MATH 466/566 SPRING 2024

CHELSEA WALTON RICE U.

LASTTIME

LECTURE #20

- · (BI) MODULES IN (C, 0, 1)
- . MONADS & EM CATEGORIES

TOPICS:

I. OPERATIONS ON ALGEBRAS & (BI) MODULES (§ 4.5)

II. GENERALIZED EILENBERG-WATTS THEOREM (54.7.1)

III. GENERALIZED MORITA'S THEOREM (§4.7.2)

GIVEN A MONOIDAL CATEGORY (8,0,1),

AN ALGEBRA IN & IS (A, M: A&A -> A, u; 11 -> A)
OBJECTING MORPHISMS IN &
SATISFYING ASSOCIATIVITY + UNITALITY AXIOMS

GIVEN A MONOIDAL CATEGORY (8,0,1),

A LEFT A-MODULE IN CO IS (M, D: A&M -> M) Control Home (A&M, M) SAT. ASSOC. + UNIT. AXIOMS A RIGHT A-MODULE IN C 1S (M, d:M@A -> M) C HONZ(M@A,M) SAT. ASSOC. + UNIT. AXIOMS

GIVEN A MONOIDAL CATEGORY (8,0,1),

A LEFT A-MODULE IN C IS (M, D: A&M -> M) C Home (A&M, M) SAT. ASSOC. + UNIT. AXIOMS A RIGHT A-MODULE IN &

1S (M, d:M&A -> M)

then (M&A,M)

SAT. ASSOC. + UNIT. AXIOMS

A
$$(B_1, B_2)$$
-BIMODULE IN & IS $(M, D: B_1 \otimes M \rightarrow M, 4: M \otimes B_2 \rightarrow M)$
.9. $(M, D) \in B_1 - Mod(A)$, $(M, A) \in Mod - B_2(A)$
 $4 (D \otimes id_{B_2}) = D(id_{B_1} \otimes A) \alpha_{B_1, M, B_2}$

I. OPERATIONS ON ALGEBRAS & (BI) MODULES GIVEN A MONOIDAL CATEGORY (8,0,1), (CREATE MORE!



AN ALGEBRA IN & IS (A, M: AQA -> A, u; 11-> A) OBJECT IN & MORPHISMS IN & SATISFYING ASSOCIATIVITY + UNITALITY AXIOMS

A LEFT A-MODULE IN & (S (M, D: A@M -> M)

(B Hone (A@M, M) SAT. ASSOC. + UNIT. AXIOMS

A RIGHT A-MODULE IN & $(M, A: M \otimes A \longrightarrow M)$ n n
tonz (MOA, M) SAT. ASSOC. + UNIT. AXIOMS

A (B1, B2)-BIMODULE IN & IS (M, D: B1@M-)M, 4:M@B2-)M) . >. (M, D) ∈ B1-Mod(C), (M, d) ∈ Mod-B2(C)

ASSUME (C, Ø, 11) } ABELIAN MONOIDAL

TAKE MON. CAT. (4,0,1).

ALGEBRA IN & =

(A, M:A&A -> A, N: L -> A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN C = (M, D: A⊗M→M) SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN & =

(M, 4: M⊗A → M)

SAT. ASSOC + UNITALITY

 $(B_{1}, B_{2}) - BIMOD. | N C =$ $(M_{1}, D : B_{1} \otimes M \rightarrow M_{2})$ $4 : M \otimes B_{2} \rightarrow M_{2}$ $3 \cdot (M_{1}, D) \in B_{1} - Mod(C),$ $(M_{1}, 4) \in Mod - B_{2}(C)$ $4 \cdot 4(D \otimes id_{B_{2}}) =$ $D(id_{B_{1}} \otimes 4) \alpha_{B_{1}, M_{1}B_{2}}$

... VIA BIPRODUCTS

TAKE (A1, M1, U1), (A2, M2, U2) & Alg(4).

DEF: $(A_1, M_1, U_1) \square (A_2, M_2, U_2)$:= $(A_1 \square A_2, M_\square, U_\square) \in Alg(\mathcal{C})$

ASSUME (C, Ø, 11) } ABELIAN MONOIDAL

TAKE MON. CAT. (4,0,1).

ALGEBRA IN & =

(A, M:A&A -> A, N:L->A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN C = (M, D: A⊗M→M) SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN & = (M, 4: M⊗A → M) SAT. ASSOC + UNITALITY

$$(B_{1}, B_{2}) - BIMOD. | N C =$$

$$(M_{1}, D : B_{1} \otimes M \rightarrow M_{2})$$

$$4 : M \otimes B_{2} \rightarrow M_{2}$$

$$3 : (M_{1}, D) \in B_{1} - Mod(C),$$

$$(M_{1}, 4) \in Mod - B_{2}(C)$$

$$4 \cdot 4(D \otimes id_{B_{2}}) =$$

$$D(id_{B_{1}} \otimes 4) \cap A_{B_{1}, M_{1}, B_{2}}$$

... VIA BIPRODUCTS

TAKE (A1, M1, U1), (A2, M2, U2) & Alg(4).

DEF:
$$(A_1, M_1, U_1) \square (A_2, M_2, U_2)$$

:= $(A_1 \square A_2, M_\square, U_\square) \in Alg(\mathcal{C})$

VIA
$$M_{\square}: (A_{1} \square A_{2}) \otimes (A_{1} \square A_{2})$$

$$\sim \downarrow$$

$$(A_{1} \otimes A_{1}) \square (A_{1} \otimes A_{2})$$

$$\square (A_{2} \otimes A_{1}) \square (A_{2} \otimes A_{2})$$

$$M_{1} \square 0 \square 0 \square A_{2}$$

$$A_{1} \square 0 \square 0 \square A_{2}$$

$$A_{1} \square A_{2}$$

(C, O, 1) }
ABELIAN MONOIDAL

TAKE MON. CAT. (4,0,1).

ALGEBRA IN & =

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LEFT A-MODULE IN C = (M, D: A⊗M→M) SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN & = (M, 4: M&A -> M) SAT. ASSOC + UNITALITY

$$(B_{1}, B_{2}) - BIMOD. | N C =$$

$$(M_{1}, D: B_{1} \otimes M \rightarrow M_{1})$$

$$4: M \otimes B_{2} \rightarrow M)$$

$$3. (M_{1}, D) \in B_{1} - Mod(C),$$

$$(M_{1}, d) \in Mod - B_{2}(C)$$

$$4 d(D \otimes id_{B_{2}}) =$$

$$D(id_{B_{1}} \otimes d) \alpha_{B_{1}, M_{1}, B_{2}}$$

... VIA BIPRODUCTS

TAKE (A1, M1, U1), (A2, M2, U2) & Alg(4).

DEF:
$$(A_1, M_1, u_1) \square (A_2, M_2, u_2)$$

:= $(A_1 \square A_2, M_\square, u_\square) \in Alg(\mathcal{C})$

VIA
$$M_{\square}: (A_{1} \square A_{2}) \otimes (A_{1} \square A_{2})$$

$$\sim \downarrow$$

$$(A_{1} \otimes A_{1}) \square (A_{1} \otimes A_{2})$$

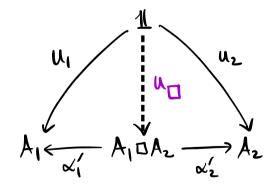
 $\Box (A_2 \otimes A_1) \Box (A_2 \otimes A_2)$

$$A_{1} \square 0 \square 0 \square A_{2}$$

$$A_{1} \square 0 \square 0 \square A_{2}$$

$$A_{1} \square A_{2}$$

UNIV. PROPERTY
OF PRODUCTS:



ASSUME (C, Ø, 1L) }
ABELIAN MONOIDAL}

TAKE MON. CAT. (4,0,1).

