III. EILENBERG-MOORE CATEGORIES

TAKE A CATEGORY $\mathcal{A}$.

$\mathrm{Monad}(\mathcal{A}) := \mathrm{Alg}(\mathrm{End}(\mathcal{A}))$

OBJECTS:

ENDOFUNCTOR $T: \mathcal{A} \to \mathcal{A}$

NAT TRANS: $\mu: T \circ T \Rightarrow T$

$\eta: \mathrm{Id}_{\mathcal{A}} \Rightarrow T$

SATISFYING $\forall X \in \mathcal{A}$:

$$\begin{array}{ccc} T^3(X) & \xrightarrow{\mu_{T(X)}} & T^2(X) \\ {\scriptstyle T(\mu_X)}\downarrow & 2 & \downarrow{\scriptstyle \mu_X} \\ T^2(X) & \xrightarrow{\mu_X} & T(X) \end{array}$$

$$\begin{array}{ccc} T(X) & \xrightarrow{\eta_{T(X)}} & T^2(X) \\ & {\scriptstyle id_{T(X)}}\searrow \; 2 & \downarrow{\scriptstyle \mu_X} \\ & & T(X) \end{array}$$

$$\begin{array}{ccc} T(X) & \xrightarrow{T(\eta_X)} & T^2(X) \\ & {\scriptstyle id_{T(X)}}\searrow \; 2 & \downarrow{\scriptstyle \mu_X} \\ & & T(X) \end{array}$$

A NICE SUBCATEGORY OF MODULES OVER MONADS

$T\text{-Mod}(\mathrm{End}(\mathcal{A}))$ FOR $T := (T, \mu, \eta)$ IS QUITE LARGE

MUCH BETTER THEORY FOR EM CATEGORIES

ADJUNCTIONS ⇄ MONADS

WILL SEE