG.

RESULTS w/ YADAV [2411.18453]

ON NONDEGENERACY

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ON NONDEGENERACY

THESE TYPES OF BRAIDED $\otimes$ CATS $\mathcal{C} := (\mathcal{C}, \otimes, \mathbb{1}, c)$

PLAY A ROLE IN STUDYING TQFTs, CFTs, ...

... KEY FEATURE OF "MODULAR" $\otimes$ CATS

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HERE, " $\otimes$ CAT" = TENSOR CATEGORY / $\mathbb{K}$

$\mathcal{C}$ HAS TRIVIAL

MÜGER CENTER $\mathcal{Z}_2(\mathcal{C})$

$$\left\{ x \in \mathcal{C} : \sum_i x \frown_i y = \left| \right| y \quad \forall y \in \mathcal{C} \right\} \xleftarrow{\text{OBS}} \left\{ \right.$$

$$\uparrow \cong \text{Vec}_{\mathbb{K}}$$

RESULTS w/ YADAV [2411.18453]

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$$\left\{ x \in \mathcal{C} : \sum_i x \smile_i y = x \parallel y \quad \forall y \in \mathcal{C} \right\} \leftarrow \text{OBJS}$$

$$\uparrow \cong \text{Vec}_{\mathbb{K}}$$

RESULTS w/ yADAV [2411.18453]

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... KEY FEATURE OF "MODULAR" @ CATS

HERE, " $\otimes$ CAT" = TENSOR CATEGORY / $\mathbb{R}$

$\mathcal{L}$ HAS TRIVIAL

MÜGER CENTER $\mathbb{Z}_2(G)$

$$\left\{ x \in \mathcal{C} : \sum_i x_i y_i = 1 \mid \forall y \in \mathcal{C} \right\} \leftarrow \text{OBS}$$

$$\uparrow \quad \mathbb{R}^n = \text{Vec}_{\mathbb{R}}$$

RESULTS w/ YADAV [2411.18453]

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THESE TYPES OF BRAIDED $\otimes$ CATS $\mathcal{C} := (\mathcal{C}, \otimes, \mathbb{1}, c)$

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HERE, " $\otimes$ CAT" = TENSOR CATEGORY / $\mathbb{K}$

$\mathcal{C}$ HAS TRIVIAL

MÜGER CENTER $\mathcal{Z}_2(\mathcal{C})$

$$\left\{ X \in \mathcal{C} : \sum_i X \cap \begin{array}{c} \text{ } \\ \text{ } \end{array} \begin{array}{c} \text{ } \\ \text{ } \end{array} = \left| \begin{array}{c} \text{ } \\ \text{ } \end{array} \right| \forall Y \in \mathcal{C} \right\} \leftarrow \text{OBS}$$

$\uparrow \cong \text{Vec}_{\mathbb{K}}$

RESULTS w/ YADAV [2411.18453]

ON NONDEGENERACY

SHIMIZU'S THEOREM: THE FOLLOWING ARE EQUIV.

FOR A BRAIDED FINITE TENSOR CATEGORY $\mathcal{C} = (\mathcal{C}, \otimes, \mathbb{1}, c)$:

$\mathcal{C}$ HAS TRIVIAL

MÜGER CENTER $Z_2(\mathcal{C})$

$$\left\{ x \in \mathcal{C} : \sum_i x \smile_i y = \left| \right| y \quad \forall y \in \mathcal{C} \right\} \xleftarrow{\text{OBS}} \left(\right)$$

$$\uparrow \simeq \text{Vec}_{\mathbb{K}}$$

RESULTS w/ YADAV [2411.18453]

ON NONDEGENERACY

SHIMIZU'S THEOREM: THE FOLLOWING ARE EQUIV.