ALGEBRA IN & =

(A, M:A & A -> A, N: L -> A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN C = (M, D: A⊗M→M) SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN & =

(M, 4: M&A -> M)

SAT. ASSOC + UNITALITY

 $(B_{1}, B_{2}) - B_{1}MOD. | N C =$ $(M_{1} D : B_{1} \otimes M \rightarrow M_{1})$ $4 : M \otimes B_{2} \rightarrow M_{1}$ $3 : (M_{1} D) \in B_{1} - Mod(C),$ $(M_{1} A) \in Mod - B_{2}(C)$ $4 : A(D \otimes id_{B_{2}}) =$ $D(id_{B_{1}} \otimes A) \cap A_{B_{1}, M_{1}, B_{2}}$

... VIA BIPRODUCTS

TAKE (A1, M1, U1), (A2, M2, U2) & Alg(4).

DEF:
$$(A_1, M_1, u_1) \square (A_2, M_2, u_2)$$
 $= (A_1 \square A_2, M_{\square}, U_{\square}) \in Alg(\mathcal{C})$

VIA $M_{\square}: (A_{1} \square A_{2}) \otimes (A_{1} \square A_{2})$ $\sim \downarrow$ $(A_{1} \square A_{1}) \square (A_{1} \square A_{2})$

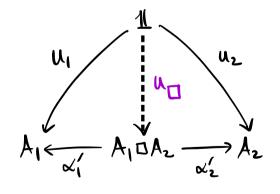
 $(A_{1} \otimes A_{1}) \square (A_{1} \otimes A_{2})$ $\square (A_{2} \otimes A_{1}) \square (A_{2} \otimes A_{2})$

$$A_{1} \square 0 \square 0 \square A_{2}$$

$$A_{1} \square 0 \square 0 \square A_{2}$$

$$A_{1} \square A_{2}$$

UNIV. PROPERTY
OF PRODUCTS:



ASSUME (C, O, 1) ABELIAN MONOIDAL,

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ALGEBRA IN & =

(A, M:A&A -> A, N: L -> A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN C = (M, D: A⊗M→M) SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN & =

(M, 4: M&A -> M)

SAT. ASSOC + UNITALITY

 $(B_{1}, B_{2}) - BIMOD. | N C_{1} =$ $(M_{1}, D : B_{1} \otimes M \rightarrow M_{2})$ $4 : M \otimes B_{2} \rightarrow M_{2}$ $3 : (M_{1}, D) \in B_{1} - Mod(C_{1}),$ $(M_{1}, d) \in Mod - B_{2}(C_{2})$ $4 \cdot d(D \otimes id_{B_{2}}) =$ $D(id_{B_{1}} \otimes d) \alpha_{B_{1}, M_{1}, B_{2}}$

... VIA BIPRODUCTS

TAKE (A, M, u) = Alg(&) & (M, D,), (M2, D2) = A-Mod(&)

DEF:
$$(M_1, D_1) \square (M_2, D_2)$$

:= $(M_1 \square M_2, \triangleright) \in A-Mod(\mathcal{E})$

VIA.

(RSUME (C, O, 1))

ABELIAN MONOIDAL

TAKE MON. CAT. (4,0,1).

ALGEBRA IN & =

(A, M:A&A -> A, N: L-> A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN C = (M, D: A⊗M→M) SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN C = (M, 4: M⊗A → M) SAT. ASSOC + UNITALITY

 $(B_{1}, B_{2}) - B_{1}MOD. | N C =$ $(M_{1}, D: B_{1} \otimes M \rightarrow M_{1})$ $4: M \otimes B_{2} \rightarrow M$ $3. (M_{1}, D) \in B_{1} - Mod(C),$ $(M_{1}, 4) \in Mod - B_{2}(C)$ $4 (D \otimes id_{B_{2}}) =$ $D(id_{B_{1}} \otimes 4) \Omega_{B_{1}, M_{1}B_{2}}$

... VIA BIPRODUCTS

TAKE (A, M, u) = Alg(&) & (M, D,), (M2, D2) = A-Mod(&)

DEF:
$$(M_1, D_1) \square (M_2, D_2)$$

:= $(M_1 \square M_2, \blacktriangleright) \in A-Mod(&)$

VIA

WORKS SIMILARLY ON

RIGHT MODULES & BIMODULES

ASSUME (C, O, 1) ABELIAN MONOIDAL

TAKE MON. CAT. (4,0,1).

ALGEBRA IN & =

(A, M:A&A -> A, N:L->A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN & =

(M, D: A⊗M→M)

SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN & =

(M, 4: M⊗A → M)

SAT. ASSOC + UNITALITY

 $(B_{1}, B_{2}) - BIMOD. (N C = (M_{1}, D): B_{1} \otimes M \rightarrow M_{1})$ $4: M \otimes B_{2} \rightarrow M)$ $3. (M_{1}, D) \in B_{1} - Mod(C),$ $(M_{1}, 4) \in Mod - B_{2}(C)$ $4 (D \otimes id_{B_{2}}) = D(id_{B_{1}} \otimes 4) \alpha_{B_{1}, M_{1}B_{2}}$

... VIA 🕸

TAKE A, B & Alg(&).

TO DEFINE ASB & Alg(E), NEED -

 $M_{\otimes}: (A \otimes B) \otimes (A \otimes B)$

 $A\otimes B$ $\int_{M_{A}\otimes M_{B}}$ $(A\otimes A)\otimes (B\otimes B)$

ASSUME (C, 0, 1) } ABELIAN MONOIDAL

TAKE MON. CAT. (4,0,1).

ALGEBRA IN & =

(A, M:A&A -> A, N:L->A) SAT. ASSOC + UNITALITY

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RIGHT A-MODULE IN & =

(M, 4: M⊗A → M)

SAT. ASSOC + UNITALITY

 $(B_{1}, B_{2}) - BIMOD. | N C =$ $(M_{1}, D : B_{1} \otimes M \rightarrow M_{1})$ $4 : M \otimes B_{2} \rightarrow M$ $3 : (M_{1}, D) \in B_{1} - Mod(C),$ $(M_{1}, 4) \in Mod - B_{2}(C)$ $4 : M \otimes id_{B_{2}} =$ $5 (id_{B_{1}} \otimes 4) \alpha_{B_{1}, M_{1}B_{2}}$

... VIA 🛇

TAKE A, B & Alg(&).

TO DEFINE ASB & Alg(E), NEED -

M&: (A&B) & (A&B)

USE ASSOCIATIVITY

(A

NEED MORPHISM

BOA - AOB IN &

... DONE WITH "BRAIDING" ON & LATER

ABELIAN MONOIDAL

TAKE MON. CAT. (4,0,1).

ALGEBRA IN & =

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TAKE
$$B_1$$
, $B_2 \in Alg(e)$ \$ $(M, D) \in B_1 - Mod(e)$
 $(N, 4) \in Mod - B_2(e)$

DEF:
$$(M, \triangleright) \otimes (N, \triangleleft)$$

:= $(M \otimes N, \triangleright, \triangleleft) \in (B_1, B_2) - Binod(\mathcal{C})$



ABELIAN MONOIDAL

TAKE MON. CAT. (4,0,1).

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... VIA 🛇

TAKE
$$B_1$$
, $B_2 \in Alg(e) \notin (M, D) \in B_1 - Mod(e)$
 $(N, 4) \in Mod - B_2(e)$

DEF:
$$(M, \triangleright) \otimes (N, \triangleleft)$$

:= $(M \otimes N, \triangleright, \triangleleft) \in (B_1, B_2) - Binod(e)$

$$>: \mathcal{B}_{1} \otimes (\mathbb{M} \otimes \mathbb{N}) \xrightarrow{\alpha_{\mathcal{B}_{1}, \mathcal{M}_{1}, \mathcal{N}}^{-1}} (\mathcal{B}_{1} \otimes \mathbb{M}) \otimes \mathbb{N} \xrightarrow{\mathcal{D} \otimes id} \mathbb{M} \otimes \mathbb{N}$$

ASSUME (C, Ø, 11)?
ABELIAN MONOIDAL

TAKE MON. CAT. (4,0,4).

