

I. MONOIDAL CATEGORIES

MONOIDAL CATEGORY

$$(\mathcal{C}, \otimes, \mathbb{1}, \alpha, \ell, r)$$

CONSISTS OF:

(a) CATEGORY \mathcal{C}

(b) BIFUNCTOR

$$\otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$$

(c) OBJECT $\mathbb{1} \in \mathcal{C}$

(d, e, f) NATURAL ISOMS:

$$\alpha = \left\{ \begin{array}{l} \alpha_{x,y,z}: (x \otimes y) \otimes z \\ \Rightarrow x \otimes (y \otimes z) \end{array} \right\}_{x,y,z \in \mathcal{C}}$$

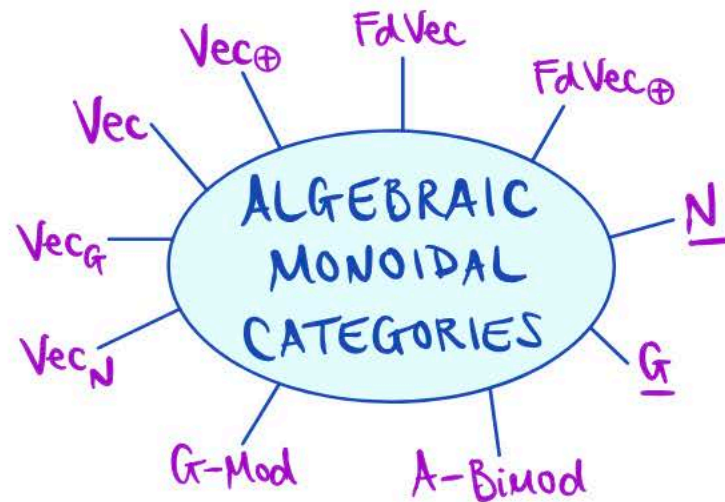
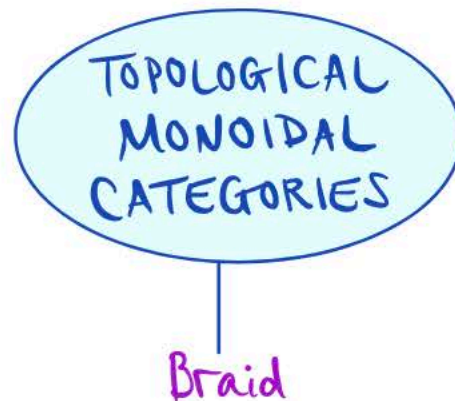
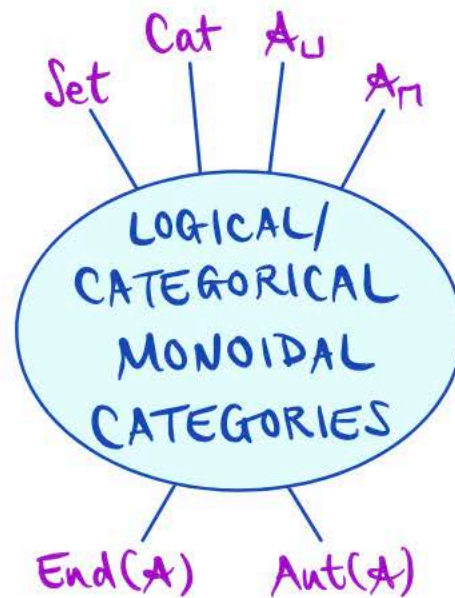
$$\ell = \{ \ell_x: \mathbb{1} \otimes x \xrightarrow{\sim} x \}_{x \in \mathcal{C}}$$

$$r = \{ r_x: x \otimes \mathbb{1} \xrightarrow{\sim} x \}_{x \in \mathcal{C}}$$

SATISFYING THE

PENTAGON AXIOM

& TRIANGLE AXIOM



AND MANY MORE!