FOR A BRAIDED FINITE TENSOR CATEGORY $\mathcal{C} = (\mathcal{C}, \otimes, \mathbb{1}, c)$:

$\mathcal{C}$ IS FACTORIZABLE:

EMBEDDINGS

$$\mathcal{C} := (\mathcal{C}, c) \longrightarrow \mathcal{Z}(\mathcal{C})$$

$$\bar{\mathcal{C}} := (\mathcal{C}, c^{-1}) \longrightarrow \mathcal{Z}(\mathcal{C})$$

$$\text{YIELD } \mathcal{C} \boxtimes \bar{\mathcal{C}} \simeq \mathcal{Z}(\mathcal{C})$$

$\mathcal{C}$ HAS TRIVIAL

MÜGER CENTER $\mathcal{Z}_2(\mathcal{C})$

$$\left\{ x \in \mathcal{C} : \sum_i x \smile_i y = \left| \right| y \quad \forall y \in \mathcal{C} \right\} \xleftarrow{\text{OBS}} \left\{ \right\}$$

$$\uparrow \simeq \text{Vec}_{\mathbb{K}}$$

RESULTS W/ YADAV [2411.18453]

ON NONDEGENERACY

SHIMIZU'S THEOREM: THE FOLLOWING ARE EQUIV.

FOR A BRAIDED FINITE TENSOR CATEGORY $\mathcal{C} = (\mathcal{C}, \otimes, \mathbb{1}, c)$:

$\mathcal{C}$ IS NONDEGENERATE:

$$C_{\mathcal{C}} := \int^{X \in \mathcal{C}} X^* \otimes X \in \text{HopfAlg}(\mathcal{C})$$

HAS A NONDEG HOPF PAIRING

$$\omega_{\mathcal{C}} : C_{\mathcal{C}} \otimes C_{\mathcal{C}} \rightarrow \mathbb{1}$$

$\mathcal{C}$ IS FACTORIZABLE:

EMBEDDINGS

$$\mathcal{C} := (\mathcal{C}, c) \longrightarrow \mathcal{Z}(\mathcal{C})$$

$$\bar{\mathcal{C}} := (\mathcal{C}, c^{-1}) \longrightarrow \mathcal{Z}(\mathcal{C})$$

$$\text{YIELD } \mathcal{C} \boxtimes \bar{\mathcal{C}} \simeq \mathcal{Z}(\mathcal{C})$$

$\mathcal{C}$ HAS TRIVIAL

MÜGER CENTER $\mathcal{Z}_2(\mathcal{C})$

$$\left\{ X \in \mathcal{C} : \begin{array}{c} X \text{ } \int^y \\ \cap \end{array} = \begin{array}{c} X \text{ } \int_y \\ \cap \end{array} \forall y \in \mathcal{C} \right\} \xleftarrow{\text{OBS}} \left. \right\}$$

$$\uparrow \simeq \text{Vec}_{\mathbb{K}}$$

RESULTS w/ YADAV [2411.18453]

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SHIMIZU'S THEOREM: THE FOLLOWING ARE EQUIV.

FOR A BRAIDED FINITE TENSOR CATEGORY $\mathcal{C} = (\mathcal{C}, \otimes, \mathbb{1}, c)$:

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HAS A NONDEG HOPF PAIRING

$$\omega_{\mathcal{C}} : C_{\mathcal{C}} \otimes C_{\mathcal{C}} \rightarrow \mathbb{1}$$

$\mathcal{C}$ IS WEAKLY FACTOR'BLE:

$$\text{Hom}_{\mathcal{C}}(\mathbb{1}, C_{\mathcal{C}}) \rightarrow \text{Hom}_{\mathcal{C}}(C_{\mathcal{C}}, \mathbb{1})$$

$$f \longmapsto [\omega_{\mathcal{C}} \circ (f \otimes \text{id})]$$

IS BIJECTIVE

$\mathcal{C}$ IS FACTORIZABLE:

EMBEDDINGS

$$\mathcal{C} := (\mathcal{C}, c) \longrightarrow \mathcal{Z}(\mathcal{C})$$

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$$\text{YIELD } \mathcal{C} \boxtimes \bar{\mathcal{C}} \simeq \mathcal{Z}(\mathcal{C})$$

$\mathcal{C}$ HAS TRIVIAL

MÜLLER CENTER $\mathcal{Z}_2(\mathcal{C})$

$$\left\{ X \in \mathcal{C} : \begin{array}{c} X \text{ } \begin{array}{c} \text{ } \end{array} \end{array} \right\}$$

$$\uparrow \simeq \text{Vec}_{\mathbb{K}}$$

RESULTS w/ YADAV [2411.18453]

ON NONDEGENERACY

SHIMIZU'S THEOREM: THE FOLLOWING ARE EQUIV.

