Preview for Lecture #18 of C. Walton's Spring 2024 course on "Symmetries of Algebras" at Rice U.
Keywords: algebra in a monoidal category, coinduced algebra, Doctrinal Adjunction, ideal in a monoidal category, monad, quotient algebra in a monoidal category, subalgebra in a monoidal category
https://math.rice.edu/~notlaw/symalgbook.html, https://www.619wreath.com/

I. ALGEBRAS IN MONOIDAL CATEGORIES

MONOIDAL CATEGORY $(\mathcal{C}, \otimes, \mathbb{1}, a, l, r)$

CONSISTS OF:

(a) CATEGORY $\mathcal{C}$

(b) BIFUNCTOR $\otimes : \mathcal{C} \times \mathcal{C} \to \mathcal{C}$

(c) OBJECT $\mathbb{1} \in \mathcal{C}$

(d,e,f) NATURAL ISOMS:

$$a = \left\{ a_{X,Y,Z} : (X \otimes Y) \otimes Z \xrightarrow{\sim} X \otimes (Y \otimes Z) \right\}_{X,Y,Z \in \mathcal{C}}$$

$$l = \{ l_X : \mathbb{1} \otimes X \xrightarrow{\sim} X \}_{X \in \mathcal{C}}$$

$$r = \{ r_X : X \otimes \mathbb{1} \xrightarrow{\sim} X \}_{X \in \mathcal{C}}$$

SATISFYING THE

PENTAGON AXIOM

& TRIANGLE AXIOM

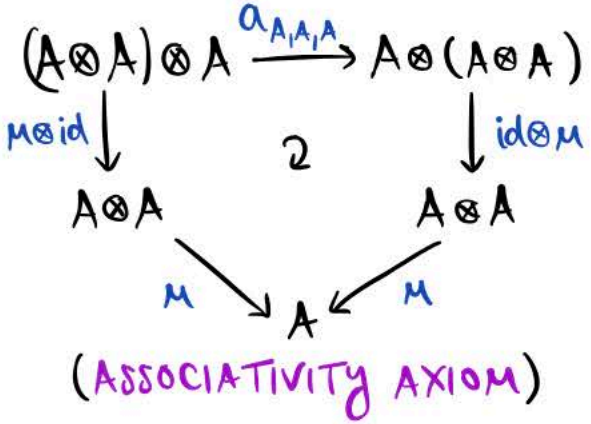
AN ALGEBRA IN $(\mathcal{C}, \otimes, \mathbb{1}, a, l, r)$ CONSISTS OF:

(a) AN OBJECT $A \in \mathcal{C}$

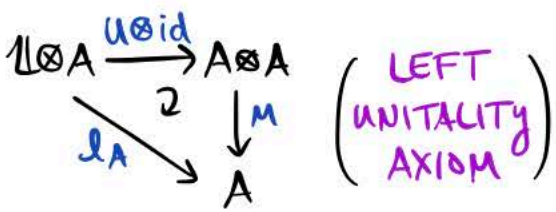
(b) A MORPHISM $m := m_A : A \otimes A \to A \in \mathcal{C}$
(MULTIPLICATION MORPHISM)

(c) A MORPHISM $u := u_A : \mathbb{1} \to A \in \mathcal{C}$
(UNIT MORPHISM)

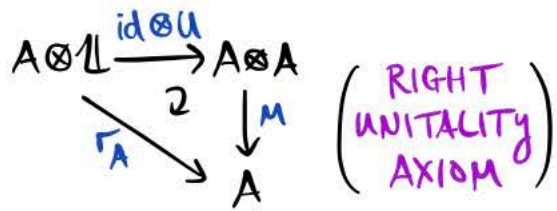
SATISFYING:



(ASSOCIATIVITY AXIOM)



(LEFT UNITALITY AXIOM)



(RIGHT UNITALITY AXIOM)