MATH 466/566 SPRING 2024

CHELSEA WALTON RICE U.

LAST TIME

- · RIGID CATEGORIES
- · PIVOTAL CATEGORIES

LECTURE #16

TOPICS:

I. FUSION CATEGORIES

(§§3.9.1, 3.9.3)

II. FUSION RULES & RANK

(§§3.9.1, 3.9.3)

III. FROBENIUS-PERRON DIMENSION

(53.9.2)



A MONOIDAL CATEGORY

- (a) & IS ABELIAN
- (b) & 18 (IR-) LINEAR
- (c) & is LOCALLY FINITE
- (d) IL IS ABSOLUTELY SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & 15 FINITE

SUPER NICE SMONOIDAL CATEGORIES

A MONOIDAL CATEGORY

&:= (&, ⊗, 1, a, 1, r)
IS FUSION IF:

- (a) & IS ABELIAN
- (b) & 18 (IR-) LINEAR
- (c) & IS LOCALLY FINITE
- (d) IL IS ABSOLUTELY SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

SUPER NICE }
MONOIDAL CATEGORIES

A MONOIDAL CATEGORY

C:= (C, ⊗, L, a, L, r)
(S FUSION IF:

(a) & IS ABELIAN

- (b) & IS (IR-) LINEAR
- (c) & IS LOCALLY FINITE
- (d) IL IS ABSOLUTELY SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (g) & IS FINITE

SUPER NICE MONOIDAL CATEGORIES

A MONOIDAL CATEGORY

&:= (&, ⊗, 1, a, 1, r)
IS FUSION IF:



SUPER NICE
MONOIDAL CATEGORIES

(a) & IS ABELIAN

- (b) & 18 (k-) LINEAR
- (c) & IS LOCALLY FINITE
- (d) IL IS ABSOLUTELY SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (g) & IS FINITE



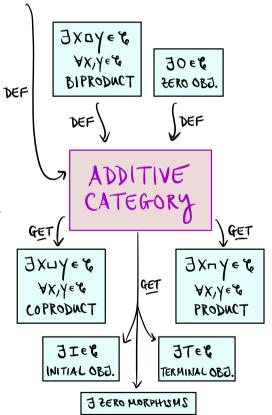
A MONOIDAL CATEGORY

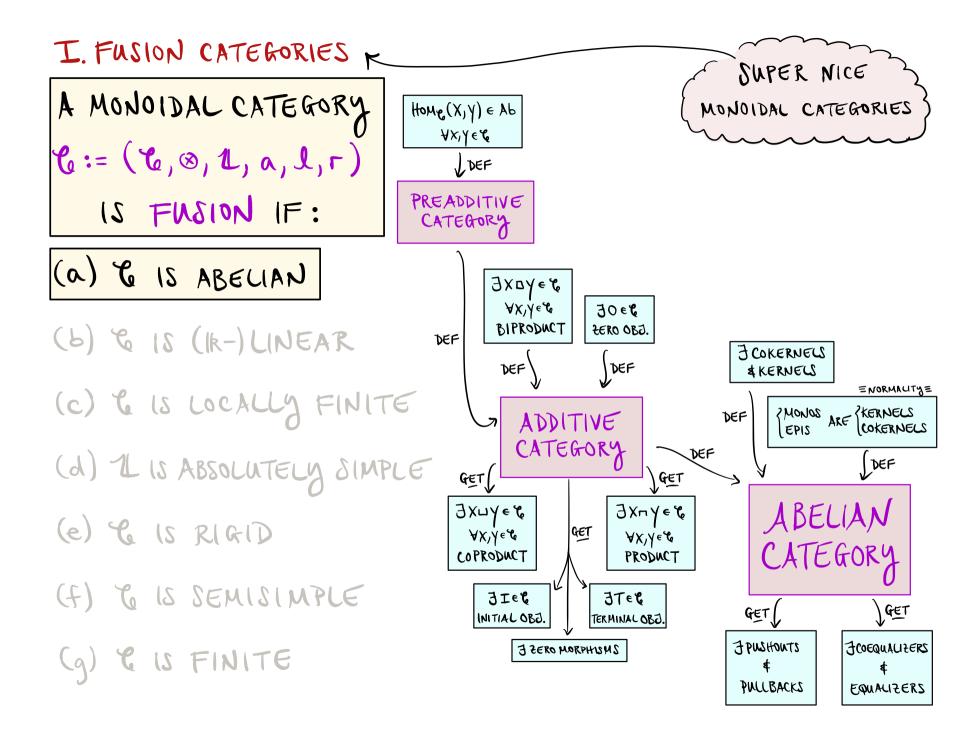
&:= (&,⊗,1,a,1,r)

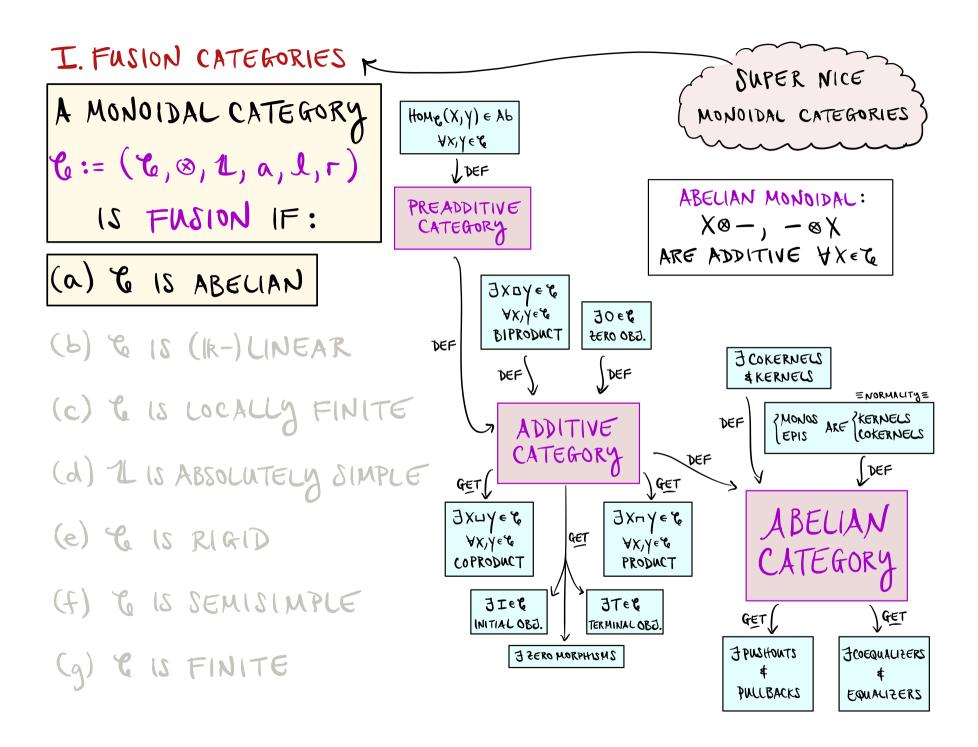
HOME(X,Y) & Ab VX,Y & & DOEF PREADDITIVE CATEGORY SUPER NICE MONOIDAL CATEGORIES

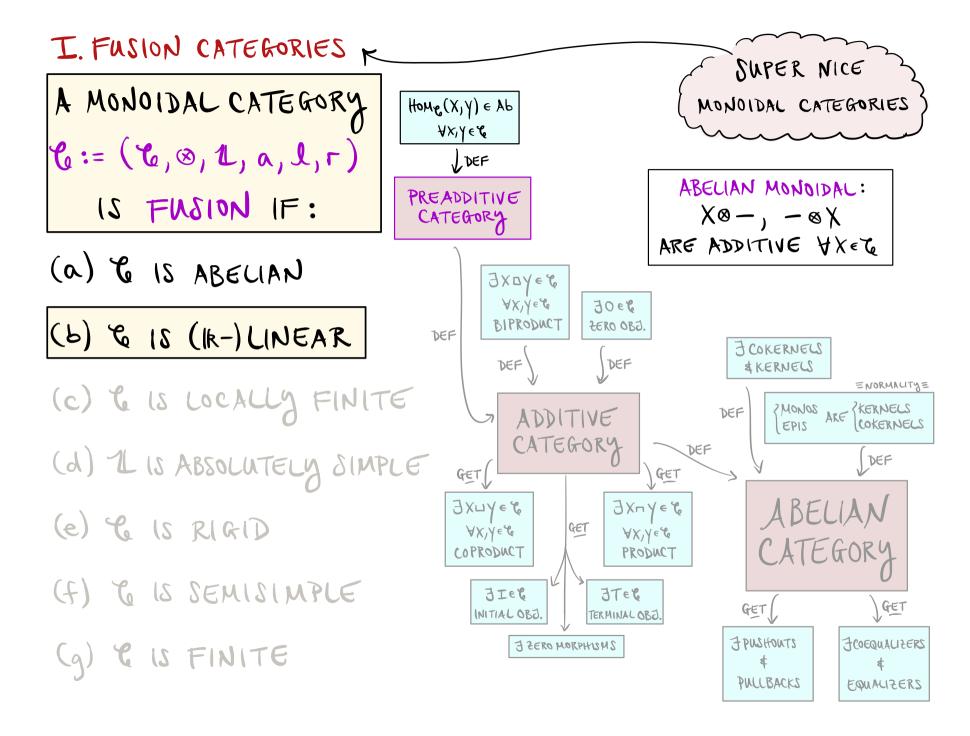
(a) & IS ABELIAN

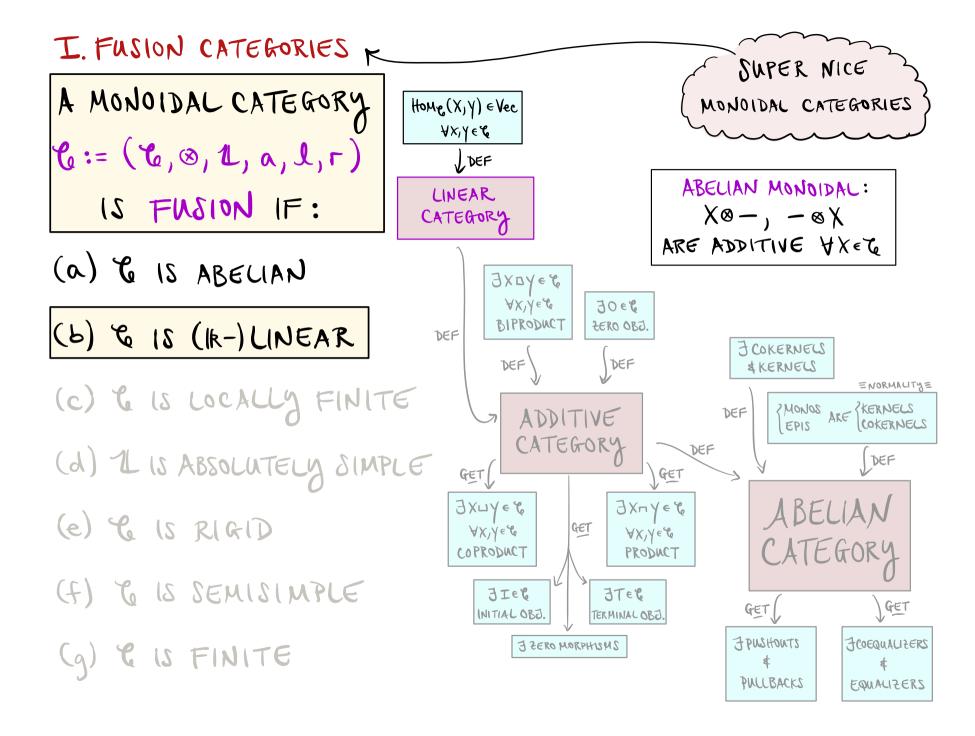
- (b) & 18 (R-) LINEAR
- (c) & IS LOCALLY FINITE
- (d) IL IS ABSOLUTELY SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (g) & IS FINITE