FOR A BRAIDED FINITE TENSOR CATEGORY $\mathcal{C} = (\mathcal{C}, \otimes, \mathbb{1}, c)$:

$\mathcal{C}$ IS NONDEGENERATE:

$$C_{\mathcal{C}} := \int^{X \in \mathcal{C}} X^* \otimes X \in \text{HopfAlg}(\mathcal{C})$$

HAS A NONDEG HOPF PAIRING

$$\omega_{\mathcal{C}} : C_{\mathcal{C}} \otimes C_{\mathcal{C}} \rightarrow \mathbb{1}$$

PF



$\mathcal{C}$ IS WEAKLY FACTOR'BLE:

$$\text{Hom}_{\mathcal{C}}(\mathbb{1}, C_{\mathcal{C}}) \rightarrow \text{Hom}_{\mathcal{C}}(C_{\mathcal{C}}, \mathbb{1})$$

$$f \longmapsto [\omega_{\mathcal{C}} \circ (f \otimes \text{id})]$$

IS BIJECTIVE



$\mathcal{C}$ IS FACTORIZABLE:

EMBEDDINGS

$$\mathcal{C} := (\mathcal{C}, c) \rightarrow \mathcal{Z}(\mathcal{C})$$

$$\bar{\mathcal{C}} := (\mathcal{C}, c^{-1}) \rightarrow \mathcal{Z}(\mathcal{C})$$

$$\text{YIELD } \mathcal{C} \boxtimes \bar{\mathcal{C}} \simeq \mathcal{Z}(\mathcal{C})$$



$\mathcal{C}$ HAS TRIVIAL

MÜGER CENTER $\mathcal{Z}_2(\mathcal{C})$

$$\left\{ X \in \mathcal{C} : \sum_i X^i \otimes Y^i = \sum_j X^j \otimes Y^j \quad \forall Y \in \mathcal{C} \right\} \xleftarrow{\text{OBS}} \left\{ \right\}$$

$$\uparrow \simeq \text{Vec}_{\mathbb{K}}$$

RESULTS W/ YADAV [2411.18453]

$\mathcal{C}$ BR.FIN. $\otimes$ CAT. $\mathcal{M} \in \mathcal{C}\text{-BrMod}$

SHIMIZU'S THEOREM: THE FOLLOWING ARE EQUIV.

FOR A BRAIDED FINITE TENSOR CATEGORY $\mathcal{C} = (\mathcal{C}, \otimes, \mathbb{1}, c)$:

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$$\omega_{\mathcal{C}} : C_{\mathcal{C}} \otimes C_{\mathcal{C}} \rightarrow \mathbb{1}$$



$\mathcal{C}$ IS WEAKLY FACTOR'BLE:

$$\begin{aligned} \text{Hom}_{\mathcal{C}}(\mathbb{1}, C_{\mathcal{C}}) &\rightarrow \text{Hom}_{\mathcal{C}}(C_{\mathcal{C}}, \mathbb{1}) \\ f &\longmapsto [\omega_{\mathcal{C}} \circ (f \otimes \text{id})] \end{aligned}$$

IS BIJECTIVE



$\mathcal{C}$ IS FACTORIZABLE:

EMBEDDINGS

$$\mathcal{C} := (\mathcal{C}, c) \rightarrow \mathcal{Z}(\mathcal{C})$$

$$\bar{\mathcal{C}} := (\mathcal{C}, c^{-1}) \rightarrow \mathcal{Z}(\mathcal{C})$$

$$\text{YIELD } \mathcal{C} \boxtimes \bar{\mathcal{C}} \simeq \mathcal{Z}(\mathcal{C})$$



$\mathcal{C}$ HAS TRIVIAL

MÜGER CENTER $\mathcal{Z}_2(\mathcal{C})$

$$\left\{ X \in \mathcal{C} : \begin{array}{c} X \\ \bigcap \\ \bigcup \end{array} \begin{array}{c} \gamma \\ \gamma \end{array} = \left\| \begin{array}{c} \gamma \\ \gamma \end{array} \right\| \forall \gamma \in \mathcal{C} \right\} \leftarrow \text{OBS}$$

$$\uparrow \simeq \text{Vec}_{\mathbb{K}}$$

RESULTS W/ YADAV [2411.18453]

$\mathcal{C}$ BR.FIN. $\otimes$ CAT. $\eta \in \mathcal{C}\text{-BrMod}$

CONSIDER THE TERMS BELOW...