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SAT. ASSOC + UNITALITY

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... VIA 💩

TAKE
$$A_1B_1$$
, $B_2 \in Alg(\mathcal{C}) \notin (M, D_M, A_M) \in (B_1, A)$ -Bimod(\mathcal{C})
 $(N, D_M, A_M) \in (A_1B_2)$ -Bimod(\mathcal{C})

DEF:
$$(M, D_M, A_M) \otimes_A (N, D_M, A_N)$$

:= $(M \otimes_A N, D_M, A_M) \in (B_1, B_2) - Bimod(&)$

$$>: B_{1} \otimes (M \otimes N) \xrightarrow{\alpha_{B_{1},M,N}^{-1}} (B_{1} \otimes M) \otimes N \xrightarrow{P_{M} \otimes id} M \otimes N$$

$$: (M \otimes N) \otimes B_2 \xrightarrow{\Lambda_{M_1N_1}B_2} M \otimes (N \otimes B_2) \xrightarrow{id \otimes \triangleleft_N} M \otimes N$$

ASSUME (C, Ø, 11)? ABELIAN MONOIDAL,

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TAKE
$$A_1B_1$$
, $B_2 \in Alg(\mathcal{C}) \notin (M, D_N, A_N) \in (B_1, A)$ -Bimod(\mathcal{C})
 $(N, D_N, A_N) \in (A_1B_2)$ -Bimod(\mathcal{C})

DEF:
$$(M, D_M, A_M) \otimes_A (N, D_N, A_N)$$

:= $(M \otimes_A N, \triangleright_A) \in (B_1, B_2) - Binod(e)$

$$M \otimes_{A} N := \text{coeq} \left((M \otimes A) \otimes N \xrightarrow{\text{d}_{M} \otimes \text{id}} M \otimes N \right)$$

$$(\text{id} \otimes P_{N}) a_{M/A,N} M \otimes N$$

$$> : \mathcal{B}_{1} \otimes (M \otimes N) \xrightarrow{\alpha_{\mathcal{B}_{1}, M, N}^{-1}} (\mathcal{B}_{1} \otimes M) \otimes N \xrightarrow{\mathcal{D}_{M} \otimes id} M \otimes N$$

$$: (M \otimes N) \otimes B_2 \xrightarrow{\Lambda_{M_1N_1B_2}} M \otimes (N \otimes B_2) \xrightarrow{id \otimes \triangleleft_N} M \otimes N$$

ASSUME (C, Ø, 11)?
ABELIAN MONOIDAL

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ALGEBRA IN & =

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... VIA 💩

TAKE
$$A_1B_1$$
, $B_2 \in Alg(\mathcal{C}) \notin (M, D_M, A_M) \in (B_1, A)$ -Bimod(\mathcal{C})
 $(N, D_M, A_M) \in (A_1B_2)$ -Bimod(\mathcal{C})

DEF:
$$(M, D_M, A_M) \otimes_A (N, D_N, A_N)$$

$$:= (M \otimes_A N, D_A) \overset{\epsilon}{\leftarrow} (B_1, B_2) - Bimod(\mathcal{C})$$

$$M \otimes_{A} N := \text{Coeq} \left((M \otimes A) \otimes N \xrightarrow{d_{M} \otimes \text{id}} M \otimes N \right)$$

$$: (M \otimes N) \otimes B_2 \xrightarrow{\alpha_{M/N/B_2}} M \otimes (N \otimes B_2) \xrightarrow{id \otimes \triangleleft_N} M \otimes N$$

ASSUME (C, Ø, 1L) } ABELIAN MONOIDAL

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... VIA 💩

TAKE A_1B_1 , $B_2 \in Alg(\mathcal{C}) \notin (M, D_M, A_M) \in (B_1,A)$ -Bimod(\mathcal{C}) $(N, D_M, A_M) \in (A_1B_2)$ -Bimod(\mathcal{C})

DEF: $(M, D_M, A_M) \otimes_A (N, D_M, A_N)$ $:= (M \otimes_A N, D, A) \in (B_1, B_2) - Bimod(2)$

EXER.4.38: TAKE ME (B1,11)-Bimod(e),
NE (11, B2)-Bimod(e).

SHOW: MOLN = MON PREV. CONSTRUCTION
AS (B1, B2)-BIMODULES IN C.

ASSUME (C, Ø, 11)?
ABELIAN MONOIDAL

TAKE MON. CAT. (4,0,4).

ALGEBRA IN & =

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... VIA 🛇

TAKE A_1B_1 , $B_2 \in Alg(\mathcal{C}) \notin (M, D_M, A_M) \in (B_1,A)$ -Bimod(\mathcal{C}) $(N, D_M, A_M) \in (A_1B_2)$ -Bimod(\mathcal{C})

DEF: $(M, D_M, A_M) \otimes_A (N, D_M, A_N)$ $:= (M \otimes_A N, D_M) \overset{\epsilon}{\leftarrow} (B_1, B_2) - Bimod(\mathcal{C})$

EXER.4.39: TAKE M,N,P & A-Binod(e).
SHOW:

- (a) $(M \otimes_A N) \otimes_A P \cong M \otimes_A (N \otimes_A P)$,
- (b) M@A Arey = M = Arey @A M,

 AS A-BIMODULES IN C.

ASSUME (C, S, 1) }

ABELIAN MONOIDAL

TAKE MON. CAT. (4,0,4).

ALGEBRA IN & =

(A, M:A&A -> A, N:L->A) SAT. ASSOC + UNITALITY

(M, D: A⊗M→M)
SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN & =

(M, 4: M⊗A → M)

SAT. ASSOC + UNITALITY

 $(B_{1}, B_{2}) - BIMOD. | N C =$ $(M_{1}, D : B_{1} \otimes M \rightarrow M_{2})$ $4 : M \otimes B_{2} \rightarrow M_{2}$ $3 : (M_{1}, D) \in B_{1} - Mod(C),$ $(M_{1}, d) \in Mod - B_{2}(C)$ $4 : M \otimes G_{2} \rightarrow G_{2}(C)$ $4 : M \otimes G_{2} \rightarrow G_{2}(C)$ $5 : M \otimes G_{2} \rightarrow G_{2}(C)$ $6 : M \otimes G_{2} \rightarrow G_{2}(C)$ $7 : M \otimes G_{2} \rightarrow G_{2}(C)$ $8 : M \otimes G_{2} \rightarrow G_{2}(C)$ $9 : M \otimes G_{2} \rightarrow G_{2}(C)$ $1 : M \otimes G_{2} \rightarrow G_{2}(C)$ $2 : M \otimes G_{2} \rightarrow G_{2}(C)$ $3 : M \otimes G_{2} \rightarrow G_{2}(C)$ $4 : M \otimes G_{2} \rightarrow G_{2}(C)$ $4 : M \otimes G_{2} \rightarrow G_{2}(C)$ $5 : M \otimes G_{2} \rightarrow G_{2}(C)$ $6 : M \otimes G_{2} \rightarrow G_{2}(C)$ $7 : M \otimes G_{2} \rightarrow G_{2}(C)$ $8 : M \otimes G_{2} \rightarrow G_{2}(C)$ $9 : M \otimes G_{2} \rightarrow G_{2}(C)$ $1 : M \otimes G_{2} \rightarrow G_{2}(C)$ $2 : M \otimes G_{2} \rightarrow G_{2}(C)$ $3 : M \otimes G_{2} \rightarrow G_{2}(C)$ $4 : M \otimes G_{2} \rightarrow G_{2}(C)$ $5 : M \otimes G_{2} \rightarrow G_{2}(C)$ $6 : M \otimes G_{2} \rightarrow G_{2}(C)$ $7 : M \otimes G_{2} \rightarrow G_{2}(C)$ $8 : M \otimes G_{2} \rightarrow G_{2}(C)$ $9 : M \otimes G_{2} \rightarrow G_{2}(C)$

... VIA HOM

OF BUT HOME (X,Y) IS

NOT NECESSARILY

AN OBJECT IN C

SO LET'S NOT CONSIDER THIS OPERATION HERE.

TAKE MON. CAT. (4,0,4).