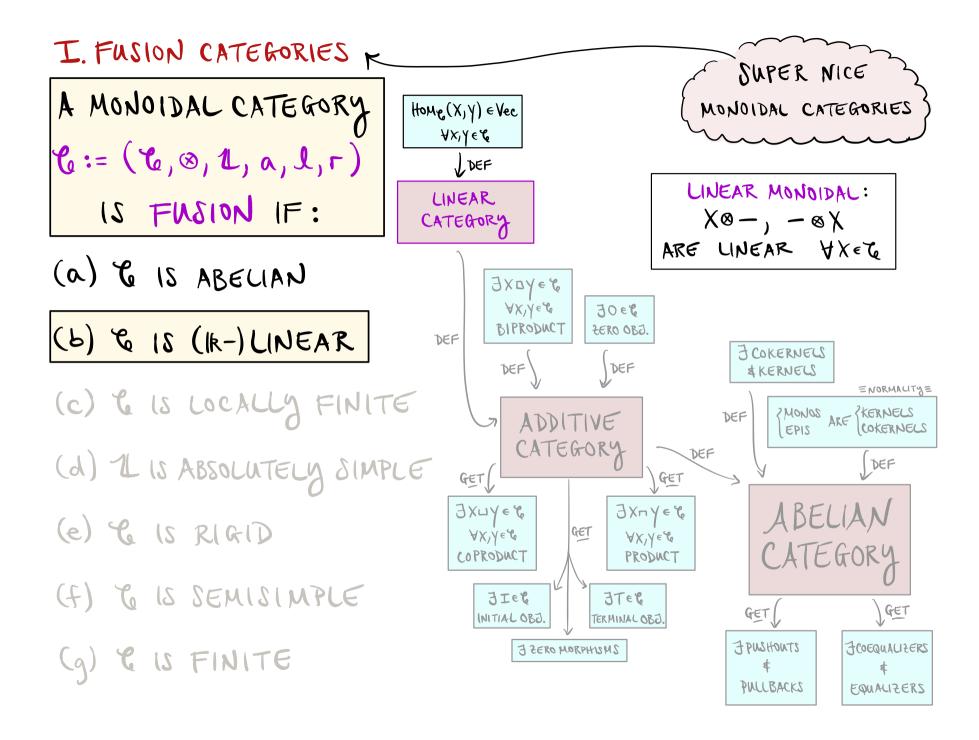








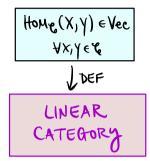




A MONOIDAL CATEGORY

C:= (C, ⊗, 1, a, 1, r)

15 FUSION IF:



SUPER NICE S MONOIDAL CATEGORIES

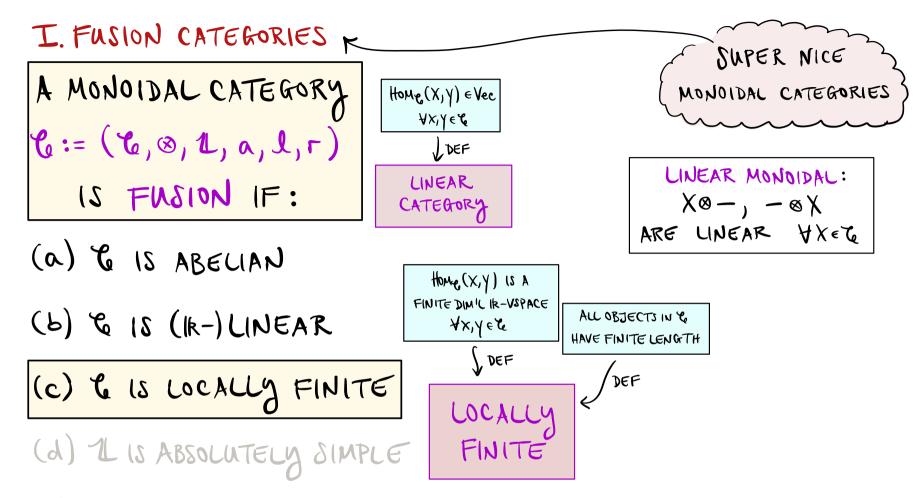
LINEAR MONOIDAL:

XO-, -OX ARE UNEAR YXEG

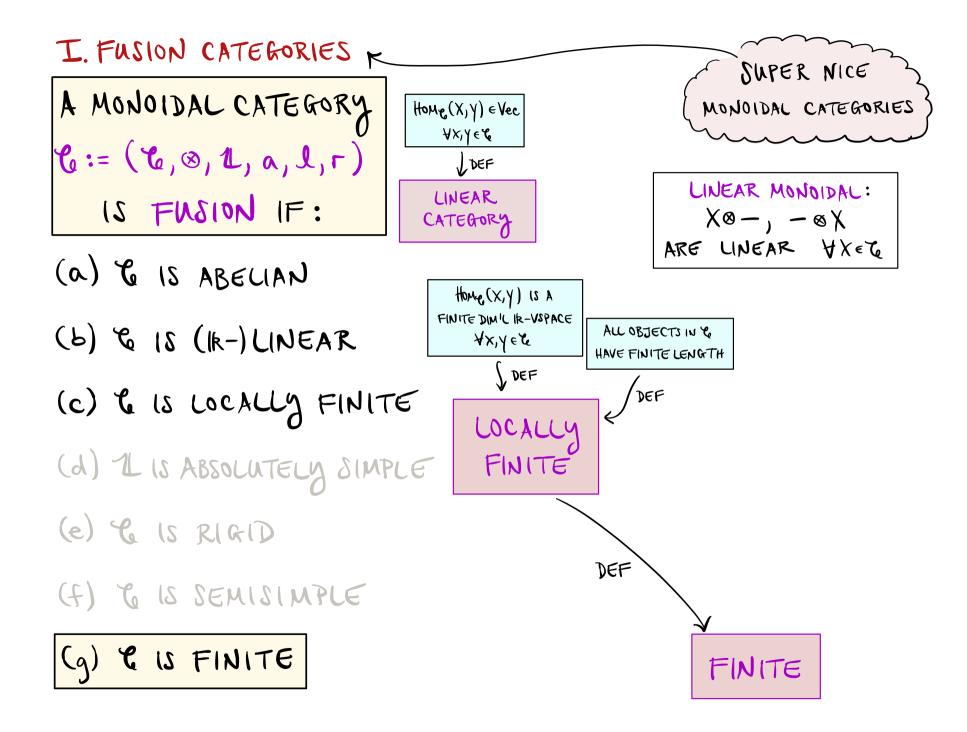
- (a) & IS ABELIAN
- (b) & 18 (IR-) LINEAR

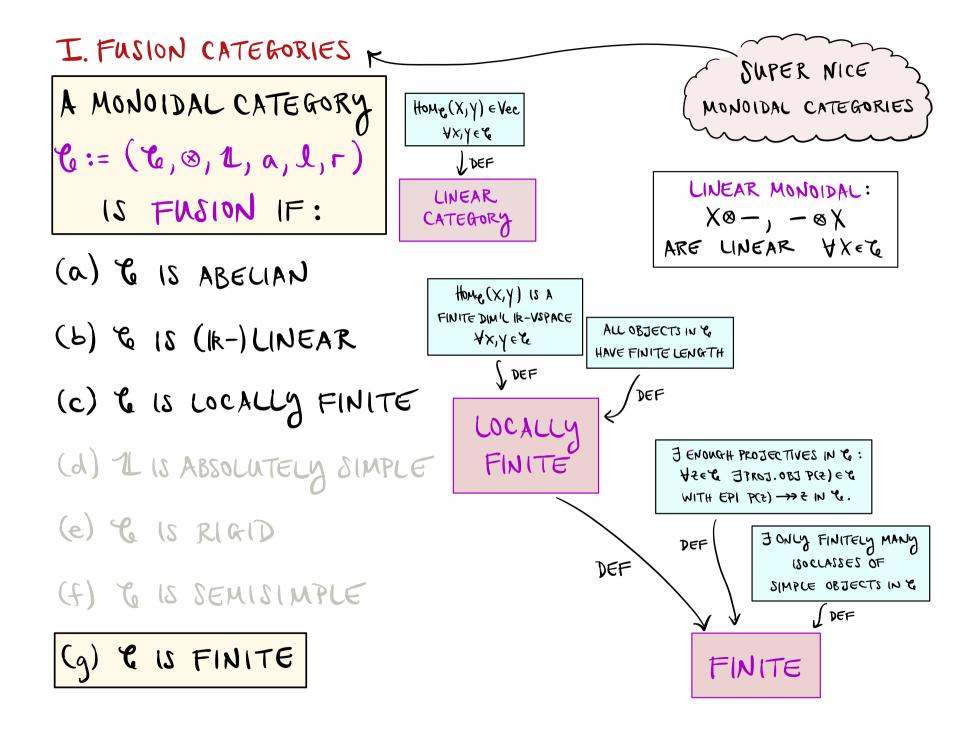
(c) & is LOCALLY FINITE

- (d) IL IS ABSOLUTELY SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (g) & IS FINITE



- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (g) & IS FINITE





A MONOIDAL CATEGORY

C:= (C, ⊗, L, a, L, r)

(S FUSION IF:

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SUPER NICE MONOIDAL CATEGORIES

A MONOIDAL CATEGORY C:= (C, S, L, a, L, r) IS FUSION IF:

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- (g) & IS FINITE

SUPER NICE MONOIDAL CATEGORIES

X * e & IS SEMISIMPLE IF X = Lie X Xi
FOR SIMPLE OBJECTS Xi.

& IS SEMISIMPLE IF ALL OBJECTS ARE SS

A MONOIDAL CATEGORY

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SUPER NICE MONOIDAL CATEGORIES

X & C IS SEMISIMPLE IF X = LieT Xi FOR SIMPLE OBJECTS Xi.

& IS SEMISIMPLE IF ALL OBJECTS ARE SS

SIMPLE OBJECTS

 $X^{\sharp e}$ C IS SIMPLE

(F THE ONLY SUBOBTS OF X

ARE $X \notin O$

INDECOMPOSABLE OBJECTS

Y * C IS INDECOMPOSABLE

IF X \neq X, \ld X_2

V NONZERO SUBOBJ. X, \lambda of X

A MONOIDAL CATEGORY

C:= (C, ⊗, L, a, L, r)
(S FUSION IF:

- (a) & IS ABELIAN
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- (g) & IS FINITE

SUPER NICE MONOIDAL CATEGORIES

X & C IS SEMISIMPLE IF X = LieI Xi FOR SIMPLE OBJECTS Xi.

& IS SEMISIMPLE IF ALL OBJECTS ARE SS

SIMPLE OBJECTS

 $X^* \in \mathcal{C}$ is simple IF THE ONLY SUBOBTS OF X ARE X & O

INDECOMPOSABLE
OBJECTS

X * 6 C IS INDECOMPOSABLE

IF X \neq X, \ld X_2

V NONZERO SUBOBJ. X, \ld X_2 OF X

A MONOIDAL CATEGORY

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(S FUSION IF:

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SUPER NICE S MONOIDAL CATEGORIES

X * & C IS SEMISIMPLE IF X = LieI Xi
FOR SIMPLE OBJECTS Xi.

& IS SEMISIMPLE IF ALL OBJECTS ARE SS

SIMPLE OBJECTS

IF THE ONLY SUBOBTS OF X

ARE X & O

INDECOMPOSABLE
OBJECTS

X * & C IS INDECOMPOSABLE

IF X \neq X, \ld X_2

Y NONZERO SUBOBJ. X, \lambda_2 OF X

FINITE LENGTH
OBJECTS

PROP:

IN SEMISIMPLE CATEGORIES,
INDECOMPOSABLE OBJECTS
OF FINITE LENGTH
ARE SIMPLE.