η IS NONDEGENERATE:

$$E_\eta := \int_{M \in \eta} \underline{\text{Hom}}(M, M) \in \text{Alg}(\mathcal{C})$$

HAS A NONDEG PAIRING

$$\omega_\eta: \mathbb{1} \longrightarrow \mathcal{C}_c^* \otimes E_\eta$$



$\mathcal{C}$ IS WEAKLY FACTOR'BLE:

$$\begin{aligned} \text{Hom}_{\mathcal{C}}(\mathbb{1}, \mathcal{C}_c) &\longrightarrow \text{Hom}_{\mathcal{C}}(\mathcal{C}_c, \mathbb{1}) \\ f &\longmapsto [\omega_c \circ (f \otimes \text{id})] \end{aligned}$$

IS BIJECTIVE



$\mathcal{C}$ IS FACTORIZABLE:

EMBEDDINGS

$$\mathcal{C} := (\mathcal{C}, c) \longrightarrow \mathcal{Z}(\mathcal{C})$$

$$\bar{\mathcal{C}} := (\mathcal{C}, c^{-1}) \longrightarrow \mathcal{Z}(\mathcal{C})$$

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$\mathcal{C}$ HAS TRIVIAL

MÜGER CENTER $\mathcal{Z}_2(\mathcal{C})$

$$\left\{ X \in \mathcal{C} : \begin{matrix} X \\ \cap \end{matrix} \begin{matrix} Y \\ \cup \end{matrix} = \begin{matrix} X \\ \cap \end{matrix} \begin{matrix} Y \\ \cup \end{matrix} \forall Y \in \mathcal{C} \right\} \xleftarrow{\text{OBS}} \left\{ \begin{matrix} \text{ } \end{matrix} \right\}$$

$$\uparrow \simeq \text{Vec}_{\mathbb{K}}$$

RESULTS W/ YADAV [2411.18453]

$\mathcal{C}$ BR.FIN. $\otimes$ CAT. $\eta \in \mathcal{C}\text{-BrMod}$

CONSIDER THE TERMS BELOW...

η IS NONDEGENERATE:

$$E_\eta := \int_{M \in \eta} \underline{\text{Hom}}(M, M) \in \text{Alg}(\mathcal{C})$$

HAS A NONDEG PAIRING

$$\omega_\eta: \mathbb{1} \longrightarrow \mathcal{C}_\mathcal{C}^* \otimes E_\eta$$



η IS WEAKLY FACTOR'BLE:

$$\text{Hom}_\mathcal{C}(\mathcal{C}_\mathcal{C}^*, \mathbb{1}) \longrightarrow \text{Hom}_\mathcal{C}(\mathbb{1}, E_\eta)$$

$$f \longmapsto [(f \otimes \text{id}) \circ \omega_\eta]$$

IS BIJECTIVE



$\mathcal{C}$ IS FACTORIZABLE:

EMBEDDINGS

$$\mathcal{C} := (\mathcal{C}, c) \longrightarrow \mathcal{Z}(\mathcal{C})$$

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$$\text{YIELD } \mathcal{C} \boxtimes \bar{\mathcal{C}} \simeq \mathcal{Z}(\mathcal{C})$$



$\mathcal{C}$ HAS TRIVIAL

MÜGER CENTER $\mathcal{Z}_2(\mathcal{C})$

$$\left\{ X \in \mathcal{C} : \begin{matrix} X \\ \cap \\ \cup \end{matrix} \begin{matrix} \gamma \\ \gamma \end{matrix} = \begin{matrix} X \\ \cap \\ \cup \end{matrix} \begin{matrix} \gamma \\ \gamma \end{matrix} \forall \gamma \in \mathcal{C} \right\} \xleftarrow{\text{OBS}} \left\{ \begin{matrix} \gamma \\ \gamma \end{matrix} \right\}$$

$$\uparrow \simeq \text{Vec}_{\mathbb{K}}$$

RESULTS w/ YADAV [2411.18453]

$\mathcal{C}$ BR.FIN. $\otimes$ CAT. $\eta \in \mathcal{C}\text{-BrMod}$

CONSIDER THE TERMS BELOW...