ALGEBRA IN & =

(A, M:A&A -> A, N:L->A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN C = (M, D: A⊗M→M) SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN & =

(M, 4: M&A -> M)

SAT. ASSOC + UNITALITY

 $(B_{1}, B_{2}) - BIMOD. | N C =$ $(M_{1}, D: B_{1} \otimes M \rightarrow M_{1})$ $4: M \otimes B_{2} \rightarrow M$ $3. (M_{1}, D) \in B_{1} - Mod(C),$ $(M_{1}, d) \in Mod - B_{2}(C)$ $4 (D \otimes id_{B_{2}}) =$ $D(id_{B_{1}} \otimes d) \alpha_{B_{1}, M_{1}, B_{2}}$

... VIA DUALS

ASSUME (C, Ø, 1L)

ABELIAN MONOIDAL

RIGID

RECALL (-)*, *(-): 6-> & ARE CONTRAVARIANT

TAKE MON. CAT. (4,0,4).

ALGEBRA IN & =

(A, M:A&A -> A, N:L->A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN C = (M, D: A⊗M→M) SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN C = (M, 4: M⊗A → M) SAT. ASSOC + UNITALITY

 $(B_{1}, B_{2}) - BIMOD. | N C =$ $(M_{1}, D: B_{1} \otimes M \rightarrow M_{1})$ $4: M \otimes B_{2} \rightarrow M)$ $3. (M_{1}, D) \in B_{1} - Mod(C),$ $(M_{1}, d) \in Mod - B_{2}(C)$ $4 d(D \otimes id_{B_{2}}) =$ $D(id_{B_{1}} \otimes d) \alpha_{B_{1}, M_{1}, B_{2}}$

... VIA DUALS

ASSUME (C, Ø, 11)

ABELIAN MONOIDAL

RIGID

RECALL (-)*, *(-): 6-> & ARE CONTRAVARIANT

GIVEN (A, M, u) \in Alg(\mathbb{C}),

THE OBJECT A* IS NATURALLY

A "COALGEBRA":

$$\begin{pmatrix} A^* & A^* & A^* \otimes A^* \end{pmatrix} \qquad \text{``CoUNIT''}$$

$$\begin{pmatrix} A^* & A^* & A^* \otimes A^* \end{pmatrix} \qquad \Sigma : A^* \longrightarrow 1$$

SATISFYING "COASSOCIATIVITY" & "COUNITALITY"

TAKE MON. CAT. (4,0,4).

ALGEBRA IN & =

(A, M:A&A -> A, N:L-A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN & = (M, D: A⊗M→M) SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN C = (M, 4: M⊗A → M) SAT. ASSOC + UNITALITY

 $(B_{1}, B_{2}) - BIMOD. | N C_{1} =$ $(M_{1}, D: B_{1} \otimes M \rightarrow M_{1})$ $4: M \otimes B_{2} \rightarrow M_{1}$ $3. (M_{1}, D) \in B_{1} - Mod(C_{1}),$ $(M_{1}, A) \in Mod - B_{2}(C_{2})$ $4 (D \otimes id_{B_{2}}) =$ $D(id_{B_{1}} \otimes A) Q_{B_{1}, M_{1}, B_{2}}$

... VIA DUALS

ASSUME (C, Ø, 11) } ABELIAN MONOIDAL RIGID

RECALL (-)*, *(-): 6-> 6 ARE CONTRAVARIANT

GIVEN (A, M, u) \in Alg(R),

THE OBJECT A* IS NATURALLY

A "COALGEBRA":

$$\begin{pmatrix}
A^* \\
A^*
\end{pmatrix}$$

$$\begin{array}{c}
\text{"CountTiplication"} \\
A^* \otimes A^*
\end{array}$$

$$\begin{array}{c}
\text{"CountT'} \\
\text{E: } A^* \longrightarrow 1 \\
\text{U*}
\end{array}$$

$$\begin{array}{c}
\text{U*}
\end{array}$$

$$\begin{array}{c}
\text{U*}
\end{array}$$

SATISFYING "COASSOCIATIVITY" & "CONNITALITY"

... MORE LATER

TAKE MON. CAT. (4,0,1).

ALGEBRA IN & =

(A, M:A⊗A → A, N: L → A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN C = (M, D: A⊗M→M) SAT. ASSOC + UNITACITY

RIGHT A-MODULE IN & =

(M, 4: M&A -> M)

SAT. ASSOC + UNITALITY

 $(B_{1}, B_{2}) - BIMOD. | N C =$ $(M_{1}, D : B_{1} \otimes M \rightarrow M_{2})$ $4 : M \otimes B_{2} \rightarrow M_{2}$ $3 : (M_{1}, D) \in B_{1} - Mod(C),$ $(M_{1}, d) \in Mod - B_{2}(C)$ $4 : M \otimes id_{B_{2}} =$ $5 (id_{B_{1}} \otimes d) \alpha_{B_{1}, M_{1}, B_{2}}$

... VIA DUALS



RECALL (-)*, *(-): 6-> 6 ARE CONTRAVARIANT

EXERCISE 4.41: TAKE (M,D) & A-Mod(&).

THEN (IN STRICT CASE)

(M*, d) ∈ Mod-A(&) VIA:

TAKE MON. CAT. (4,0,4).

ALGEBRA IN & =

(A, M:A&A -> A, N: L-> A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN & =

(M, D: A⊗M→M)

SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN & =

(M, 4: M⊗A → M)

SAT. ASSOC + UNITALITY

 $(B_{1}, B_{2}) - BIMOD. | N C =$ $(M_{1}, D: B_{1} \otimes M \rightarrow M_{1})$ $4: M \otimes B_{2} \rightarrow M)$ $3. (M_{1}, D) \in B_{1} - Mod(C),$ $(M_{1}, d) \in Mod - B_{2}(C)$ $4 (D \otimes id_{B_{2}}) =$ $D(id_{B_{1}} \otimes d) \alpha_{B_{1}, M_{1}, B_{2}}$

... VIA DUALS

ASSUME (C, O, 11) }
ABELIAN MONOIDAL

RIGID

RECALL (-)*, *(-): 6-> 6 ARE CONTRAVARIANT

EXERCISE 4.41: TAKE (M,D) & A-Mod(C).

THEN (IN STRICT CASE)

(M*, d) ∈ Mod-A(G) VIA:

4: M*& A idn* & idn & coevm > M* & A & M & M*

M*.

TAKE MON. CAT. (4,0,1).

ALGEBRA IN & =

(A, M:A&A -> A, N:L-A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN & =

(M, D: A⊗M→M)

SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN & =

(M, 4: M⊗A → M)

SAT. ASSOC + UNITALITY

 $(B_{1}, B_{2}) - BIMOD. | N C =$ $(M_{1}, D: B_{1} \otimes M \rightarrow M_{1})$ $4: M \otimes B_{2} \rightarrow M$ $3. (M_{1}, D) \in B_{1} - Mod(C),$ $(M_{1}, d) \in Mod - B_{2}(C)$ $4 (D \otimes id_{B_{2}}) =$ $D(id_{B_{1}} \otimes d) \alpha_{B_{1}, M_{1}, B_{2}}$

... VIA DUALS

ASSUME (C, Ø, 11)

ABELIAN MONOIDAL

RIGID

RECALL (-)*, *(-): 6-> 4 ARE CONTRAVARIANT

EXERCISE 4.41: TAKE (M,D) & A-Mod(&).

THEN (IN STRICT CASE)

(M*, d) ∈ Mod-A(%) VIA:

4: M*& A idm* oid A & Coev m M* & A & M & M*

idm* & D & idm* M* & M & M & M*

M*.

TAKE MON. CAT. (4,0,1).