SEMISIMPLE
CATEGORIES

A MONOIDAL CATEGORY

&:= (&, ⊗, 1, a, 1, r)
IS FUSION IF:

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SUPER NICE SMONOIDAL CATEGORIES

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- (g) & IS FINITE

SUPER NICE MONOIDAL CATEGORIES

AXEG:

$$\exists \chi^* \in \mathcal{C} \text{ with}$$

$$ev_{\chi} : \chi^* \otimes \chi \longrightarrow \mathcal{L} \quad \chi^* \chi$$

$$coev_{\chi} : \mathcal{L} \longrightarrow \chi \otimes \chi^* \quad \chi^* \chi^*$$

$$\chi^* = |\chi^* \downarrow \chi^* \downarrow \chi^* \downarrow \chi^* \downarrow \chi^*$$

$$-AND -$$

A MONOIDAL CATEGORY C:= (C, S, L, a, L, r) IS FUSION IF:

- (a) & IS ABELIAN
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SUPER NICE MONOIDAL CATEGORIES

AXEG:

$$\exists X^* \in \mathcal{C} \text{ with}$$

$$ev_X^* : X^* \otimes X \to \mathcal{L} \quad X^* \downarrow X$$

$$coev_X^* : \mathcal{L} \to X \otimes X^* \quad x^* \times X^*$$

$$x \downarrow^X = |X|^* \neq x^* \downarrow f_{X^*} = |X|^*$$

-AND-

$$\exists^* \chi \in \mathcal{C} \text{ WITH} \\
ev_{\chi}^{R} : \chi \otimes^* \chi \to \mathcal{L} \quad \chi^* \chi^* \chi$$

$$coev_{\chi}^{R} : \mathcal{L} \to^* \chi \otimes \chi *_{\chi} \chi^* \chi$$

$$\chi \downarrow \uparrow_{\chi} = |\chi + \chi \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow$$

A MONOIDAL CATEGORY

C:= (C, ⊗, L, a, L, r)

(S FUSION IF:

- (a) & IS ABELIAN
- (b) & 18 (IR-) LINEAR
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- (d) 1 is absolutely simple
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SUPER NICE SMONOIDAL CATEGORIES

A MONOIDAL CATEGORY

&:= (&, ⊗, 1, a, 1, r)
IS FUSION IF:

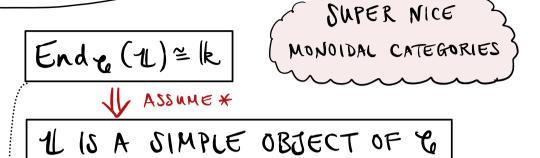
- (a) & IS ABELIAN
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End (1) = k

SUPER NICE >

A MONOIDAL CATEGORY C:= (C, S, L, a, L, r) IS FUSION IF:

- (a) & IS ABELIAN X
- (b) & 18 (IR-) LINEAR X
- (c) & is LOCALLY FINITE
- (d) IL IS ABSOLUTELY SIMPLE
- (e) & IS RIGID *
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- (g) & IS FINITE

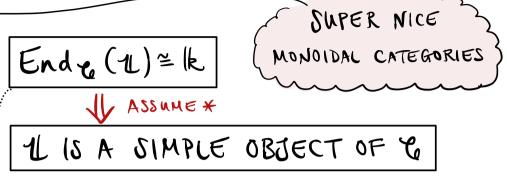


A MONOIDAL CATEGORY

C:= (C, 8, 1, a, 1, r)

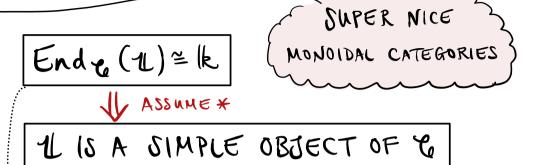
IS FUSION IF:

- (a) & IS ABELIAN X
- (b) & 18 (IR-) LINEAR X
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A MONOIDAL CATEGORY C:= (C, S, L, a, L, r) IS FUSION IF:

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- (g) & IS FINITE



PF/TAKE A SUBOBJECT OF 1L: $(X, {}^{(MONO)}, 1: X \rightarrow L)$ WLOG MONZERO, SIMPLE GET S.E.S. $0 \rightarrow X \stackrel{\iota}{\rightarrow} 1 \longrightarrow coker(\iota) \rightarrow 0$.

A MONOIDAL CATEGORY C:= (C, ⊗, L, a, L, r) IS FUSION IF:

- (a) & IS ABELIAN X
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SUPER NICE MONOIDAL CATEGORIES Endr (11) = 1k JI ASSUMEX

1 IS A SIMPLE OBJECT OF &

PF/ TAKE A SUBOBJECT OF 1L: (X, (MONO) X -> 1L) WLOG MONZERO, SIMPLE GET S.E.S. 0 -> X -> 1 -> coker(1) -> 0. (-)* IS EXACT ⇒

GET S.E.S. O -> Ox ker(1)* -> 11 -> X* -> O.

A MONOIDAL CATEGORY C:= (C, o, L, a, L, r) IS FUSION IF:

- (a) & IS ABELIAN *
- (b) & 18 (IR-) LINEAR X
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- (d) IL IS ABSOLUTELY SIMPLE
- (e) & IS RIGID *
- (f) & IS SEMISIMPLE
- (g) & 13 FINITE

Ende (11) = lk N ASSUME * SUPER NICE S MONOIDAL CATEGORIES

1 IS A SIMPLE OBJECT OF &

PF/ TAKE A SUBOBJECT OF 1L:

(X, (MONO) X -> 1L) WLOG MONZERO, SIMPLE

GET S.E.S. 0 -> X -> 1 -> coker(1) -> 0.

(-)* IS EXACT ⇒

GET S.E.S. $0 \rightarrow c_0 \ker(\iota)^* \rightarrow 1 \stackrel{\iota^*}{\longrightarrow} \chi^* \rightarrow 0$.

 $(X \otimes -)$ EXACT \Rightarrow GET S.E.S. :

 $0 \to X \otimes \operatorname{coker}(\iota)^* \longrightarrow X \xrightarrow{\operatorname{id} \otimes \iota^*} X \otimes X^* \longrightarrow 0$

A MONOIDAL CATEGORY C:= (C, o, L, a, L, r) IS FUSION IF:

- (a) & IS ABELIAN *
- (b) & 18 (IR-) LINEAR X
- (c) & IS LOCALLY FINITE
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- (e) & IS RIGID *
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

Ende (11) = lk

N ASSUME *

SUPER NICE >

IL IS A SIMPLE OBJECT OF &

PF/ TAKE A SUBOBJECT OF 1L:

(X, (MONO) (X -> 1L) WLOG MONZERO, SIMPLE

GET S.E.S. 0 -> X -> 1 -> coker(1) -> 0.

(-)* IS EXACT ⇒

GET S.E.S. $0 \rightarrow c_0 \ker(\iota)^* \rightarrow 1 \xrightarrow{\iota^*} X^* \rightarrow 0$.

 $(X \otimes -)$ EXACT \Rightarrow GET 8.E.S. :

 $0 \rightarrow \times \otimes \operatorname{coker}(\iota)^* \longrightarrow \times \xrightarrow{\operatorname{id} \otimes \iota^*} \times \otimes \times^* \longrightarrow 0$

X SIMPLE => idx & 1* MONIC

A MONOIDAL CATEGORY C:= (C, o, L, a, L, r) IS FUSION IF:

- (a) & IS ABELIAN *
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Ende (11) = lk IL ASSUME* SUPER NICE >

IL IS A SIMPLE OBJECT OF &

PF/TAKE A SUBOBJECT OF 1L:

(X, (MONO) X -> 1L) WLOG MONZERO, SIMPLE

GET S.E.S. 0 -> X -> 1 -> coker(1) -> 0.

(-)* IS EXACT ⇒

GET S.E.S. $0 \rightarrow c_0 \ker(\iota)^* \rightarrow 1 \xrightarrow{\iota^*} X^* \rightarrow 0$.

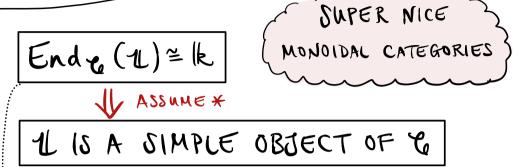
 $(X \otimes -) \in XACT \Rightarrow GET S.E.S.$

 $0 \to \chi \otimes \operatorname{coker}(\iota)^* \longrightarrow \chi \xrightarrow{\operatorname{id} \otimes \iota^*} \chi \otimes \chi^* \longrightarrow 0$

 $X SIMPLE \Rightarrow id_X \otimes \iota^* MONIC$ $\Rightarrow id_X \otimes \iota^* 180.$

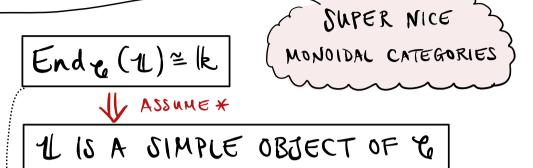
A MONOIDAL CATEGORY C:= (C, o, l, a, l, r) IS FUSION IF:

- (a) & IS ABELIAN X
- (b) & 18 (IR-) LINEAR X
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- (g) & IS FINITE



A MONOIDAL CATEGORY C:= (C, o, l, a, l, r) IS FUSION IF:

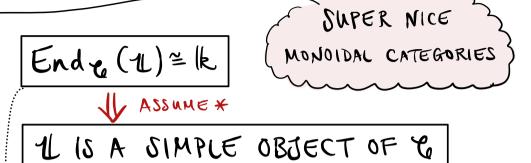
- (a) & IS ABELIAN *
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- (c) & is LOCALLY FINITE
- (d) IL IS ABSOLUTELY SIMPLE
- (e) & IS RIGID *
- (f) & IS SEMISIMPLE
- (g) & IS FINITE



PF/ TAKE A SUBOBJECT OF 1L: $(X, \overset{(MONO)}{1}: X \to L) \text{ WLOG MONZERO, SIMPLE}$ GET $id_X \otimes i^* : X \to X \otimes X^* \text{ AN (SO.}$ $NOW \otimes : L \xrightarrow{Coev_X} X \otimes X^* \xrightarrow{(id \otimes i^*)^{-1}} X$ IS A NONZERO MORPHISM.

A MONOIDAL CATEGORY C:= (C, 8, 1, a, 1, r) IS FUSION IF:

- (a) & IS ABELIAN X
- (b) & IS (IR-) LINEAR X
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PF/TAKE A SUBOBJECT OF 1L: $(X, \overset{(MONO)}{1}: X \to L) \text{ WLOG MONZERO, SIMPLE}$ GET $id_X \otimes i^* : X \to X \otimes X^* \text{ AN (SO.}$ $NOW \otimes : L \xrightarrow{Coev_X} X \otimes X^* \xrightarrow{(id \otimes i^*)^{-1}} X$ IS A NONZERO MORPHISM. $SO, L \xrightarrow{\varnothing} X \xrightarrow{L} L \text{ IS NONZERO}$

A MONOIDAL CATEGORY C:= (C, o, L, a, L, r) IS FUSION IF:

- (a) & IS ABELIAN *
- (b) & IS (IR-) LINEAR X
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- (g) & IS FINITE

Ende (1) = 1k MONOIDAL CATEGORIES

WASSUME *

1 IS A SIMPLE OBJECT OF &

PF/TAKE A SUBOBJECT OF 1: $(X)^{(MONO)}(X) \rightarrow U$ WLOG MONZERO, SIMPLE

GET $id_X \otimes i^* : X \rightarrow X \otimes X^*$ AN (SO).

NOW $\otimes : U \xrightarrow{Coev_X} X \otimes X^* \xrightarrow{(id \otimes i^*)^{-1}} X$ SO, $U \xrightarrow{\otimes} X \xrightarrow{U} U$ IS NONZERO

1 ABSOLUTELY SIMPLE => 18 = 2 ide

A MONOIDAL CATEGORY C:= (C, o, L, a, L, r) IS FUSION IF:

- (a) & IS ABELIAN X
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- (g) & 15 FINITE

Ende (1) = |k | MONOIDAL CATEGORIES

WASSUME *

IL IS A SIMPLE OBJECT OF &

PF/ TAKE A SUBOBJECT OF 1L:

(X, (MONO) X -> 1L) WLOG MONZERO, SIMPLE

GET $id_X \otimes i^* : X \longrightarrow X \otimes X^* AN (50.$

NOW Ø: 11 Coevx X X X (id@i*)-1 X

15 A NONZERO MORPHISM.

SO, 1 × X - 1 IS NONZERO

1 ABSOLUTELY SIMPLE => 18 = 2 ide

:. 1\$ 15 AN 180 => 1\$ EPIC

A MONOIDAL CATEGORY C:= (C, o, L, a, L, r) IS FUSION IF:

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- (f) & IS SEMISIMPLE
- (g) & 15 FINITE

Ende (1) = lk

SUPER NICE >

1 IS A SIMPLE OBJECT OF &

PF/TAKE A SUBOBJECT OF 1L:

(X, 1: X -> 1L) WLOG MONZERO, SIMPLE

GET $id_X \otimes i^* : X \longrightarrow X \otimes X^* AN (50.$

NOW Ø: 11 Coevx X X X (id@i*)-1 X

15 A NONZERO MORPHISM.

SO, 1 × X - 1 IS NONZERO

1 ABSOLUTELY SIMPLE => 18 = 2 ide

: 1\$ IS AN ISO ⇒ 1\$ EPIC ⇒ 1 EPIC :: X=16

A MONOIDAL CATEGORY C:= (C, 8, 1, a, 1, r) IS FUSION IF:

- (a) & IS ABELIAN X
- (b) & IS (IR-) LINEAR X
- (c) & is LOCALLY FINITE **
- (d) 1 is absolutely simple
- (e) & IS RIGID *
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

