II. UNIVERSALITY REVISITED

$F: \mathcal{C} \to \mathcal{D}$ & $G: \mathcal{D} \to \mathcal{C}$ FORM AN ADJUNCTION $F \dashv G$ IF $\exists$ NAT. TRANS'NS

$\eta : Id_{\mathcal{C}} \Rightarrow GF$ UNIT

& $\varepsilon : FG \Rightarrow Id_{\mathcal{D}}$ COUNIT

$\ni$. Δ IDENTITIES HOLD:

$\varepsilon_{F(X)} \circ F(\eta_X) = id_{F(X)} \quad \forall X \in \mathcal{C}$

$G(\varepsilon_Y) \circ \eta_{G(Y)} = id_{G(Y)} \quad \forall Y \in \mathcal{D}$

$\equiv$ OR EQUIVALENTLY $\equiv$

$\exists$ BIJECTIONS

$\tau_{X,Y} : Hom_{\mathcal{D}}(F(X), Y) \xrightarrow{\sim} Hom_{\mathcal{C}}(X, G(Y))$

NATURAL IN X & Y $\forall X \in \mathcal{C}, Y \in \mathcal{D}$

FORM I