II. EXACTNESS

$\mathcal{C}$ ≡ ABELIAN CATEG.

EXAMPLE TAKE $\Bbbk$-ALGS. A, B.
& $Q = {}_BQ_A$ BIMODULE.

GET ADDITIVE FUNCTORS:

RIGHT EX. $\boxed{Q \otimes_A -} : A\text{-Mod} \longrightarrow B\text{-Mod}$

$\boxed{\mathrm{Hom}_{B\text{-Mod}}(Q, -)} : B\text{-Mod} \longrightarrow A\text{-Mod}$
LEFT EX.

WITH $(Q \otimes_A -) \dashv (\mathrm{Hom}_{B\text{-Mod}}(Q, -))$

COOL FACTS: TAKE AN ADDITIVE FUNCTOR
$F : \mathcal{C} \longrightarrow \mathcal{D}$

- $\exists$ LEFT ADJOINT TO $F \Longrightarrow$
  F IS LEFT EXACT

- $\exists$ RIGHT ADJOINT TO $F \Longrightarrow$
  F IS RIGHT EXACT

EILENBERG–WATTS THEOREM

TAKE FINITE DIM'L $\Bbbk$-ALGEBRAS A, B.
FOR $\Bbbk$-LINEAR
$F : A\text{-FdMod} \longrightarrow B\text{-FdMod}$, GET:

F LEFT EXACT	F RIGHT EXACT
$\Updownarrow$ ✓	$\Updownarrow$ ✓
F HAS A LEFT ADJOINT	F HAS A RIGHT ADJOINT
$\Updownarrow$ ✓	$\Updownarrow$ ✓
$F \cong \mathrm{Hom}_{A\text{-FdMod}}(P, -)$ FOR SOME BIMOD. $P = {}_AP_B$.	$F \cong Q \otimes_A -$ FOR SOME BIMOD. $Q = {}_BQ_A$.