η IS NONDEGENERATE:

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HAS A NONDEG PAIRING

$$\omega_\eta: \mathbb{1} \longrightarrow \mathcal{C}_\mathcal{C}^* \otimes E_\eta$$



η IS WEAKLY FACTOR'BLE:

$$\text{Hom}_\mathcal{C}(\mathcal{C}_\mathcal{C}^*, \mathbb{1}) \longrightarrow \text{Hom}_\mathcal{C}(\mathbb{1}, E_\eta)$$

$$f \longmapsto [(f \otimes \text{id}) \circ \omega_\eta]$$

IS BIJECTIVE



η IS FACTORIZABLE:

$$\eta \boxtimes \text{Fun}_\mathcal{C}(\eta, \eta)$$

$$\simeq \mathcal{Z}_\mathcal{C}(\eta)$$

(IN A CERTAIN WAY)



$\mathcal{C}$ HAS TRIVIAL

MÜLLER CENTER $\mathcal{Z}_2(\mathcal{C})$

$$\left\{ x \in \mathcal{C} : \sum_i x_i y = x \parallel y \quad \forall y \in \mathcal{C} \right\} \leftarrow \text{OBJS}$$

$$\uparrow \simeq \text{Vec}_{\mathbb{K}}$$

RESULTS W/ YADAV [2411.18453]

$\mathcal{C}$ BR.FIN. $\otimes$ CAT. $\mathcal{M} \in \mathcal{C}\text{-BrMod}$

CONSIDER THE TERMS BELOW...

$\mathcal{M}$ IS NONDEGENERATE:

$$E_{\mathcal{M}} := \int_{M \in \mathcal{M}} \underline{\text{Hom}}(M, M) \in \text{Alg}(\mathcal{C})$$

HAS A NONDEG PAIRING

$$\omega_{\mathcal{M}}: \mathbb{1} \longrightarrow \mathcal{C}_{\mathcal{C}}^* \otimes E_{\mathcal{M}}$$



$\mathcal{M}$ IS WEAKLY FACTOR'BLE:

$$\begin{aligned} \text{Hom}_{\mathcal{C}}(\mathcal{C}_{\mathcal{C}}^*, \mathbb{1}) &\longrightarrow \text{Hom}_{\mathcal{C}}(\mathbb{1}, E_{\mathcal{M}}) \\ f &\longmapsto [(f \otimes \text{id}) \circ \omega_{\mathcal{M}}] \end{aligned}$$

IS BIJECTIVE



$\mathcal{M}$ IS FACTORIZABLE:

$$\begin{aligned} \mathcal{M} \boxtimes \text{Fun}_{\mathcal{C}}(\mathcal{M}, \mathcal{M}) \\ \simeq \mathcal{E}_{\mathcal{C}}(\mathcal{M}) \end{aligned}$$

(IN A CERTAIN WAY)



$\mathcal{M}$ HAS TRIVIAL
SYMMETRIC CENTER $\mathcal{Z}_2(\mathcal{M})$

$$\left\{ x \in \mathcal{C} : x \bigcap^M = x \big| \big|^M \forall M \in \mathcal{M} \right\} \xleftarrow{\text{OBJS}}$$

$\uparrow \simeq \text{Vec}_{\mathbb{K}}$

RESULTS W/ YADAV [2411.18453]

$\mathcal{C}$ BR.FIN. $\otimes$ CAT. $\eta \in \mathcal{C}\text{-BrMod}$

CONSIDER THE TERMS BELOW.

WE OBTAIN THE IMPLICATIONS...

η IS NONDEGENERATE:

$$E_\eta := \int_{M \in \eta} \underline{\text{Hom}}(M, M) \in \text{Alg}(\mathcal{C})$$

HAS A NONDEG PAIRING

$$\omega_\eta: \mathbb{1} \longrightarrow \mathcal{C}_c^* \otimes E_\eta$$



η IS WEAKLY FACTOR'BLE:

$$\text{Hom}_{\mathcal{C}}(\mathcal{C}_c^*, \mathbb{1}) \rightarrow \text{Hom}_{\mathcal{C}}(\mathbb{1}, E_\eta)$$

$$f \longmapsto [(f \otimes \text{id}) \circ \omega_\eta]$$

IS BIJECTIVE



η IS FACTORIZABLE:

$$\eta \boxtimes \text{Fun}_{\mathcal{C}}(\eta, \eta)$$

$$\simeq \mathcal{E}_{\mathcal{C}}(\eta)$$

(IN A CERTAIN WAY)