Ende (1)=k

MONOIDAL CATEGORIES

SUPER NICE

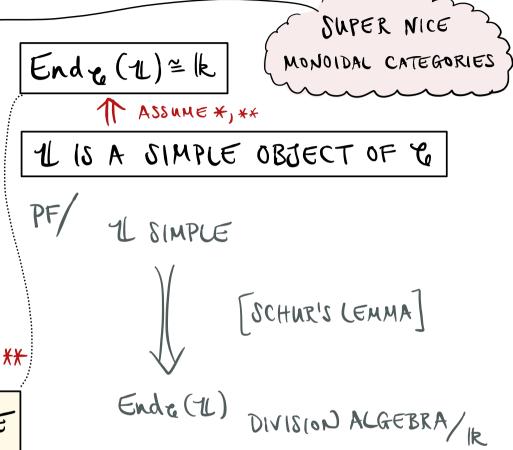
ASSUME *, **

1 IS A SIMPLE OBJECT OF &

PF/

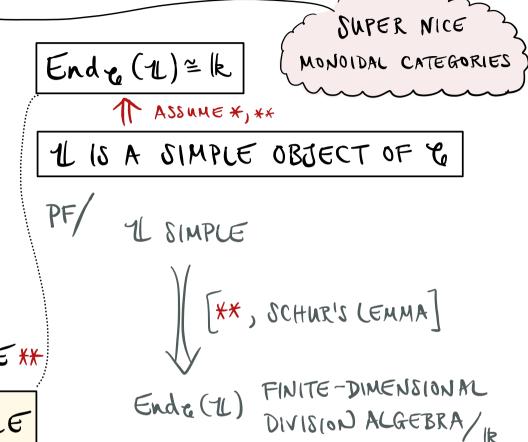
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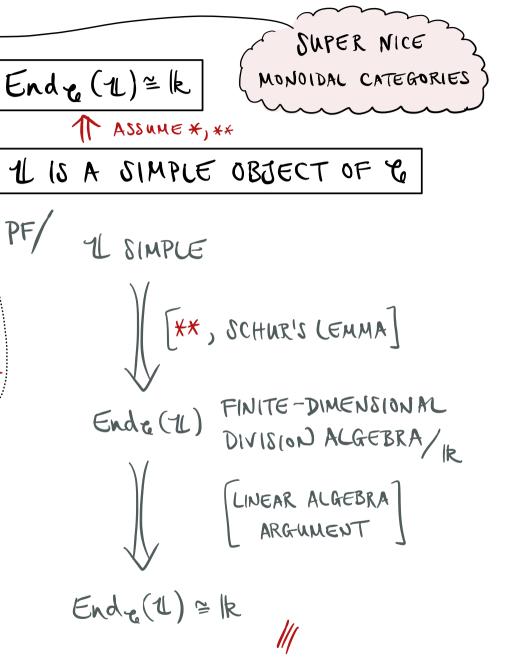
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A MONOIDAL CATEGORY C:= (C, ⊗, L, a, L, r) IS FUSION IF:

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A MONOIDAL CATEGORY

C:= (C, ⊗, 1, a, 1, r)
(S FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (q) & IS FINITE

WHEN ARE MONOIDAL CATEGORIES

TWO FUSION CATEGORIES

CONSIDERED THE SAME?

A MONOIDAL CATEGORY

C:= (C, ⊗, 1, a, 1, r)
(S FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) 1 is ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

WHEN ARE MONOIDAL CATEGORIES

TWO FUSION CATEGORIES

CONSIDERED THE SAME?

SUPER NICE

0

WHICH ITEMS ARE STRUCTURAL ??

AND WHICH ARE PROPERTIES ??

A MONOIDAL CATEGORY 6:= (6, 8, 1, a, 1, r) | STRUCTURE IS FUSION IF: (a) & IS ABELIAN - PROPERTY (6) & 18 (IR-) LINEAR + STRUCTURE (c) & is LOC. FINITE - PROPERTY (d) 1 is ABS. SIMPLE - PROPERTY (e) & IS RIGID - PROPERTY (f) & IS SEMISIMPLE - PROPERTY (q) & IS FINITE - PROPERTY

WHEN ARE

MONOIDAL CATEGORIES

SUPER NICE

TWO FUSION CATEGORIES

CONSIDERED THE SAME?

A MONOIDAL CATEGORY

6:= (6,8,1,a,1,r)

STRUCTURE

IS FUSION IF:

(a) & IS ABELIAN — PROPERTY

(b) & IS (IR-) LINEAR—STRUCTURE

(c) & IS LOC. FINITE—PROPERTY

(d) 1 LIS ABS. SIMPLE—PROPERTY

(e) & IS RIGID—PROPERTY

(f) & IS SEMISIMPLE PROPERTY

(9) & IS FINITE - PROPERTY

WHEN ARE

SUPER NICE >

TWO FUSION CATEGORIES
CONSIDERED THE SAME?

THE ANSWER ??

A MONOIDAL CATEGORY 6:= (6, 8, 1, a, 1, r) | STRUCTURE IS FUSION IF: (a) & IS ABELIAN - PROPERTY (6) & 18 (IR-) LINEAR + STRUCTURE (c) & is LOC. FINITE - PROPERTY (d) 1 is ABS. SIMPLE - PROPERTY (e) & IS RIGID - PROPERTY (f) & IS SEMISIMPLE PROPERTY (9) & IS FINITE - PROPERTY

WHEN ARE

SUPER NICE >

TWO FUSION CATEGORIES
CONSIDERED THE SAME?

IF J UNEAR FUNCTOR

BETWEEN THE TWO

THAT'S AN ISOM./EQUIV.

OF MONOIDAL CATEGORIES

A MONOIDAL CATEGORY

C:= (C, ⊗, 1, a, 1, r)

(S FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) 1 Is ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (q) & IS FINITE

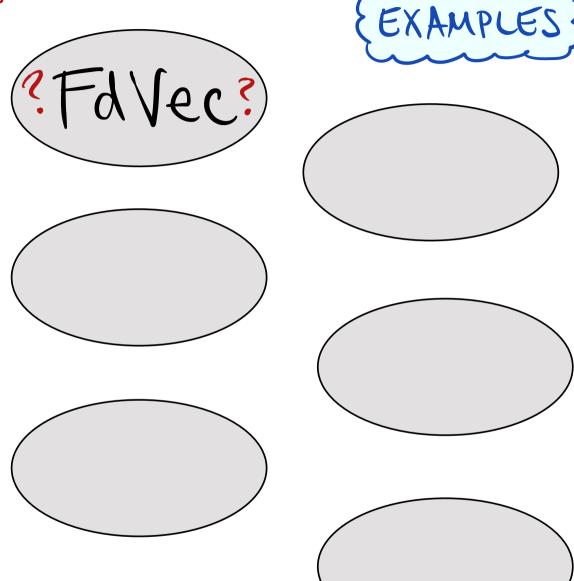


A MONOIDAL CATEGORY

C:= (C, ⊗, 1, a, 1, r)

15 FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) 1 is ABS. SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (q) & IS FINITE

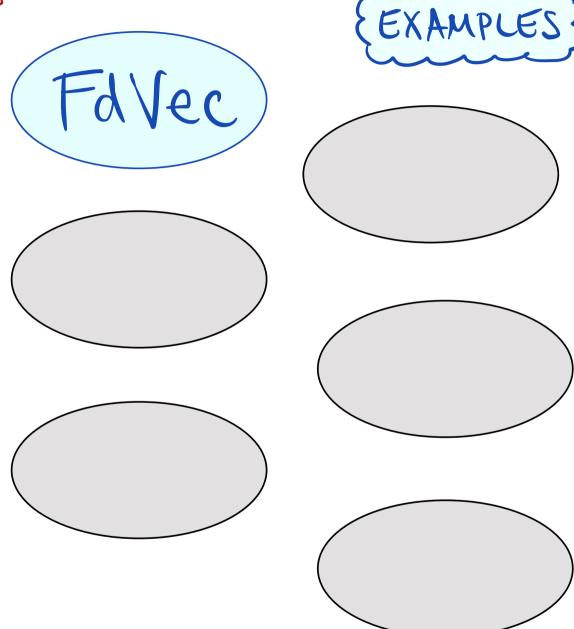


A MONOIDAL CATEGORY

& := (&, ⊗, 1, a, 1, r)

(S FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (q) & IS FINITE



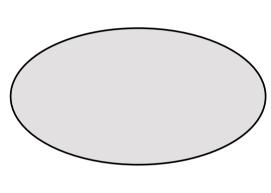
A MONOIDAL CATEGORY

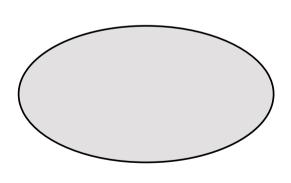
C:= (C, ⊗, 1, a, 1, r)

15 FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (q) & IS FINITE

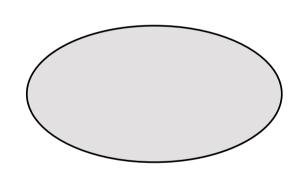


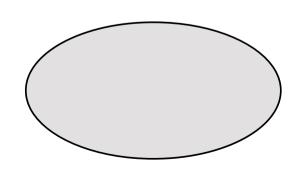












A MONOIDAL CATEGORY

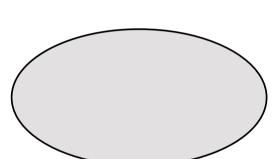
C:= (C, ⊗, 1, a, 1, r)

(S FUSION IF:

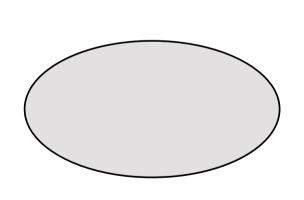
- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (q) & IS FINITE

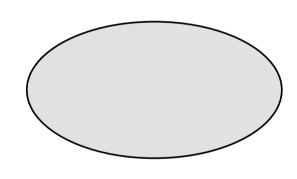




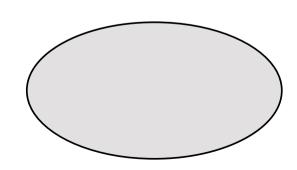








(JUST ONE REASON)



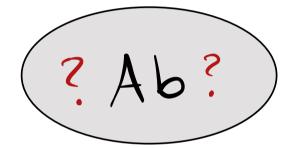
A MONOIDAL CATEGORY

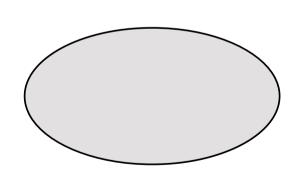
C:= (C, ⊗, 1, a, 1, r)

15 FUSION IF:

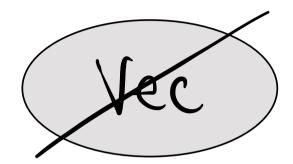
- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) 1 is ABS. SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (q) & IS FINITE

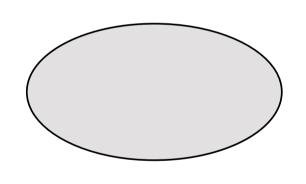


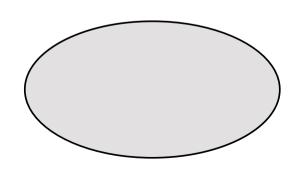










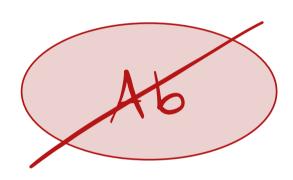


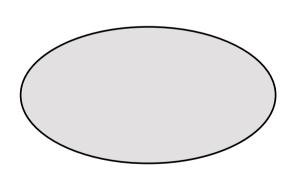
A MONOIDAL CATEGORY

C:= (C, ⊗, 1, a, 1, r)
(S FUSION IF:

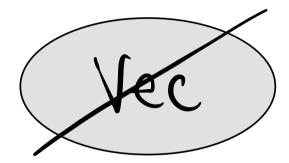
- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (q) & IS FINITE

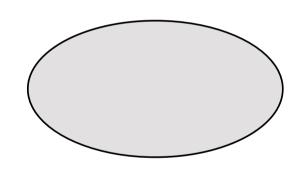
FdVec

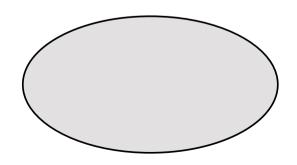












(JUST ONE REASON)

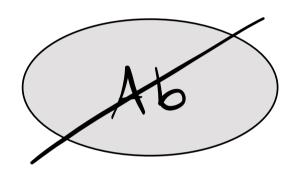
A MONOIDAL CATEGORY

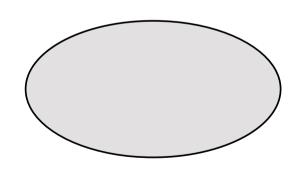
C:= (C, ⊗, 1, a, 1, r)

15 FUSION IF:

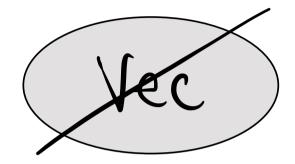
- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) 1 is ABS. SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (q) & IS FINITE



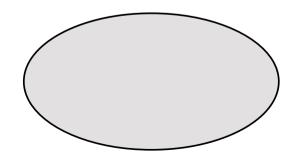








? Set?