ALGEBRA IN & =

(A, M:A&A - A, N: L - A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN & =

(M, D: A⊗M→M)

SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN & =

(M, 4: M⊗A → M)

SAT. ASSOC + UNITALITY

 $(B_{1}, B_{2}) - BIMOD. | N C =$ $(M_{1}, D: B_{1} \otimes M \rightarrow M_{1})$ $4: M \otimes B_{2} \rightarrow M)$ $3. (M_{1}, D) \in B_{1} - Mod(C),$ $(M_{1}, d) \in Mod - B_{2}(C)$ $4 (D \otimes id_{B_{2}}) =$ $D(id_{B_{1}} \otimes d) \alpha_{B_{1}, M_{1}, B_{2}}$

... VIA DUALS

ASSUME (C, Ø, 11) ABELIAN MONOIDAL RIGID

RECALL (-)*, *(-): 6-> 6 ARE CONTRAVARIANT

EXERCISE 4.41: TAKE (M,D) & A-Mod(&).

THEN (IN STRICT CASE)

(M*, d) ∈ Mod-A(6) VIA:

 $\begin{array}{c}
\bullet: M^{*} \otimes A \xrightarrow{id_{M^{*}} \otimes id_{A} \otimes coeV_{M}^{\perp}} & M^{*} \otimes A \otimes M \otimes M^{*} \\
& \xrightarrow{id_{M^{*}} \otimes D \otimes id_{M^{*}}} & M^{*} \otimes M \otimes M^{*} \\
& \xrightarrow{eV_{M}^{\perp} \otimes id_{M^{*}}} & M^{*}
\end{array}$

TAKE MON. CAT. (4,0,1).

ALGEBRA IN & =

(A, M:A&A -> A, N:L->A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN & =

(M, D: A⊗M→M)

SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN & =

(M, 4: M&A -> M)

SAT. ASSOC + UNITALITY

 $(B_{1}, B_{2}) - BIMOD. | N C_{1} =$ $(M_{1}, D : B_{1} \otimes M \rightarrow M_{1})$ $4 : M \otimes B_{2} \rightarrow M)$ $3 \cdot (M_{1}, D) \in B_{1} - Mod(C_{1}),$ $(M_{1}, 4) \in Mod - B_{2}(C)$ $4 \cdot (D \otimes id_{B_{2}}) =$ $5 \cdot (id_{B_{1}} \otimes 4) a_{B_{1}, M_{1}, B_{2}}$

... VIA DUALS

ASSUME (C, Ø, 11)

ABELIAN MONOIDAL

RIGID

RECALL (-)*, *(-): 6-> 6 ARE CONTRAVARIANT

EXERCISE 4.41: TAKE (M,D) & A-Mod(C).

THEN (IN STRICT CASE)

(M*, d) ∈ Mod-A(&) VIA:

 $\begin{array}{c} \mathbf{4}: \ \mathcal{M}^{*} \otimes \mathbf{A} \xrightarrow{\mathrm{id}_{\mathcal{M}^{*}} \otimes \mathrm{id}_{\mathbf{A}} \otimes \mathrm{coev}_{\mathcal{M}^{*}}} & \mathcal{M}^{*} \otimes \mathbf{A} \otimes \mathcal{M} \otimes \mathcal{M}^{*} \\ \xrightarrow{\mathrm{id}_{\mathcal{M}^{*}} \otimes \mathbf{b} \otimes \mathrm{id}_{\mathcal{M}^{*}}} & \mathcal{M}^{*} \otimes \mathcal{M} \otimes \mathcal{M}^{*} \\ \xrightarrow{\mathrm{ev}_{\mathcal{M}^{*}} \otimes \mathrm{id}_{\mathcal{M}^{*}}} & \mathcal{M}^{*} & \\ & \xrightarrow{\mathrm{ev}_{\mathcal{M}^{*}} \otimes \mathrm{id}_{\mathcal{M}^{*}}} & \mathcal{M}^{*} & . \end{array}$

LIKEWISE, (N, a) & Mod-A(e)

~> (*N,) & A-Mod(C) FOR SOME .

TAKE MON. CAT. (4,0,4).

ALGEBRA IN & =

(A, M:A&A - A, N:L-A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN & = (M, D: A⊗M→M)

SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN C = (M, 4: M⊗A → M) SAT. ASSOC + UNITALITY

 $(B_{1}, B_{2}) - BIMOD. | N C =$ $(M_{1}, D: B_{1} \otimes M \rightarrow M_{2})$ $4: M \otimes B_{2} \rightarrow M$ $3. (M_{1}, D) \in B_{1} - Mod(C),$ $(M_{1}, d) \in Mod - B_{2}(C)$ $4 d(D \otimes id_{B_{2}}) =$ $D(id_{B_{1}} \otimes d) \alpha_{B_{1}, M_{1}, B_{2}}$

... VIA DUALS



RECALL (-)*, *(-): 6-> 6 ARE CONTRAVARIANT

EXERCISE 4.41: TAKE (M,D) & A-Mod(&).

THEN (IN STRICT CASE)

(M*, d) ∈ Mod-A(%) VIA:

LET'S EXPLORE THS U

 $4: \mu^{*} \otimes A \xrightarrow{id_{\mu^{*}} \otimes id_{A} \otimes coeV_{\mu}} \mu^{*} \otimes A \otimes \mu \otimes \mu^{*}$ $\xrightarrow{id_{\mu^{*}} \otimes b \otimes id_{\mu^{*}}} \mu^{*} \otimes \mu \otimes \mu^{*}$ $\xrightarrow{eV_{\mu}} \otimes id_{\mu^{*}} \mu^{*}$ $\mu^{*} \otimes \mu \otimes \mu^{*}$

LIKEWISE, (N, d) & Mod-A(e)

~> (*N,) & A-Mod(C) FOR SOME D.

TAKE MON. CAT. (4,0,1).

ALGEBRA IN & =

(A, M:A&A -> A, N:L->A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN & =

(M, D: A⊗M→M) SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN & =

(M, 4: M⊗A →M) SAT. ASSOC + UNITALITY

(B1, B2)-BIMOD. IN & =

 $(M, D: B_1 \otimes M \rightarrow M)$ $4: M \otimes B_2 \rightarrow M)$

- $(M, b) \in B_1 Mod(\mathcal{C}),$ $(M, d) \in Mod B_2(\mathcal{C})$

... VIA DUALS

ASSUME (C, Ø, 11) } ABELIAN, MONOIDAL } RIGID

IM

M

EXER. 4.41: TAKE (M, D) & A-Mod(4).

THEN (M*, 1) & Mod-A(8) VIA:

4: M*& A idn* coev M M* A & M & M*

idn* & D & idn* M* M & M M M*

ev M & idn* M*

M*

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TAKE MON. CAT. (4,0,1).

ALGEBRA IN & =

(A, M:A&A -> A, N:L->A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN & =

(M, D: A⊗M→M)

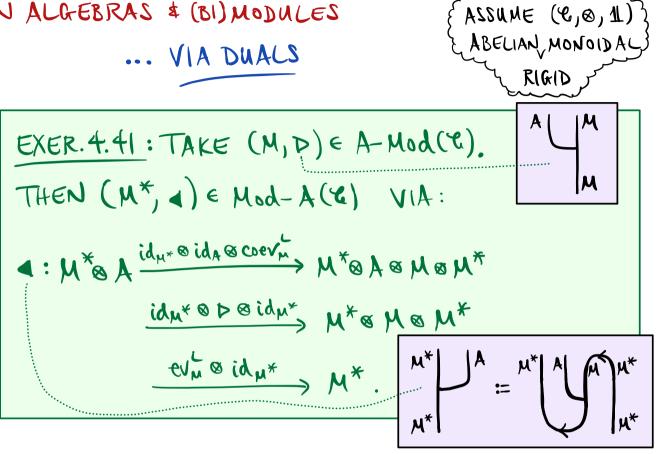
SAT. ASSOC + UNITACITY

RIGHT A-MODULE IN & =

(M, 4: M&A -> M)

SAT. ASSOC + UNITALITY

 $(B_{1}, B_{2}) - BIMOD. | N C =$ $(M_{1}, D : B_{1} \otimes M \rightarrow M_{1})$ $4 : M \otimes B_{2} \rightarrow M)$ $3 \cdot (M_{1}, D) \in B_{1} - Mod(C),$ $(M_{1}, A) \in Mod - B_{2}(C)$ $4 \cdot (D \otimes id_{B_{2}}) =$ $7 \cdot (id_{B_{1}} \otimes A) \cap A_{B_{1}, M_{1}, B_{2}}$



TAKE MON. CAT. (4,0,1).