η HAS TRIVIAL
SYMMETRIC CENTER $\mathcal{Z}_2(\eta)$

$$\left\{ x \in \mathcal{C}: x \bigcap^M = x \big| \big|_M \forall M \in \eta \right\} \xleftarrow{\text{OBJS}}$$

$$\uparrow \simeq \text{Vec}_{\mathbb{K}}$$

RESULTS w/ yADAV [2411.18453]

$$\mathcal{C} \text{ BR.FIN.} \otimes \text{CAT.} \quad \mathcal{M} \in \mathcal{C}\text{-BrMod}$$

CONSIDER THE TERMS BELOW.

FOR you!

WE OBTAIN THE IMPLICATIONS & HAVE THIS QUESTION

η IS NON DEGENERATE:

$$E_\eta := \int_{M \in \eta} \underline{\text{Hom}}(M, M) \in \text{Alg}(\mathcal{C})$$

HAS A NONDEG PAIRING

$$\omega_n: \mathbb{L} \longrightarrow C_c^* \otimes E_n$$



m IS FACTORIZABLE:

$$\mathcal{M} \boxtimes \text{Func}(\mathcal{M}, \mathcal{M}) \cong \mathcal{E}(\mathcal{M})$$

(IN A CERTAIN WAY)



η IS WEAKLY FACTOR'BLE:

$$\begin{aligned} \text{Hom}_\mathcal{C}(C_i^*, \mathbb{1}) &\rightarrow \text{Hom}_\mathcal{C}(\mathbb{1}, E_m) \\ f &\longmapsto [(f \otimes \text{id}) \circ \omega_m] \end{aligned}$$

IS BIJECTIVE



η HAS TRIVIAL
SYMMETRIC CENTER $\mathbb{Z}_2(\eta)$

$$\left\{ x \in \mathcal{C} : x \underset{\mathcal{R}}{\parallel}^M = x \underset{\mathcal{R}}{\parallel}^M \quad \forall m \in \mathcal{M} \right\} \leftarrow \text{OBS}$$



RESULTS W/ YADAV [2411.18453]

$\mathcal{C}$ BR. FIN. $\otimes$ CAT.

$\eta \in \mathcal{C}\text{-BrMod}$

WE OBTAIN THAT:

η IS NON DEGENERATE:

$$E_\eta := \int_{M \in \eta} \underline{\text{Hom}}(M, M) \in \text{Alg}(\mathcal{C})$$

HAS A NONDEG PAIRING

$$\omega_\eta: \mathbb{1} \longrightarrow \mathcal{C}_c^* \otimes E_\eta$$



η IS FACTORIZABLE:

$$\eta \boxtimes \text{Fun}_{\mathcal{C}}(\eta, \eta) \simeq \mathcal{E}_{\mathcal{C}}(\eta)$$

(IN A CERTAIN WAY)

RESULTS W/ YADAV [2411.18453]

$\mathcal{C}$ BR.FIN. $\otimes$ CAT.

$\mathcal{M} \in \mathcal{C}\text{-BrMod}$

WE OBTAIN THAT:

$\mathcal{M}$ IS NONDEGENERATE:

$$E_{\mathcal{M}} := \int_{M \in \mathcal{M}} \underline{\text{Hom}}(M, M) \in \text{Alg}(\mathcal{C})$$

HAS A NONDEG PAIRING

$$\omega_{\mathcal{M}}: \mathbb{1} \longrightarrow \mathcal{C}_{\mathcal{C}}^* \otimes E_{\mathcal{M}}$$



$\mathcal{M}$ IS FACTORIZABLE:

$$\mathcal{M} \boxtimes \text{Fun}_{\mathcal{C}}(\mathcal{M}, \mathcal{M}) \simeq \mathcal{E}_{\mathcal{C}}(\mathcal{M})$$

(IN A CERTAIN WAY)

HOPF CASE:

$(H, R) = \text{FIN.DIM. QUASITRI. HOPF ALG.}$

$(B, K) = \text{FIN.DIM. QUASITRI. H-COMOD. ALG}$

$\exists$ NOTION OF (B, K) BEING FACTORIZABLE

RESULTS W/ YADAV [2411.18453]

$\mathcal{C}$ BR.FIN. $\otimes$ CAT.