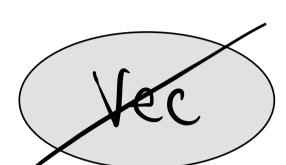


A MONOIDAL CATEGORY

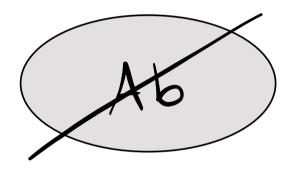
C:= (C, ⊗, 1, a, 1, r)
(S FUSION IF:

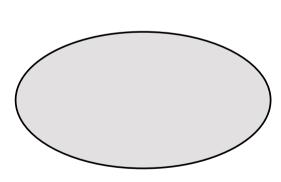
- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (q) & IS FINITE



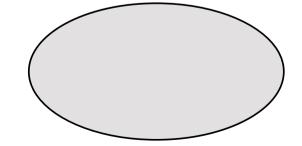


EXAMPLES









(JUST ONE REASON)

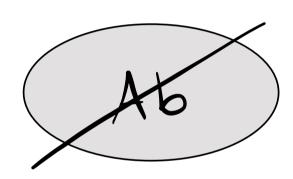
A MONOIDAL CATEGORY

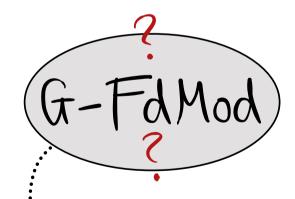
C:= (C, ⊗, 1, a, 1, r)

15 FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (q) & IS FINITE

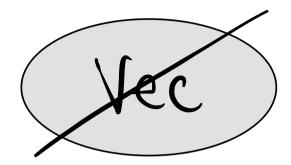


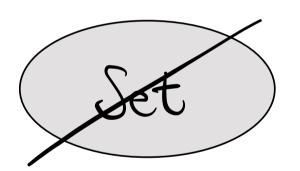


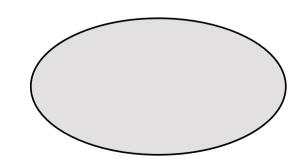












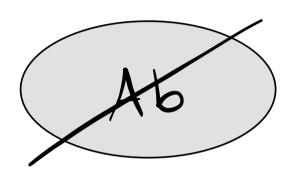
A MONOIDAL CATEGORY

C:= (C, ⊗, 1, a, 1, r)
(S FUSION IF:

- (a) & IS ABELIAN
- (b) & 18 (1R-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (q) & IS FINITE

(JUST ONE REASON)

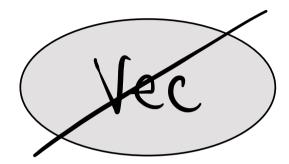


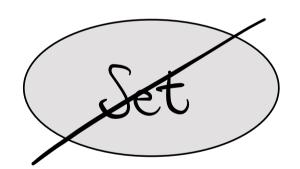


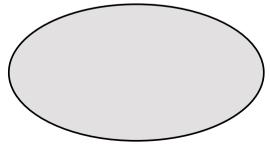












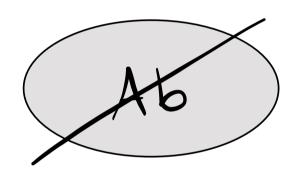
A MONOIDAL CATEGORY

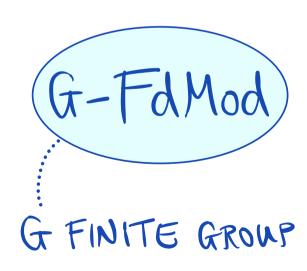
C:= (C, ⊗, 1, a, 1, r)

15 FUSION IF:

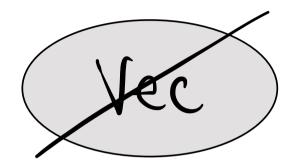
- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (9) & IS FINITE

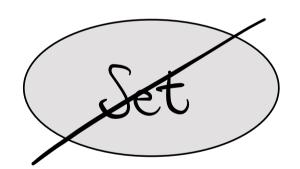


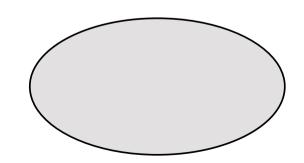








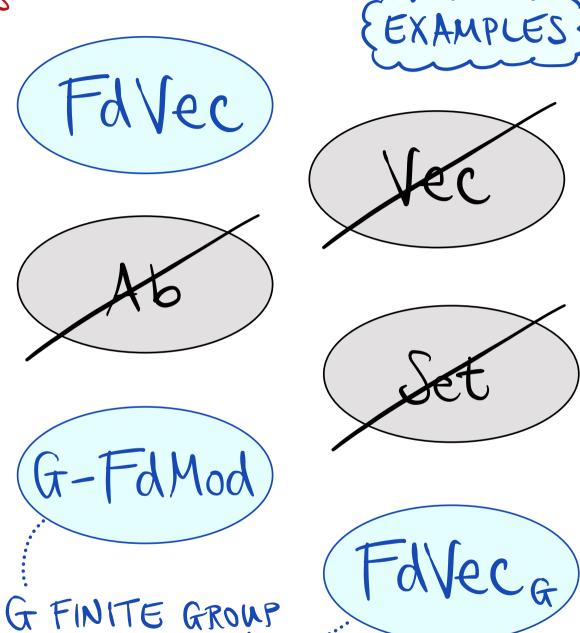




A MONOIDAL CATEGORY

C:= (C, ⊗, 1, a, 1, r)
(S FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (9) & IS FINITE



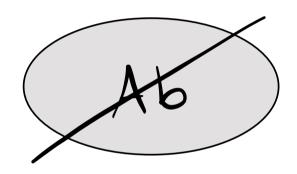
A MONOIDAL CATEGORY

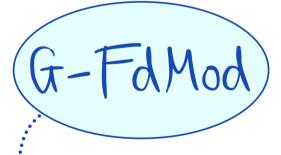
C:= (C, ⊗, 1, a, 1, r)
(S FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (IR-) LINEAR
- (c) 6 IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (9) & IS FINITE

GOVERNED BY SIMPLE OBJECTS

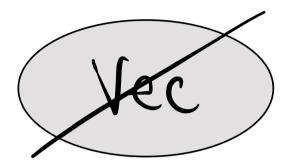


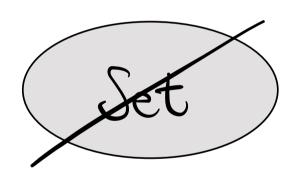




G FINITE GROUP







FdVeca

A MONOIDAL CATEGORY

C:= (C, 8, 1, a, 1, r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) 1 is ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

GOVERNED BY SIMPLE OBJECTS ITT(%) := SET OF ISOCLASSES [X]
OF SIMPLE OBJECTS OF &

C FINITE => [Irr(C)] < 00

RANK OF C

A MONOIDAL CATEGORY

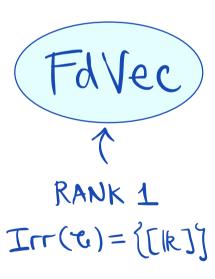
C:= (C, S, L, a, L, r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & 18 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

GOVERNED BY SIMPLE OBJECTS ITT (%) := SET OF ISOCLASSES [X]
OF SIMPLE OBJECTS OF &

& FINITE => [Irr(&)] < 00 RANK OF &



A MONOIDAL CATEGORY

C:= (6,8,1,a,l,r)

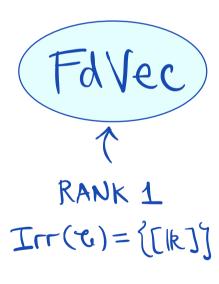
IS FUSION IF:

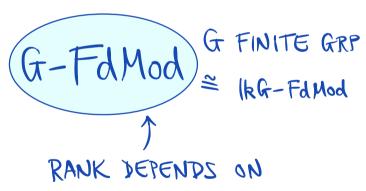
- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (q) & IS FINITE

GOVERNED BY SIMPLE OBJECTS ITT(%) := SET OF ISOCLASSES [X]
OF SIMPLE OBJECTS OF &

C FINITE => [Irr(C)] < 00

RANK OF C





RANK DEPENDS ON
ARTIN-WEDDERBURN
DECOMPOSITION OF IRG

A MONOIDAL CATEGORY

C:= (C, 8, 1, a, 1, r)

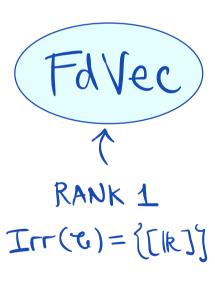
IS FUSION IF:

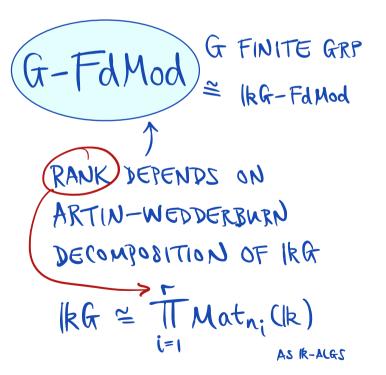
- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (q) & IS FINITE

GOVERNED BY SIMPLE OBJECTS ITT(%) := SET OF ISOCLASSES [X]
OF SIMPLE OBJECTS OF &

C FINITE => [Irr(C)] < 00

RANK OF C





A MONOIDAL CATEGORY

C:= (C, S, L, a, L, r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & 18 (1R-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

GOVERNED BY SIMPLE OBJECTS ITT((%) := SET OF ISOCLASSES [X]
OF SIMPLE OBJECTS OF &

C FINITE => [Irr(C)] < 00

RANK OF C

Ex. G FINITE ABELIAN

$$-k(G-Fd Mod) = ??$$

G-FdMod & FINITE GRP

| IkG-FdMod

RANK DEPENDS ON
ARTIN-WEDDERBURN
DECOMPOSITION OF IRG

IRG = TT Matn; (Ik)
AS IR-ALGS

A MONOIDAL CATEGORY

C:= (6, 8, 1, a, 1, r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & 18 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

GOVERNED BY SIMPLE OBJECTS ITT(() := SET OF ISOCLASSES [X]
OF SIMPLE OBJECTS OF &

C FINITE => [Irr(C)] < 00

RANK OF C

EX.

G FINITE ABECIAN

W

IRG COMMUTATIVE

W

TK (G-Fd Mod) = IGI

G-FdMod & FINITE GRP

RANK DEPENDS ON

ARTIN-WEDDERBURN

DECOMPOSITION OF IKG

IKG = TT Matn; (IK)

i=1

AS R-ALGS

A MONOIDAL CATEGORY

C:= (C, S, L, a, L, r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (q) & IS FINITE

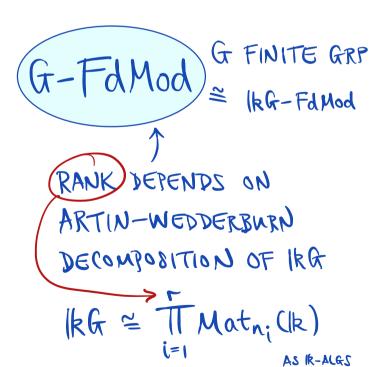
GOVERNED BY SIMPLE OBJECTS ITT(%) := SET OF ISOCLASSES [X]
OF SIMPLE OBJECTS OF &

C FINITE => [Irr(C)] < 00

RANK OF C

$$\exists x$$
.

 $G \cong S_3$



A MONOIDAL CATEGORY

C:= (C, S, L, a, L, r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & 18 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

GOVERNED BY SIMPLE OBJECTS ITT(() := SET OF ISOCLASSES [X]
OF SIMPLE OBJECTS OF 6

C FINITE => [Irr(C)] < 00

RANK OF C

 GFINITE GRP

RANK DEPENDS ON

ARTIN-WEDDERBURN

DECOMPOSITION OF IKA

IKG = TT Matn; (IK)

i=1

AS IR-ALGS

A MONOIDAL CATEGORY

C:= (C, S, L, a, L, r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (IR-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

ITT (%) := SET OF 150CLASSES [X]
OF SIMPLE OBJECTS OF &

& FINITE => [Irr(&)] < 00 RANK OF &

RESEARCH PROBLEM

CLASSIFY ALL FUSION CATEGORIES OF A GIVEN RANK, UP TO EQUIV.

GOVERNED BY SIMPLE OBJECTS

A MONOIDAL CATEGORY

C:= (6,0,1,a,1,r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (IR-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

ITT (%) := SET OF ISOCLASSES [X]
OF SIMPLE OBJECTS OF &

& FINITE => [Irr(&)] < 00 RANK OF &

RESEARCH PROBLEM

CLASSIFY ALL FUSION CATEGORIES OF A GIVEN RANK, UP TO EQUIV.