ALGEBRA IN & =

(A, M:A&A -> A, N:L->A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN & =

(M, D: A⊗M→M)

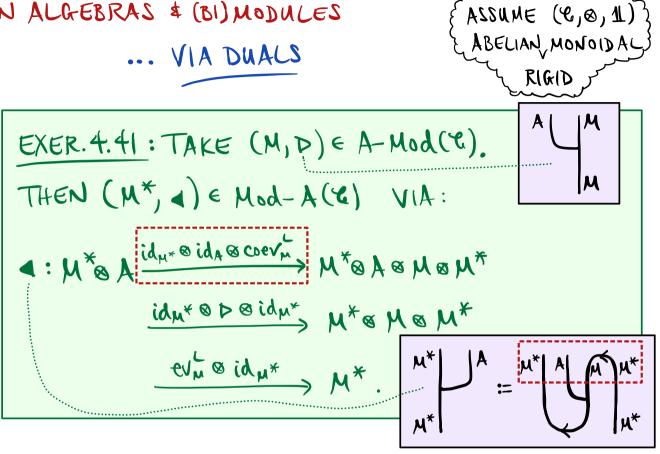
SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN & =

(M, 4: M&A -> M)

SAT. ASSOC + UNITALITY

 $(B_{1}, B_{2}) - B_{1}MOD. | N C_{1} =$ $(M_{1}, D: B_{1} \otimes M \rightarrow M_{1})$ $4: M \otimes B_{2} \rightarrow M_{1}$ $3. (M_{1}, D) \in B_{1} - Mod(C_{1}),$ $(M_{1}, A) \in Mod - B_{2}(C_{1})$ $4 (D \otimes id_{B_{2}}) =$ $7 (id_{B_{1}} \otimes A) Q_{B_{1}, M_{1}, B_{2}}$



TAKE MON. CAT. (4,0,1).

ALGEBRA IN & =

(A, M:A&A -> A, N:L->A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN & = (M, D: A⊗M→M)

SAT. ASSOC + UNITALITY

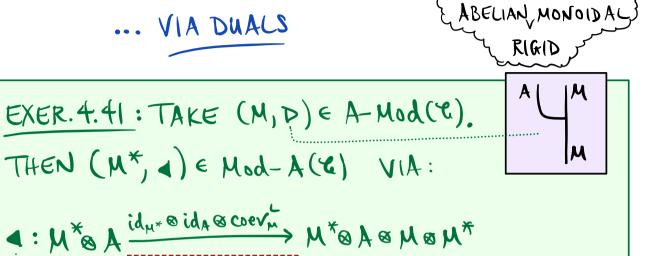
RIGHT A-MODULE IN & =

(M, 4: M⊗A →M) SAT. ASSOC + UNITALITY

(B1, B2)-BIMOD. IN & =

 $(M, D: B_1 \otimes M \rightarrow M)$ $4: M \otimes B_2 \rightarrow M)$

 $(M, b) \in B_1 - Mod(C),$ $(M, d) \in Mod - B_2(C)$



idn* & D & idn* M* & M & M*

evm & idm* M* M*

ASSUME (C, O, 1)

TAKE MON. CAT. (4,0,1).

ALGEBRA IN & =

(A, M:A&A - A, N: L - A) SAT. ASSOC + UNITALITY

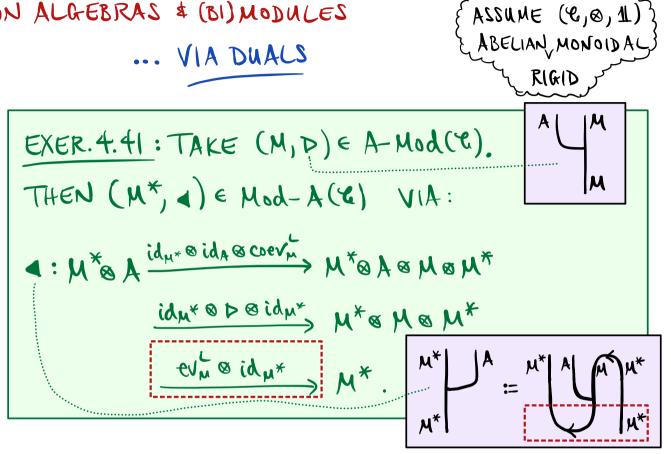
(M, D: A⊗M→M)
SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN & =

(M, 4: M⊗A → M)

SAT. ASSOC + UNITALITY

 $(B_{1}, B_{2}) - BIMOD. | N C =$ $(M_{1}, D: B_{1} \otimes M \rightarrow M_{2})$ $4: M \otimes B_{2} \rightarrow M_{2}$ $3. (M_{1}, D) \in B_{1} - Mod(C)_{2}$ $(M_{1}, d) \in Mod - B_{2}(C)$ $4 (D \otimes id_{B_{2}}) =$ $D(id_{B_{1}} \otimes d) \Omega_{B_{1}, M_{1}, B_{2}}$



I. OPERATIONS ON ALGEBRAS & (BI) MODULES

TAKE MON. CAT. (4,0,1).

ALGEBRA IN & =

(A, M:A⊗A → A, N: L → A) SAT. ASSOC + UNITALITY

LEFT A-MODULE IN & =

(M, D: A⊗M→M) SAT. ASSOC + UNITALITY

RIGHT A-MODULE IN & =

 $(M, 4: M \otimes A \rightarrow M)$

SAT. ASSOC + UNITALITY

(B1, B2)-BIMOD. IN & =

 $(M, D: B_1 \otimes M \rightarrow M)$

4: M⊗Bz→M)

 $(M, D) \in B_1 - Mod(C),$

(M, 4) ∈ Mod-B2(8)

\$ 4(>&idb2) =

> (idb, & d) ab,, M, B.

... VIA DUALS

ASSUME (C, Ø, 11)

ABELIAN MONOIDAL

RIGID

M

EXER. 4.41: TAKE (M, D) & A-Mod(4).

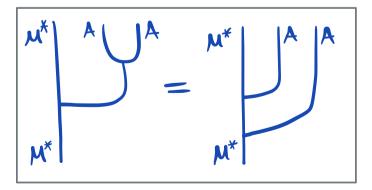
THEN (M*,) & Mod-A(&) VIA:

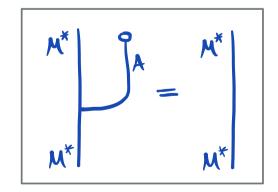
4: M* & A idn* & idn & coev ~ M* & A & M & M*

idn* & D & idn* M* & M & M & M*

evi & idu* M* M*

SHOW:





I. OPERATIONS ON ALGEBRAS & (BI) MODULES

I. OPERATIONS ON ALGEBRAS & (BI) MODULES TAKE MON. CAT. (4,0,1). ALGEBRA IN 4 =					ASSUME (C, Ø, 11) } ABELIAN, MONOIDAL RIGID	
(A, M:A&A -> A, N: L -> A) SAT. ASSOC + UNITALITY	BIPROD.	MON.PROD. ⊗	TENJ. PROD. ØA	ttoM	DUAL (-)*,*(-)	OF
LEFT A-MODULE IN & = (M, D: A⊗M→M) SAT. ASSOC + UNITALITY		NOTYET	Notyet	NIA	NIA	ALGS IN Y
RIGHT A-MODULE IN & = (M, 4: M&A -> M) SAT. ASSOC + UNITALITY		/		. (/	MoDS
$(B_{1}, B_{2}) - B_{1}MOD. N C_{1} =$ $(M_{1} D : B_{1} \otimes M \rightarrow M_{1}$ $4 : M \otimes B_{2} \rightarrow M$				NA		N 61
$ \begin{array}{ll} \exists. & (M,D) \in B_1 - Mod(\mathcal{C}), \\ & (M,d) \in Mod - B_2(\mathcal{C}) \\ & & & & & & & \\ & & & & & & & \\ & & & & $				NIA	THINK ABOUT THIS	BIMODS. N &

TAKE IR-ALGS. A, B. $Q = Q_A$ BIMODINCE.