$\mathcal{M} \in \mathcal{C}\text{-BrMod}$

WE OBTAIN THAT:

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$$E_{\mathcal{M}} := \int_{M \in \mathcal{M}} \underline{\text{Hom}}(M, M) \in \text{Alg}(\mathcal{C})$$

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(IN A CERTAIN WAY)

HOPF CASE:

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$\exists$ NOTION OF (B, K) BEING **FACTORIZABLE**

• (H, Δ) IS A FACTORIZABLE H-COMOD ALG.

$\Leftrightarrow H$ IS A FACTORIZABLE HOPF ALG.

RESULTS W/ YADAV [2411.18453]

$\mathcal{C}$ BR.FIN. $\otimes$ CAT.

$\mathcal{M} \in \mathcal{C}\text{-BrMod}$

WE OBTAIN THAT:

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$\mathcal{M}$ IS FACTORIZABLE:

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(IN A CERTAIN WAY)

HOPF CASE:

$(H, R) = \text{FIN.DIM. QUASITRI. HOPF ALG.}$

$(B, K) = \text{FIN.DIM. QUASITRI. H-COMOD. ALG}$

$\exists$ NOTION OF (B, K) BEING **FACTORIZABLE**

- (H, Δ) IS A FACTORIZABLE H-COMOD ALG.
 $\iff H$ IS A FACTORIZABLE HOPF ALG.
- $(R_H(A), K_{\text{REF}})$ IS A FACTOR'BLE H-COMOD ALG.
 WHEN $R_H(A)$ IS H-SIMPLE

RESULTS w/ YADAV [2411.18453]

$\mathcal{C}$ BR.FIN. $\otimes$ CAT.

$\mathcal{M} \in \mathcal{C}\text{-BrMod}$

WE OBTAIN THAT:

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$$E_{\mathcal{M}} := \int_{M \in \mathcal{M}} \underline{\text{Hom}}(M, M) \in \text{Alg}(\mathcal{C})$$

HAS A NONDEG PAIRING

$$\omega_{\mathcal{M}}: \mathbb{1} \rightarrow \mathcal{C}_{\mathcal{C}}^* \otimes E_{\mathcal{M}}$$



$\mathcal{M}$ IS FACTORIZABLE:

$$\mathcal{M} \boxtimes \text{Fun}_{\mathcal{C}}(\mathcal{M}, \mathcal{M}) \simeq \mathcal{E}_{\mathcal{C}}(\mathcal{M})$$

(IN A CERTAIN WAY)

HOPF CASE:

$(H, R) = \text{FIN.DIM. QUASITRI. HOPF ALG.}$

$(B, K) = \text{FIN.DIM. QUASITRI. H-COMOD. ALG}$

$\exists$ NOTION OF (B, K) BEING FACTORIZABLE

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 $\iff H$ IS A FACTORIZABLE HOPF ALG.
- $(R_H(A), K_{\text{REF}})$ IS A FACTOR'BLE H-COMOD ALG.
~~WHEN $R_H(A)$ IS H-SIMPLE~~ QUESTION

RESULTS W/ YADAV [2411.18453]

$\mathcal{C}$ BR.FIN. $\otimes$ CAT.

$\mathcal{M} \in \mathcal{C}\text{-BrMod}$

WE OBTAIN THAT:

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HAS A NONDEG PAIRING

$$\omega_{\mathcal{M}}: \mathbb{1} \rightarrow \mathcal{C}_{\mathcal{C}}^* \otimes E_{\mathcal{M}}$$



$\mathcal{M}$ IS FACTORIZABLE:

$$\mathcal{M} \boxtimes \text{Fun}_{\mathcal{C}}(\mathcal{M}, \mathcal{M})$$

$$\simeq \mathcal{C}_{\mathcal{C}}(\mathcal{M})$$

(IN A CERTAIN WAY)

HOPF CASE:

$(H, R) = \text{FIN.DIM. QUASITRI. HOPF ALG.}$

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$\exists$ NOTION OF (B, K) BEING FACTORIZABLE

- (H, Δ) IS A FACTORIZABLE H-COMOD ALG.
 $\iff H$ IS A FACTORIZABLE HOPF ALG.
- $(R_H(A), K_{\text{REF}})$ IS A FACTOR'BLE H-COMOD ALG.
~~WHEN $R_H(A)$ IS H-SIMPLE~~ QUESTION

THM: $\mathcal{B}\text{-Mod}$ IS A NONDEG $(H\text{-Mod})\text{-Mod}$ CAT

$\iff \mathcal{B}$ IS A FACTORIZABLE H-COMOD ALG.