VRANK 1: FdVec

GOVERNED BY SIMPLE OBJECTS

A MONOIDAL CATEGORY

C:= (6,0,1,a,1,r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & IS (IR-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

ITT (%) := SET OF ISOCLASSES [X]
OF SIMPLE OBJECTS OF &

& FINITE => [Irr(&)] < 00 RANK OF &

RESEARCH PROBLEM

CLASSIFY ALL FUSION CATEGORIES OF A GIVEN RANK, UP TO EQUIV.

VRANK 1: FdVec

VRANK 2 [OSTRIK] 34EQUIV. CLASSES

GOVERNED BY SIMPLE OBJECTS

A MONOIDAL CATEGORY

C:= (C, S, L, a, L, r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & 18 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (g) & IS FINITE

GOVERNED BY SIMPLE OBJECTS ITT(() := SET OF ISOCLASSES [X]
OF SIMPLE OBJECTS OF 6

C FINITE => [Irr(C)] < 00 RANK OF C

RESEARCH PROBLEM = RATHER IMPOSSIBLE =

CLASSIFY ALL FUSION CATEGORIES

OF A GIVEN RANK, UP TO EQUIV.

VRANK 1: FdVec

INVOLVES KNOWING THE

YRANK 2 [OSTRIK] ARTIN-WEDDERBURN DECOMP. 34 EQUIV. CLASSES OF ALL FINITE GROUPS

A MONOIDAL CATEGORY

C:= (6, 8, 1, a, 1, r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & IS (IR-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

GOVERNED BY
SIMPLE OBJECTS

ITT (%) := SET OF ISOCLASSES [X]
OF SIMPLE OBJECTS OF &

& FINITE => [Irr(&)] < 00 RANK OF &

RESEARCH PROBLEM = RATHER IMPOSSIBLE =

CLASSIFY ALL FUSION CATEGORIES OF A GIVEN RANK, UP TO EQUIV.

A MONOIDAL CATEGORY

C:= (C, 0, 1, a, 1, r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

GOVERNED BY SIMPLE OBJECTS ITT (%) := SET OF ISOCLASSES [X]
OF SIMPLE OBJECTS OF &

& FINITE => [Irr(&)] < 00 RANK OF &

=WEAKEN=

RESEARCH PROBLEM SHOW THAT

JONLY FINITELY MANY FUSION CATEGORIES OF A GIVEN RANK, UP TO EQUIV.

A MONOIDAL CATEGORY

C:= (C, S, L, a, L, r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & IS (IR-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

GOVERNED BY SIMPLE OBJECTS ITT (%) := SET OF ISOCLASSES [X]
OF SIMPLE OBJECTS OF &

& FINITE => [Irr(&)] < 00 RANK OF &

RANK-FINITENESS CONJECTURE

JONLY FINITELY MANY FUSION CATEGORIES OF A GIVEN RANK, UP TO EQUIV.

A MONOIDAL CATEGORY

C:= (6,0,1,a,1,r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

GOVERNED BY SIMPLE OBJECTS ITT (%) := SET OF ISOCLASSES [X]
OF SIMPLE OBJECTS OF &

& FINITE => [Irr(&)] < 00 RANK OF &

RANK-FINITENESS CONJECTURE

JONLY FINITELY MANY FUSION CATEGORIES OF A GIVEN RANK, UP TO EQUIV.

SETTLED IN SPECIAL CASES

[BRUILLARD-NG-ROWELL-WANG IN "MODULAR FUSION" CASE (Vol3)]

A MONOIDAL CATEGORY

C:= (6, 8, 1, a, 1, r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & IS (IR-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

ITT (%) := SET OF 180CLASSES [X]
OF SIMPLE OBJECTS OF &

& FINITE => [Irr(&)] < 00 RANK OF &

RANK-FINITENESS CONJECTURE

JONLY FINITELY MANY FUSION CATEGORIES OF A GIVEN RANK, UP TO EQUIV.

GOVERNED BY SIMPLE OBJECTS

HAVE A FINITENESS RESULT USING THIS FACT

A MONOIDAL CATEGORY

C:= (C, S, L, a, L, r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (IR-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

GOVERNED BY SIMPLE OBJECTS ITT(():= SET OF ISOCLASSES [X]
OF SIMPLE OBJECTS OF &

& FINITE => [Irr(&)] < 00

RANK OF &

TAKE (Xi)ie Irr(c) ISOCLASS REPRESENTATIVES
OF SIMPLE OBJECTS OF &

A MONOIDAL CATEGORY

C:= (C, S, L, a, L, r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

GOVERNED BY SIMPLE OBJECTS ITT(%) := SET OF 150CLASSES [X]
OF SIMPLE OBJECTS OF &

C FINITE => [Irr(C)] < 00

RANK OF C

TAKE (Xi)ie Irr(c) ISOCLASS REPRESENTATIVES
OF SIMPLE OBJECTS OF &

GET Xi & Xj = 11 Xk IINK FOR SOME NK & EZzo

? NK Sijke Irr(): FUSION RULES OF &

Nk := [Xi & Xj : Xk] MULTIPLICITY OF XK IN Xi & Xj

A MONOIDAL CATEGORY

C:= (C, ⊗, L, a, L, r)
(S FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

GOVERNED BY SIMPLE OBJECTS OCNEANU RIGIDITY [ETINGOF-NIKSHYCH-OSTRIK]

JONLY FINITELY MANY FUSION CATEGORIES

WITH A GIVEN FUSION RULE.

TAKE (Xi)i: Irr(e) ISOCLASS REPRESENTATIVES
OF SIMPLE OBJECTS OF &

? NK Sijke Irr(): FUSION RULES OF &

Nij := [Xi & Xj : Xk] MULTIPLICITY OF XK IN Xi & Xj

A MONOIDAL CATEGORY

& := (&, ⊗, 1, a, 1, r)
(S FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

OCNEANU RIGIDITY (ETINGOF-NIKSHYCH-OSTRIK)

JONLY FINITELY MANY FUSION CATEGORIES WITH A GIVEN FUSION RULE.

LET'S RETURN TO

THE RANK 2 CASE

GOVERNED BY SIMPLE OBJECTS & FINITE ⇒ | Irr(c) | < ∞ RANK OF &

Xi & Xj = 11 Ke Irr(6) XK "INK" FOR SOME NK'j & Zo

? NK Sij, KEIM(e): FUSION RULES OF C

A MONOIDAL CATEGORY

C:= (C, ⊗, 1, a, 1, r)
(S FUSION IF:

- (a) & IS ABELIAN
- (b) & 18 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) 1 is ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

COSTRIK] 34 EQUIV. CLASSES OF RANK 2 FWSION CATS
LET'S LOOK AT TWO OF THEM...

GOVERNED BY SIMPLE OBJECTS C FINITE => | Irr(c) | < 00 RANK OF C Xi & Xj = $\coprod_{k \in Irr(c)} X_k \coprod_{k'}^{i,j}$ FOR SOME $N_k^{i,j} \in \mathbb{Z}_{>0}$ $\{N_k^{i,j}\}_{i,j,k \in Irr(c)}: FUSION RULES OF C$

A MONOIDAL CATEGORY

& := (&, ⊗, 1, a, 1, r)
(S FUSION IF:

- (a) & IS ABELIAN
- (b) & 18 (1R-) LINEAR
- (c) & IS LOC. FINITE
- (d) 1 Is ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

COSTRIK] 34 EQUIV. CLASSES OF RANK 2 FWSION CATS
LET'S LOOK AT TWO OF THEM...

GOVERNED BY SIMPLE OBJECTS C FINITE => | Irr(c) | < 00 RANK OF C Xi & Xj = $\coprod_{k \in Irr(c)} X_k \coprod_{k \in Irr(c)} X_k \coprod$

A MONOIDAL CATEGORY

- (a) & IS ABELIAN
- (b) & 18 (IR-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

COSTRIK] 34 EQUIV. CLASSES OF RANK 2 FWSION CATS
LET'S LOOK AT TWO OF THEM...

TRIVIAL REP SIGN REP

| Kv = g · v = v | | kv = g · v = -v

$$\bigwedge^{l} \otimes \bigwedge^{0} \stackrel{\sim}{\sim} \bigwedge^{l}$$

$$V_1 \otimes V_1 \cong V_0$$

GOVERNED BY SIMPLE OBJECTS & FINITE ⇒ | Irr(&) | < ∞ RANK OF &

?NK SijiKEIm(e): FUSION RULES OF &

A MONOIDAL CATEGORY

& := (&, ⊗, 1, a, 1, r)
(S FUSION IF:

- (a) & IS ABELIAN
- (b) & 18 (IR-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (g) & IS FINITE

COSTRIK] 34 EQUIV. CLASSES OF RANK 2 FWSION CATS
LET'S LOOK AT TWO OF THEM...

TRIVIAL REP SIGN REP

| KV 3. g.v=v | Kv 3. g.v=-v

$$\Lambda' \otimes \Lambda^0 \stackrel{2}{\sim} \Lambda'$$

$$V_1 \otimes V_1 \cong V_0$$

FIBONACCI FUSION CATEG.



DEFINED WITH FWION RUCES

GOVERNED BY SIMPLE OBJECTS & FINITE => | Irr(&) | < 00 RANK OF &

Xi & Xj = II KE ITY (6) XK INK FOR SOME NK EZO

?NK SijiKEIm(e): FUSION RULES OF &

A MONOIDAL CATEGORY

6 := (6, 8, 1, a, 1, r)
(5 FUSION IF:

- (a) & IS ABELIAN
- (b) & 18 (IR-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

COSTRIK] 34 EQUIV. CLASSES OF RANK 2 FWSION CATS
LET'S LOOK AT TWO OF THEM...

TRIVIAL REP SIGN REP

KV 3. g.v=v KV 3. g.v=-v

$$\Lambda' \otimes \Lambda^0 \stackrel{2}{\sim} \Lambda'$$

$$V_1 \otimes V_1 \cong V_0$$

FIBONACCI FUSION CATEG.



DEFINED WITH FWION RUCES

GOVERNED BY SIMPLE OBJECTS C FINITE ⇒ | Irr(c) | < ∞ RANK OF C

?NK Sij, KEIM(e): FUSION RULES OF &

A MONOIDAL CATEGORY

& := (&, ⊗, 1, a, 1, r)
(S FUSION IF:

- (a) & IS ABELIAN
- (b) & 18 (1R-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

COSTRIK] 34 EQUIV. CLASSES OF RANK 2 FINSION CATS
LET'S LOOK AT TWO OF THEM...

TRIVIAL REP SIGN REP

| Kr + g + v = v | Kr + g + v = -v

$$V_1 \otimes V_1 \cong V_0$$

DIFFERENT FUSION RULES J INEQUIVALENT



$$Irr(e) = \{1, X\}$$

DEFINED WITH FWION RUCES

GOVERNED BY SIMPLE OBJECTS & FINITE => | Irr(E) | < 00 RANK OF &

?NK Sij, KEIM(e): FUSION RULES OF &

II. FUSION RULES & RANK RELATED TO THE "SIZE" OF X & FIL

A MONOIDAL CATEGORY

C:= (C, ⊗, L, a, L, r) IS FUSION IF:

- (a) & IS ABELIAN
- (b) & IS (IR-) LINEAR
- (c) & IS LOC. FINITE
- (d) 1 is ABS. SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (q) & IS FINITE

34 EQUIV. CLASSES

Irr(%)=[[Vo],[Vi]]

TRIVIAL REP SIGN REP RV 3.9.5=V RV 3.9.5=-5

VI & VI = Vo

DIFFERENT FUSION RULES INEQUIVALENT

FIBONACCI FUSION CATEG.

OF RANK 2 FWION CATS

Fib

Irr(6) = { 1, X]

DEFINED WITH FWION RUCES

10121

10 X = X

Xel=X

XOX = 1UX

GOVERNED BU SIMPLE OBJECTS & FINITE ⇒ | Irr(C) | < 00 RANK OF C Xi & Xj = II KE IT (E) XK INK FOR SOME NK EZO ENK SijiKEIM(E): FUSION RULES OF &

A MONOIDAL CATEGORY C:= (C, S, L, a, L, r) IS FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) 1 Is ABS. SIMPLE
- (e) & IS RIGID
- (f) 6 IS SEMISIMPLE
- (g) & IS FINITE

GOVERNED BY SIMPLE OBJECTS & FINITE ⇒ | Irr(&) | < ∞ RANK OF &

Xi ⊗ Xj = 11 | Ke Irr(&) | Xk HNK FOR SOME NK ∈ Zo.

?NK Sijike Irr(&): FUSION RULES OF &

A MONOIDAL CATEGORY

C:= (C, S, L, a, L, r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & 15 (k-) LINEAR
- (c) & IS LOC. FINITE
- (d) 1 is ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

FROBENIUS-PERRON THEOREM TAKE ME MATN (1R70).