GET ADDITIVE FUNCTORS:

RIGHTEX. $Q = Q_A$ BIMODINCE.

Hombord $Q = Q_A$ BIMODINCE.

Hombord $Q = Q_A$ BIMODINCE.

WITH (Q& -) - (Hom B-mod (Q,-))

RECALL FROM
LECTURE #11

EILENBERG-WATTS THEOREM

TAKE FINITE DIM'L IR-ALGEBRAS A, B. FOR IR-LINEAR

F: A-FdMod -> B-FdMod, GET:

FLEFTEXACT

1

F HAS A LEFT ADJOINT

1

F = HOM (P,-)

FOR SOME BIMOD.

 $P = A P_B$.

FRIGHT EXACT

FHAS A RIGHT ADJOINT

1

F=QQ-

FOR SOME BIMOD.

 $Q = {}_{B}Q_{A}$.

TAKE 1R-ALGS. A, B. & Q=BQA BIMODULE.

GET ADDITIVE FUNCTORS:

RIGHTEX. QQ -: A-Mod --- B-Mod

HomB-Mod (Q1-): B-Mod -> A-Mod

WITH $(Q \otimes_A -) - (\text{thom}_{B-\text{mod}}(Q_1 -))$

RECALL FROM LECTURE #11

WE WILL GENERALIZE
THIS IN THE
MONOIDAL SETTING

EILENBERG-WATTS THEOREM

TAKE FINITE DIM'L IR-ALGEBRAS A, B.

FOR IK-LINEAR

F: A-FdMod -> B-FdMod, GET:

FLEFTEXACT

1

F HAS A LEFT ADJOINT

1

F = HOM (P,-)

FOR SOME BIMOD.

P=APB.

F RIGHT EXACT

1

FHAS A RIGHT ADJOINT

1

F=QQ-

FOR SOME BIMOD.

 $Q = {}_{B}Q_{A}$.

ASSUME & ABELIAN MONOIDAL & X&-, -&X ARE RIGHT EXACT YXER

TAKE IR-ALGS. A, B. $Q = Q_A$ BIMODINE.

GET ADDITIVE FUNCTORS:

RIGHTEX. $Q = Q_A$ BIMODINE.

HOMB-Mod $Q = Q_A$ BIMODINE.

HOMB-Mod $Q = Q_A$ BIMODINE.

WITH $Q = Q_A$ BIMODINE.

HOMB-Mod $Q = Q_A$ BIMODINE.

HOMB-Mod $Q = Q_A$ BIMODINE.

PAY ATTENTION
TO HYPOTHESES

ASSUME & ABELIAN MONOIDAL & X&-, -&X ARE RIGHT EXACT YXE'S

TAKE IR-ALGS. A, B. $\ Q =_{B} Q_{A} \ Bimodule.$ GET ADDITIVE FUNCTORS:

RIGHTEX. $Q \otimes_{A} = : A - Mod \longrightarrow B - Mod$ Hom_{B-Mod} $(Q_{1} -) : B - Mod \longrightarrow A - Mod$ LEFT EX. $A = (Q_{1} -) : B - Mod \longrightarrow A - Mod$ WITH $(Q \otimes_{A} -) - (Hom_{B-Mod}(Q_{1} -))$

LEMMA: TAKE A, B \in Alg(\mathcal{C}).

THEN, $Q \otimes_A - : A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C})$ IS RIGHT EXACT, $\forall Q \in (B, A) - Binod(\mathcal{C}).$

ASSUME & ABELIAN MONOIDAL & X&-, -&X ARE RIGHT EXACT YXEC)

TAKE 1R-ALGS. A, B. & Q=BQA BIMODILE.

GET ADDITIVE FUNCTORS:

Hom_{B-Mod} (Q₁-): B-Mod -> A-Mod LEFT EX.

WITH (Q& -) - (thom B-Mod (Q1-))

LEMMA: TAKE A, B & Alg(&).
THEN,

 $Q \otimes_{A} -: A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C})$

IS RIGHT EXACT,

YQ ∈ (B,A)-Bimod(E).

PF/ STS (QOA-) PRESERVES COKERNELS.

TAKE Ø: M -> N & A-Mod(&).

WANT: Q@ coker(\$) = coker(id @ \$)

TAKE IR-ALGS. A, B. $Q = Q_A$ BIMODULE.

GET ADDITIVE FUNCTORS:

RIGHTEX. $Q = Q_A$ B-Mod \longrightarrow B-Mod

Hom_{B-Mod} $(Q_{1}-)$: B-Mod \longrightarrow A-Mod LEFT EX. \mathcal{A} WITH $(Q \otimes_{A} -)$ \longrightarrow $(\text{ttom}_{B-\text{Mod}}(Q_{1}-))$

LEMMA: TAKE A, B \in Alg(\mathcal{C}).

THEN, $Q \otimes_A - : A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C})$ IS RIGHT EXACT, $\forall Q \in (B, A) - Bimod(\mathcal{C}).$

PF/ STS (QOA-) PRESERVES COKERNELS.

TAKE Ø: M -> N & A-Mod(&).

WANT: Q@A Coker(\$) = (oker (id @ \$)

QQAM (idqq) QQAN (idqq) coker (idqq)

TAKE 1R-ALGS. A, B. & Q=BQA BIMODILE.

GET ADDITIVE FUNCTORS:

RIGHTEX. QQ -: A-Mod -> B-Mod

HomB-Mod (Q1-): B-Mod -> A-Mod LEFT EX.

WITH (QQ -) - (HomB-Mod (Q1-))

PF/ STS (QOA-) PRESERVES COKERNELS.

TAKE Ø: M -> N & A-Mod(&).

WANT: Q@A Coker(\$) = (oker (id @ \$)

LEMMA: TAKE A, B \in Alg(\mathcal{C}).

THEN, $Q \otimes_A - : A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C})$ IS RIGHT EXACT, $\forall Q \in (B, A) - Bimod(\mathcal{C}).$

TAKE IR-ALGS. A, B. & Q=BQA BIMODULE.

GET ADDITIVE FUNCTORS:

RIGHTEX. QQ -: A-Mod -> B-Mod

Hombond (Q1-): B-Mod -> A-Mod
LEFT EX.

WITH (Q& -) - (Hom B-Mod (Q1-))

LEMMA: TAKE A, B \in Alg(\mathcal{C}).

THEN, $Q \otimes_{A} -: A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C})$ IS RIGHT EXACT, $\forall Q \in (B, A) - Bimod(\mathcal{C}).$

PF/ STS (QOA-) PRESERVES COKERNELS.

TAKE Ø: M -> N & A-Mod(&).

WANT: Q@A Coker(\$) = (oker (id@ & \$)

QQAM ~ QQAN ~ Coker(idQQX)

BY UNIV. PROP. idQxQ

OF COKERNELS

QQA COKER(X)

ASSUME & ABELIAN MONOIDAL & X&-, -&X ARE RIGHT EXACT YXE'S

TAKE 1R-ALGS. A, B. & Q=BQA BIMODILE.

GET ADDITIVE FUNCTORS:

Hom_{B-Mod} (Q₁-): B-Mod -> A-Mod LEFT EX.

WITH (Q& -) - (HomB-mod (Q1-))

LEMMA: TAKE A, B & Alg(&).
THEN,

 $Q \otimes_{A} -: A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C})$ IS RIGHT EXACT,

YQ ∈ (B,A)-Bimod(E).

PF/ STS (QOA-) PRESERVES COKERNELS.

TAKE Ø: M -> N e A-Mod(&).

WANT: Q@A Coker(\$) = coker (id @ \$)

QQAM (dQB)
QQAN (dQB)
QQAN (idQB)
Coker(idQQB)

OF COKERNELS
QQA COKER(B)

ALSO GET & E B-Mod(C) SINCE BOO IS RIGHT EXACT.