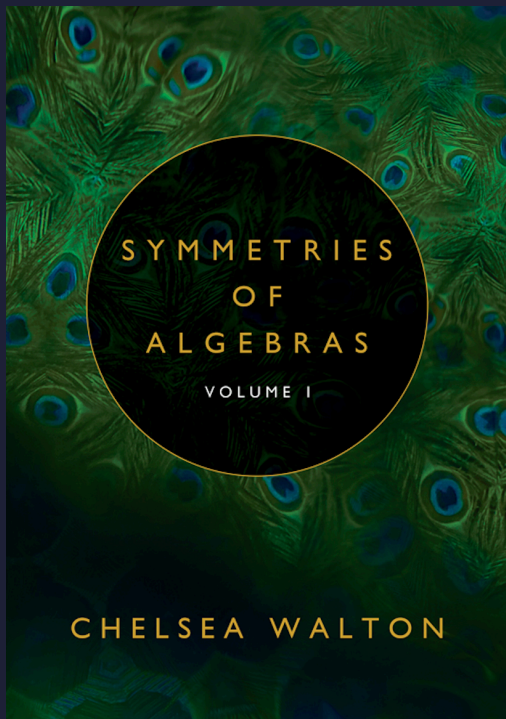
REPRESENTATIONS OF BRAIDED CATEGORIES

⋮

MORE TO COME
STAY TUNED ☺

REPRESENTATIONS OF BRAIDED CATEGORIES

COMMERCIAL
BREAK



←→
PLEASE CHECK OUT
My NEW BOOK
I HOPE THAT you LIKE IT!



$$\begin{array}{c} x \otimes y \\ \swarrow \searrow \\ z \quad x \otimes y \end{array} = \begin{array}{c} x \quad y \\ \swarrow \searrow \\ z \quad x \quad y \end{array}$$

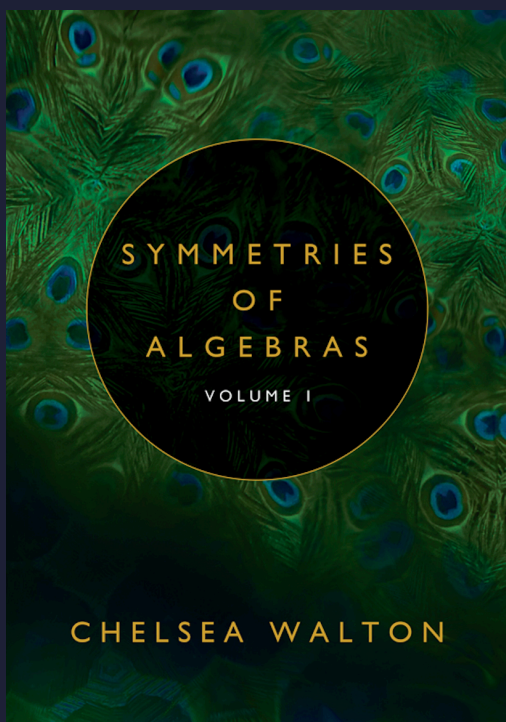
$$\begin{array}{c} x \quad y \otimes z \\ \swarrow \searrow \\ y \otimes z \quad x \end{array} = \begin{array}{c} x \quad y \\ \swarrow \searrow \\ y \quad z \quad x \end{array}$$

REPRESENTATIONS OF BRAIDED CATEGORIES

$$\begin{array}{c} x \otimes y \\ \swarrow \searrow \\ \text{red line} \end{array} = \begin{array}{c} x \quad y \\ \swarrow \searrow \\ \text{red line} \end{array}$$

$$\begin{array}{c} x \quad y \otimes \mu \\ \swarrow \searrow \\ \text{red line} \end{array} = \begin{array}{c} x \quad y \\ \swarrow \searrow \\ \text{red line} \end{array}$$

Thanks for
listening!



← →
PLEASE CHECK OUT
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