THEN 3 NON-NEGATIVE REAL ELGENVALUE

FP(M) THAT IS > ABSOLUTE VALUE OF ALL

OTHER ELGENVALUES OF M.

GOVERNED BY SIMPLE OBJECTS C FINITE => | Irr(&) | < 00 RANK OF &

Xi & Xj = $\coprod_{k \in Irr(a)} X_k \coprod_{k \in Irr(a)} X_k \coprod$

A MONOIDAL CATEGORY

C:= (C, S, L, a, L, r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & IS (IR-) LINEAR
- (c) & IS LOC. FINITE
- (d) 1 is ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

FROBENIUS-PERRON THEOREM TAKE ME MATN (1R30).

THEN 3 NON-NEGATIVE REAL ELGENVALUE

FP(M) THAT IS > ABSOLUTE VALUE OF ALL

OTHER ELGENVALUES OF M.

FP-DIMENSION OF Xi & Irr(C):

GOVERNED BY SIMPLE OBJECTS C FINITE => | Irr(&) | < 00 RANK OF &

Xi & Xj = 11 ke Irr(&) Xk Ink FOR SOME NK & EZO.

ZNK Sij, ke Irr(&): FUSION RULES OF &

A MONOIDAL CATEGORY

C:= (C, S, L, a, L, r)

IS FUSION IF:

- (a) & IS ABELIAN
- (b) & IS (IR-) LINEAR
- (c) & IS LOC. FINITE
- (d) IL IS ABS. SIMPLE
- (e) & IS RIGID
- (f) & IS SEMISIMPLE
- (g) & IS FINITE

FROBENIUS-PERRON THEOREM TAKE ME MATA (1R30).

THEN 3 NON-NEGATIVE REAL ELGENVALUE

FP(M) THAT IS > ABSOLUTE VALUE OF ALL

OTHER ELGENVALUES OF M.

FP-DIMENSION OF Xi & Irr(C):

FP-DIMENSION OF & ITSELF:

GOVERNED BY SIMPLE OBJECTS C FINITE => | Irr(&) | < 00 RANK OF &

Xi & Xj = 11 ke Irr(&) Xk UNK FOR SOME NK & EZO.

? NK Sij ke Irr(&): FUSION RULES OF &

FP THEOREM

TAKE ME MATN (1R70).

→ J E.VALUE

FP(M) & IR 30

THAT IS

> ABS VALUE OF ALL E.VALUES OF M.

FPding (Xi)

ii

FP(Ni)j,keIrr(&)

FPdin(e)

II

FPding(Xi)²
i = Irr(c)

& FINITE ⇒ | Irr(c) | < ∞ RANK OF &

Xi ⊗ Xj = ⊥ | Ke Irr(c) Xk LINK' FOR SOME NK' ∈ Z>0

? NK Jijke Irr(c): FUSION RULES OF &

FP THEOREM

TAKE ME MATN (1R70).

⇒ J E.VALUE FP(M) ∈ IR 20

THAT IS

> ABS VALUE OF ALL E.VALUES OF M.

FPding (Xi)

ii

FP(Nij)

C2-Fd Mod

Irr(%)= [[V0], [V1]]

Vo & Vo = Vo

Vo & Vi = Vi

 $V_1 \otimes V_0 \cong V_1$

VI & VI = Vo

WHAT IS

FPdime (Vo)?

FPdime (V1)?

FPdim (6)?

FPdim (C)

W

FPding(Xi)²
i = Irr(c)

C FINITE ⇒ | Irr(C) | < ∞ RANK OF C

Xi & Xj = II Ke Irr(c) Xk IINK FOR SOME NK EZO.

?NK SijiKEIM(E): FUSION RULES OF &

FP THEOREM

TAKE ME MATN (1R70).

→ J E.VALUE

FPCM) & IR 20 THAT IS

> ABS VALUE OF ALL EVALUES OF M.

FPding(Xi)

FP(N')j, K & Irr(C)

FPdim (c)

II

FPding(Xi)²
i = Irr(c)

Vo & Vo = Vo

Vo & Vi = Vi

 $V_1 \otimes V_0 \cong V_1$

VI & VI = Vo

$$\frac{i=0:}{N_0^{0,0}} \left(\begin{array}{c} N_0^{0,0} \\ N_0^{0,1} \end{array} \right)$$
HAS EVACUES = $\frac{1}{2}$

=> FPdime (Vo) =

$$\frac{i=1}{N_0^{1/0}} \cdot \frac{N_1^{1/0}}{N_0^{1/1}}$$
HAS EVACUES = {

⇒ FPdime (V1) =

C FINITE => | Irr(C) | < 00 RANK OF C Xi & Xj = $\coprod_{k \in Irr(C)} X_k \coprod_{k \in Irr(C)} X_k \coprod_{k \in Irr(C)} X_k \coprod_{k \in Irr(C)} X_k \coprod_{k \in Irr(C)} X_k \bigcup_{k \in Irr(C)} X_k \bigcup$

FP THEOREM

TAKE ME Matn(1R70).

→ J E.VALUE

FP(M) ∈ IR 30

THAT IS

> ABS VALUE OF ALL E.VALUES OF M.

FPding (Xi)

ij

FP(Nijó)

FPdim (c)

11

 \mathbb{Z} FPdim_c(X_i)² i=Irr(c)

C2-Fd Mod

Irr(%)= [[V0], [V1]]

Vo & Vo = Vo

Vo & Vi = Vi

 $\bigwedge^{l} \otimes \bigwedge^{0} \stackrel{2}{\sim} \bigwedge^{l}$

VI & VI = Vo

: FPdim(C)=

 $\underbrace{i=0:}_{\mathcal{N}_{0}^{0,1}} \left(\begin{array}{cc} 1 & O \\ \mathcal{N}_{0}^{0,1} & \mathcal{N}_{1}^{0,1} \end{array} \right)$

HAS EVACUES = {

⇒ FPdime (Vo) =

 $\underline{i=1}:\begin{pmatrix} \mathcal{N}_{0}^{1/0} & \mathcal{N}_{1}^{1/0} \\ \mathcal{N}_{0}^{1/1} & \mathcal{N}_{1}^{1/1} \end{pmatrix}$

HAS EVACUES = {

=> FPdime (V1)=

& FINITE => | Irr(C) | < 00 RANK OF &

Xi & Xj = II KE ITT(C) XK HNK FOR SOME NK EZO

[NK SijKEIM(E): FUSION RULES OF C

FP THEOREM

TAKE ME MATN (1R70).

⇒ J E.VALUE FP(M) ∈ IR 30

THAT IS

> ABS VALUE OF ALL E.VALUES OF M.

FPding (Xi)

ij

FP(Nij)

FPdim (c)

II

 \mathbb{Z} FPding $(X_i)^2$ if Irr(c)

Vo & Vo = Vo

VI & Vo = VI

VI &VI = Vo

$$\frac{i=0}{0}$$
: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

HAS EVACUES = {

⇒ FPdime (Vo) =

$$\underline{i=1}:\begin{pmatrix} \mathcal{N}_{0}^{1/0} & \mathcal{N}_{1}^{1/0} \\ \mathcal{N}_{0}^{1/1} & \mathcal{N}_{1}^{1/1} \end{pmatrix}$$

HAS EVACUES = {

=> FPdime (V1)=

C FINITE ⇒ | Irr(c) | < ∞ RANK OF C Xi ⊗ Xj = ∐_{K∈Irr(c)} X_K ^{∐N_Kij} FOR SOME N_Kij ∈ Z_j o ?N_K ∫_{i,j,K∈Irr(c)}: FUSION RULES OF C

FP THEOREM

TAKE ME MATN (1R30).

⇒ J E.VALUE FP(M) ∈ IR 30

THAT IS

> ABS VALUE OF ALL E.VALUES OF M.

FPding (Xi)

FP(Nijó)

FPdim (C)

II

FPding(Xi)²
i = Irr(c)

Irr(%)= [[V0], [V1]]

 $V_1 \otimes V_0 \cong V_1$

VI &VI = Vo

$$\frac{i=0}{0}$$
: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

HAS EVACUES = {1]

⇒ FPdime (Vo) = 1

$$\underline{i=1}:\begin{pmatrix} \mathcal{N}_{0}^{1/0} & \mathcal{N}_{1}^{1/0} \\ \mathcal{N}_{0}^{1/1} & \mathcal{N}_{1}^{1/1} \end{pmatrix}$$

HAS EVACUES = {

⇒ FPdime (V1) =

& FINITE ⇒ | Irr(E) | < ∞ RANK OF E

Xi & Xj = II KE Irr(6) XK INK FOR SOME NK EZO

?NK Sij, K∈ Irr(e): FUSION RULES OF €

FP THEOREM

TAKE ME Matn (1R>0).

→ J E.VALUE FP(M) & IR zo

THAT IS

> ABS VALUE OF ALL EVALUES OF M.

Irr(%)= {[Vo], [Vi] }

Vo & Vo = Vo

VI & Vo = VI

HAS EVACUES =
$$\{1\}$$

$$\Rightarrow \text{FPdime}_{\bullet}(V_{\bullet}) = 1$$

$$V_{1} \otimes V_{0} \cong V_{1}$$

$$V_{1} \otimes V_{1} \cong V_{0}$$

$$V_{1} \otimes V_{1} \cong V_{0}$$

$$V_{1} \otimes V_{1} \cong V_{0}$$

$$V_{2} \otimes V_{3} \cong V_{0}$$

$$V_{3} \otimes V_{4} \cong V_{0}$$

$$V_{4} \otimes V_{5} \cong V_{0}$$

$$V_{5} \otimes V_{7} \cong V_{0}$$

$$V_{7} \otimes V_{7} \cong V_{0}$$

$$V_{8} \otimes V_{7} \cong V_{7}$$

$$V_{8} \otimes V_{7} \cong V_{7} \cong V_{7}$$

$$V_{8} \otimes V_{7} \cong V_{7} \cong V_{7}$$

$$V_{8} \otimes V_{7} \cong V_{7} \cong V_{7} \cong V_{7}$$

$$V_{8} \otimes V_{7} \cong V_{7$$

& FINITE => | Irr(&) | < 00 RANK OF & Xi & Xj = 11 Ke Irr(6) XK HNK FOR SOME NK EZO [NK Sijike In(e): FUSION RULES OF &

FP THEOREM

TAKE ME MATN (1R70).

→ J E.VALUE FP(M) & IR 20

THAT IS

> ABS VALUE OF ALL E.VALUES OF M.

FPding (Xi)

FP(Nijó), KEIr(C).

C2-Fd Mod

Irr(%)= [[V0], [V1]]

Vo & Vo = Vo

Vo & Vi = Vi

V₁ ⊗ V₀ ≥ V₁

VI & VI = Vo

· FPdim(C)=

 $\frac{i=0}{0}$: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

HAS EVACUES = {1]

⇒ FPdime (Vo) = 1

 $\underline{i=1}: \left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right)$

HAS EVACUES = {

⇒ FPdime (V1) =

FPdim(t)

11

 \mathbb{Z} FPding $(X_i)^2$ in \mathbb{Z}

FP THEOREM

TAKE ME MATN (1R70).

→ J E.VALUE

FP(M) ∈ IR 30

THAT IS

> ABS VALUE OF ALL E. VALUES OF M.

FPding (Xi)

FP(Nijó), KEIrr(C).

C2-Fd Mod

Irr(%)= [[V0], [V1]]

Vo & Vo = Vo

Vo & Vi = Vi

 $V_1 \otimes V_0 \cong V_1$

 $V_1 \otimes V_1 \cong V_0$

· FPdim(C)=

 $\frac{i=0}{0}$: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

HAS EVACUES = {1]

⇒ FPdime (Vo) = 1

| i=1: (0 1)

HAS EVACUES = [1,-1]

⇒ FPdime (V1) = 1

FPdin(e)

11

 \mathbb{Z} FPding $(X_i)^2$ in \mathbb{Z}

& FINITE ⇒ | Irr(C) | < ∞ RANK OF C Xi ⊗ Xj = II Ke Irr(C) Xk Inkij FOR SOME NK EZ; o ?NK Sijike Irr(C): FUSION RULES OF C

FP THEOREM

TAKE ME MATN (1R70).

→ J E.VALUE

FP(M) & IR 30

THAT IS

> ABS VALUE OF ALL E.VALUES OF M.

FPdine (Xi)

ij

FP(Nijó)

FPdim (c)

11

 \mathbb{Z} FPding(X_i)² i=Irr(α)

Vo & Vo = Vo

Vo & Vi = Vi

 $V_1 \otimes V_0 \cong V_1$

VI & VI = Vo

: FPdim(c) = 2

$$\frac{i=0}{0}$$
: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

HAS EVACUES = [1]

⇒ FPdime (Vo) = 1

HAS EVACUES = [1,-1]

⇒ FPding (V1) = 1

& FINITE ⇒ | Irr(E) | < ∞ RANK OF &

Xi & Xj = II KE Irr(6) XK HNK FOR SOME NK EZO

[NK SijKEIM(E): FUSION RULES OF C

FP THEOREM

TAKE ME MATN (1R70).

