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★**Symmetries of algebras. Vol. 1.**

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In our quest to understand the universe, a main task is to uncover its symmetries. The symmetries of objects that can be observed in the sense of classical physics can be described by using groups. For the study of symmetries of the quantum world, a larger framework is necessary. The book under review is the first in a series of three volumes whose aim is to present such a framework. It introduces some key objects, algebras over a field, and categories, with special emphasis on monoidal categories.

The book contains four chapters: 1. Algebras over a field; 2. Categories; 3. Monoidal categories; 4. Algebras in monoidal categories. Each chapter is constructed such that the reader is provided with complete definitions of the concepts and numerous examples, then learns about constructions and key theorems, but also finds interesting comments about the history and applications of these concepts. A list of references for further exploration is provided, and also links to video presentations related to the material are indicated. The chapters are supplied with exercises, some of them illustrating the theory on concrete examples, while some others open the view to fundamental theoretical facts.

The contents of the chapters are briefly summarized as follows. Chapter 1 recalls basic facts about groups, rings and vector spaces; then it defines algebras over a field and provides several fundamental examples of such objects. The idea of representation is explained, and modules are introduced on this line. Operations on algebras and modules are presented, and several classes of algebras (simple, semisimple, separable) are discussed. Chapter 2 is an introduction to category theory, presenting categories, functors, natural transformations, abelian categories, equivalence of categories, adjoint functors, representable functors, special objects (indecomposable, simple, semisimple, finite length) in abelian categories, a few basic elements of homological algebra, finite linear abelian categories, the Morita Equivalence Theorem and the Eilenberg-Watts Theorem. Chapter 3 deals with monoidal categories, which are fundamental structures in many fields, including representation theory and mathematical physics. A wealth of examples is provided, and then monoidal functors, monoidal equivalences and module categories over monoidal categories are presented. Other topics considered here are strictness and coherence of monoidal categories, graphical calculus, several kinds (rigid, pivotal, spherical, fusion, tensor) of monoidal categories and module categories over them, the Frobenius-Perron dimension, and enriched categories. Chapter 4 discusses algebras, subalgebras and quotient algebras in monoidal categories, (bi)modules over such algebras, Eilenberg-Moore categories, operations on algebras and (bi)modules in monoidal categories, graded algebras and graded modules in monoidal categories, versions of the Morita Equivalence Theorem and the Eilenberg-Watts Theorem in the monoidal setting, internal Homs and Ends and Ostrik's Theorem, and several classes (including simple, semisimple, separable) of algebras in monoidal categories.

The only requirements are basic knowledge of linear algebra and algebraic structures like monoids, groups and rings; thus the book is accessible to newcomers. The book is written in a very friendly style, containing detailed explanations, plenty of interesting examples, figures and diagrams, and exercises, all together offering the reader a fascinating journey through a beautiful abstract world.

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