→ J E.VALUE FP(M) ∈ IR =0

THAT IS

> ABS VALUE OF ALL E.VALUES OF M.

FPding (Xi)

ij

FP(Nij)j, KeIn(E).

FPdin(e)

11

 \mathbb{Z} FPding $(X_i)^2$ i=Irr(c)

WHAT IS

FPdime (11)?

FPdime(X)?

FPdim (6)?

FIBONACCI FUSION CATEG.

Fib

Irr (e) = { 11, X }

DEFINED WITH FWION RUCES

10121

10 X = X

X ® 1 = X

XOX = 1UX

& FINITE => | Irr(E) | < 00 RANK OF &

Xi & Xj = II KE Irr(6) XK INK FOR SOME NK EZO

{NK Sij, K∈ Irr(e): FUSION RULES OF €

FP THEOREM

TAKE ME MATN (1R30).

⇒ J E.VALUE

FP(M) ∈ IR ≥ 0

THAT IS

> ABS VALUE OF ALL E.VALUES OF M.

$\underbrace{i=0:}_{\mathcal{N}_{0}^{0}} \left(\begin{array}{ccc} \mathcal{N}_{0}^{0} & \mathcal{N}_{0}^{1} \\ \mathcal{N}_{0}^{0} & \mathcal{N}_{0}^{1} \end{array} \right)$

HAS EVACUES = {

⇒ FPdime (1) =

FPding (Xi)

FP(Nijo)

$\underline{i=1}: \left(\begin{array}{cc} \mathcal{N}_0^{1/0} & \mathcal{N}_1^{1/0} \\ \mathcal{N}_0^{1/1} & \mathcal{N}_1^{1/1} \end{array}\right)$

HAS EVACUES = {

⇒ FPdime (X) =

FIBONACCI FUSION CATEG.

Trr(e) = { 11, X 1

DEFINED WITH FWION RUCES

10121

10 X = X

X ® 1 = X

XXX = 1UX

: FPdim(C)=

FPdin(e)

II

FPding(Xi)²
i = Irr(c)

C FINITE \Rightarrow $|\text{Irr}(\mathcal{C})| < \infty$ RANK OF \mathcal{C} $Xi \otimes Xj \cong \coprod_{k \in \text{Irr}(\mathcal{C})} X_k \coprod_{k \in \mathcal{C}} For \text{ some } N_k^{i,j} \in \mathcal{C}_{>0}$ $\{N_k^{i,j}\}_{i,j,k \in \text{Irr}(\mathcal{C})}: \text{ Fusion Rules of } \mathcal{C}_{>0}\}$

FP THEOREM

TAKE ME Matn (1R30).

⇒ J E.VALUE FP(M) ∈ IR =0

THAT IS

> ABS VALUE OF ALL E. VALUES OF M.

i=0:
$$(1 \ 0)$$

HAS EVACUES = (1)
 \Rightarrow FPdime (1) =

FPding (Xi) ii FP(N^{i,j}) j, k e Irr(c).

FPdin(e)

11

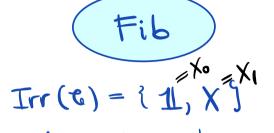
FPding(Xi)²
i = Irr(c)

$$i=1: \begin{pmatrix} N_0^{1/0} & N_1^{1/0} \\ N_0^{1/1} & N_1^{1/1} \end{pmatrix}$$

$$HAS EVACUES = \{ \}$$

$$\Rightarrow FPdime(X) = \{ \}$$

FIBONACCI FUSION CATEG.



DEFINED WITH FWION RUCES

C FINITE => | Irr(c) | < 00 RANK OF C
Xi & Xj =
$$\coprod_{k \in Irr(c)} X_k \stackrel{\coprod_{N_k^{i,j}}}{\longrightarrow}$$
 FOR SOME $N_k^{i,j} \in \mathbb{Z}_{>0}$
 $\{N_k^{i,j}\}_{i,j,k \in Irr(c)}: FUSION RULES OF C$

FP THEOREM

TAKE ME MATN (1R70).

⇒ J E.VALUE FP(M) ∈ IR =0

THAT IS

> ABS VALUE OF ALL E.VALUES OF M.

i=0: (1 0)

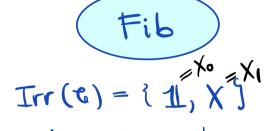
HAS EVACUES = { 1]

⇒ FPdime (1) = 1

$\underline{i=1}: \begin{pmatrix} N_0^{1/0} & N_1^{1/0} \\ N_0^{1/1} & N_1^{1/1} \end{pmatrix}$ HAS EVACUES = $\frac{3}{2}$

⇒ FPdime. (X) =

FIBONACCI FUSION CATEG.



DEFINED WITH FWION RUCES

Xex = 1ux

FPdin(e)

II

 \mathbb{Z} FPding $(X_i)^2$ in \mathbb{Z}

C FINITE \Rightarrow $|\text{Irr}(\mathcal{C})| < \infty$ RANK OF \mathcal{C} $Xi \otimes Xj \cong \coprod_{k \in \text{Irr}(\mathcal{C})} X_k \coprod_{k \in \mathcal{C}} For \text{ some } N_k^{i,j} \in \mathcal{C}_{>0}$ $\{N_k^{i,j}\}_{i,j,k \in \text{Irr}(\mathcal{C})} : \text{Fusion Rules of } \mathcal{C}_{>0}$

FP THEOREM

TAKE ME MATN (1R70).

⇒ J E.VALUE FP(M) ∈ IR 20

THAT IS > ABS VALUE OF ALL E. VALUES OF M.

FPdin(e) ii El FPding(Xi)²

i∈ Irr(C)

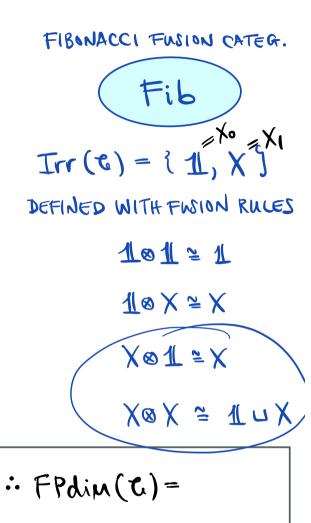
i=0:
$$(1 \ 0)$$

$$(0 \ 1)$$
HAS EVACUES = (1)

$$\Rightarrow \text{FPdime}_{(1)}(1) = 1$$

$$i=1:\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$
HAS EVACUES = $\{ \}$

$$\Rightarrow \text{FPdime}(X) = \{ \}$$



C FINITE
$$\Rightarrow$$
 $|\text{Irr}(\mathcal{C})| < \infty$ RANK OF \mathcal{C}

Xi \otimes Xj \cong $\coprod_{k \in \text{Irr}(\mathcal{C})}$ $X_k \stackrel{\text{Lin}_{k,j}}{\longrightarrow}$ FOR SOME $N_k^{i,j} \in \mathcal{C}_{>0}$.

 $\{N_k^{i,j}\}_{i,j,k \in \text{Irr}(\mathcal{C})}: \text{Fusion Rules of } \mathcal{C}_{>0}\}$

FP THEOREM

TAKE ME Matn(1R30).

→ J E.VALUE FP(M) E IR 30

THAT IS

> ABS VALUE OF ALL EVALUES OF M.

HAS EVACUES = { 1]

⇒ FPdime. (1) = 1

FPding (Xi)

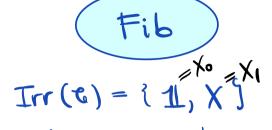
FPdin(e)

2 FPding(Xi)2 i∈ Irr(C)

$$i=1:\begin{pmatrix}0&1\\1&1\end{pmatrix}$$
HAS EVACUES = $\{\frac{1}{2}(1\pm J5)\}$

$$\Rightarrow$$
 FPdime (X) = $\frac{1}{2}(1+55)$: FPdim(C) =

FIBONACCI FUSION CATEG.



DEFINED WITH FWION RUCES

FP THEOREM

TAKE ME Matn(1R30).

→ J E.VALUE FP(M) E IR 30

THAT IS

> ABS VALUE OF ALL EVALUES OF M.

$$i=0: (1 0)$$

HAS EVACUES = { 1]

=> FPding (1) = 1

FIBONACCY FUSION CATEG. Fib Irr(6) = { 1, x 3/1 DEFINED WITH FWION RUCES 10121

GOLDEN 18X = X

X × 1 = X

XOX = 1LUX/

RATIO

FPding (Xi)

HAS EVACUES = {\frac{1}{2}(1\pm J\sigma)}

⇒ FPdime (X) = [(HJ5)]: FPdim(C) =

FPdin(e) 2 FPding(Xi)2

i∈ Irr(C)

& FINITE => (Irr(E)) < 00 RANK OF & Xi & Xj = II KE ITY(E) XK IINK FOR SOME NK EZO ENK SijiKEIM(E): FUSION RULES OF &

FP THEOREM

TAKE ME Matn(1R30).

→ J E.VALUE FP(M) E IR 30

THAT IS

> ABS VALUE OF ALL EVALUES OF M.

$$\frac{i=0}{0}$$
: $\left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right)$

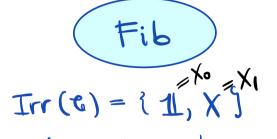
HAS EVACUES = { 1]

⇒ FPdime. (1) = 1

$$i=1:\begin{pmatrix}0&1\\1&1\end{pmatrix}$$
HAS EVACUES = $\{\frac{1}{2}(1\pm \sqrt{5})\}$

$$\Rightarrow$$
 FPdime (X) = $\frac{1}{2}$ (1+55)

FIBONACCI FUSION CATEG.



DEFINED WITH FWION RUCES

$$\Rightarrow \text{FPdime}(X) = \frac{1}{2}(1+55) \quad \text{: FPdim}(C) = \frac{5+55}{2}$$

FPdin(e) E FPding(Xi)2 i∈ Irr(C)

& FINITE ⇒ | Irr(E) | < ∞ RANK OF & Xi & Xj = II KE Irr(6) XK INK FOR SOME NK EZO ENK SijiKEIM(E): FUSION RULES OF &

FP THEOREM

TAKE ME MATN (1R70).

→ J E.VALUE

FP(M) ∈ IR =0

THAT IS

> ABS VALUE OF ALL E. VALUES OF M.

FPding (Xi)

ii

FP(Nij)j, KeIr-(E).

FPdim (C)

II

 \mathbb{Z} FPding(X_i)² i=Irr(α)

Irr(%)= [[V0], [V1]]

$$V_1 \otimes V_0 \cong V_1$$

VI & VI = Vo

FIBONACCI FUSION CATEG.



Irr (6) = { 11, X]

DEFINED WITH FWION RUCES

$$\therefore FPdim(C) = \frac{5+\sqrt{5}}{2}$$

& FINITE => | Irr(E) | < 00 RANK OF &

Xi & Xj = II KE Irr(6) XK INK FOR SOME NK EZO

[NK SijKEIM(E): FUSION RULES OF &

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FP-DIMENSION

TOPICS:

. FUSION CATEGORIES

(§§3.9.1, 3.9.3)

II. FUSION RULES & RANK

(§§3.9.1, 3.9.3)

II. FROBENIUS - PERRON DIMENSION

(f3.9.2)

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MORE EXAMPLES OF FUSION CATEGORIES,

& ABOUT THEIR MODULE CATEGORIES

TOPICS:

. FUSION CATEGORIES

(§§3.9.1, 3.9.3)

JE. FUSION RULES & RANK

(§§3.9.1, 3.9.3)

II. FROBENIUS - PERRON DIMENSION (\$3.9.2)

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& ABOUT THEIR MODULE CATEGORIES

HAS A RICH

TOPICS:

£. FUSION CATEGORIES <

(383.9.1, 3.9.3)

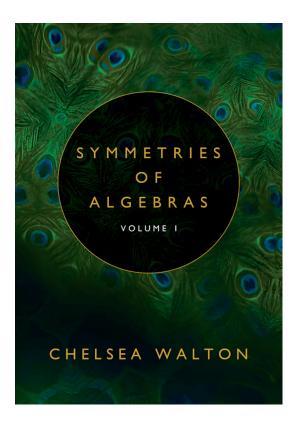
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<u>Lecture #16 keywords</u>: Frobenius-Perron dimension, Frobenius-Perron Theorem, fusion category, fusion rules, multiplicity, Ocneanu rigidity, rank, Rank-Finiteness Conjecture