TAKE 1R-ALGS. A, B. & Q=BQA BIMODILE.

GET ADDITIVE FUNCTORS:

RIGHTEX. QQ -: A-Mod -> B-Mod

Hom_{B-Mod} (Q₁-): B-Mod -> A-Mod LEFT EX.

WITH (Q& -) - (HomB-mod (Q1-))

LEMMA: TAKE A, B & Alg(&).
THEN,

 $Q \otimes_{A} -: A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C})$ IS RIGHT EXACT,

 $\forall Q \in (B,A) - Binod(4).$

PF/ STS (QOA-) PRESERVES COKERNELS.

TAKE Ø: M -> N e A-Mod(&).

WANT: Q@A Coker(\$) = (oker (id @ \$)

QQAM (dQX)
QQAN (dQX)

BY UNIV. PROP. idQXX

OF COKERNELS

QQA COKER(X)

ALSO GET & E B-Mod(E) SINCE BOO IS RIGHT EXACT.

ON THE OTHER HAND, GET

82: Q⊗A coker(Ø) → coker(ida⊗AØ) ∈ B-Mod(€)

VIA THE UNIV. PROP. OF COEQUALIZERS.

ASSUME & ABELIAN MONOIDAL \$
\$\(\times \) \(\times \) \(

TAKE 1R-ALGS. A, B. & Q=BQA BIMODILE.

GET ADDITIVE FUNCTORS:

RIGHTEX. QQ -: A-Mod -> B-Mod

Hom_{B-Mod} (Q₁-): B-Mod -> A-Mod LEFT EX.

WITH (Q& -) - (ttom B-mod (Q1-))

LEMMA: TAKE A, B & Alg(&).
THEN,

 $Q \otimes_{A} -: A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C})$ IS RIGHT EXACT,

 $\forall Q \in (B,A) - Binod(e).$

PF/ STS (QOA-) PRESERVES COKERNELS.

TAKE Ø: M -> N e A-Mod(&).

WANT: Q@A Coker(\$) = (oker (id @ \$)

QQAM (dQX)
QQAN (dQXX)

BY UNIV. PROP. idQXX

OF COKERNELS

QQA COKER(X)

ALSO GET & E B-Mod(E) SINCE BOO IS RIGHT EXACT.

ON THE OTHER HAND, GET

82: Q⊗A coker(Ø) → coker(ida⊗AØ) ∈ B-Mod(e)

VIA THE UNIV. PROP. OF COEQUALIZERS.

CHECK 81, 82 MUTHALLY INVERSE.

ASSUME & ABELIAN MONOIDAL & X&-, -&X ARE RIGHT EXACT YXER

EILENBERG-WATTS THEOREM

TAKE FINITE DIM'L IR-ALGEBRAS A, B. FOR IR-LINEAR

F: A-FdMod → B-FdMod, GET:

FLEFTEXACT

1

F HAS A LEFT ADJOINT

1

F = HOM (P,-)

FOR SOME BIMOD.

P=APB.

FRIGHT EXACT

1

FHAS A RIGHT ADJOINT

1

F=Q&-

FOR SOME BIMOD.

 $Q = {}_{B}Q_{A}$.

LEMMA: TAKE A, B \in Alg(\mathcal{C}). THEN, $Q \otimes_{A} -: A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C})$ IS RIGHT EXACT, $\forall Q \in (B, A) - Binod(\mathcal{C}).$

ASSUME & ABELIAN MONOIDAL & X&-, -&X ARE RIGHT EXACT YXEY

GENERALIZED EW THEOREM

TAKE A, B & Alg(&).

RECALL A-Mod(2), B-Mod(2) & Mod-2

VIA $(M, D) \triangleleft_{A \circ RB} X := (M \otimes X, D \otimes id_X).$

LEMMA: TAKE A, B \in Alg(\mathcal{C}). THEN, $Q \otimes_{A} -: A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C})$ IS RIGHT EXACT, $\forall Q \in (B, A) - Bimod(\mathcal{C}).$

EILENBERG-WATTS THEOREM TAKE FINITE DIM'L IR-ALGEBRAS A, B. FOR IR-LINEAR

F: A-FdMod -> B-FdMod, GET:

FLEFTEXACT

F HAS A LEFT ADJOINT

1

F = HOM (P,-) A-Famod

FOR SOME BIMOD.

P=APB.

FRIGHT EXACT

FHAS A
RIGHT ADJOINT

F=QQA

FOR SOME BIMOD.

Q=BQA.

ASSUME & ABELIAN MONOIDAL & X&-, -&X ARE RIGHT EXACT YXEY

GENERALIZED EW THEOREM

TAKE A, B & Alg(&).

RECALL A-Mod(2), B-Mod(2) & Mod-2

VIA $(M, D) \triangleleft_{A \text{ or } B} X := (M \otimes X, D \otimes id_X).$

THEN, WE GET AN EQUIV. OF CATS:

Rexmode (A-Mod(&), B-Mod(&))

(B,A)-Bimod(e).

RIGHTEXACT

MOD. CAT. FUNCTORS

LEMMA: TAKE A, B \in Alg(\mathcal{C}). THEN, $\mathbb{Q} \otimes_{\mathbb{A}} -: \mathbb{A} - \mathbb{M} \circ \mathbb{A}(\mathcal{C}) \longrightarrow \mathbb{B} - \mathbb{M} \circ \mathbb{A}(\mathcal{C})$

IS RIGHT EXACT,

YQ ∈ (B,A)-Bimod(E).

EILENBERG-WATTS THEOREM

TAKE FINITE DIM'L IR-ALGEBRAS A, B.

FOR IR-LINEAR

F: A-FdMod -> B-FdMod, GET:

FLEFTEXACT

1

F HAS A

LEFT ADJOINT

F = HOM (P,-)

FOR SOME BIMOD.

 $P = A P_B$.

FRIGHT EXACT

1

FHAS A RIGHT ADJOINT

1

F=QQ-

FOR SOME BIMOD.

 $Q = {}_{B}Q_{A}$.

ASSUME & ABELIAN MONOIDAL 3 * X®-, -®X ARE RIGHT EXACT YXEY

GENERALIZED EW THEOREM

TAKE A, B & Alg(&).

RECALL A-Mod(2), B-Mod(2) & Mod-2

VIA $(M, D) \triangleleft_{A \circ R} X := (M \otimes X, D \otimes id_X).$

THEN, WE GET AN EQUIV. OF CATO:

Rex_{Mod-e} (A-Mod(e), B-Mod(e))

(B, A)-Bimod(e).

RIGHTEXACT

MOD. CAT. FUNCTORS

LEMMA: TAKE A, B \in Alg(\mathcal{C}). THEN, $Q \otimes_{A} -: A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C})$ IS RIGHT EXACT, $\forall Q \in (B, A) - Bimod(\mathcal{C}).$ PF/

ASSUME & ABELIAN MONOIDAL & X&-, -&X ARE RIGHT EXACT YXER

GENERALIZED EW THEOREM

TAKE A, B & Alg(&).

RECALL A-Mod(2), B-Mod(2) & Mod-2

VIA $(M, D) \underset{A \text{ or } B}{\triangleleft} X := (M \otimes X, D \otimes id_X).$

THEN, WE GET AN EQUIV. OF CATS:

Rex_{Mod-e} (A-Mod(e), B-Mod(e))

(B,A)-Bimod(C).

RIGHTEXACT

MOD. CAT. FUNCTORS

LEMMA: TAKE A, B & Alg (&). THEN,

 $Q \otimes_{A} -: A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C})$

IS RIGHT EXACT,

YQ ∈ (B,A)-Bimod(e).

[F: A-Mod(C) -> B-Mod(C)]

F(A Areg)

ASSUME & ABELIAN MONOIDAL & X&-, -&X ARE RIGHT EXACT YXER

GENERALIZED EW THEOREM

TAKE A, B & Alg(&).

RECALL A-Mod(2), B-Mod(2) & Mod-2

VIA $(M, D) \triangleleft_{A \text{ or } B} X := (M \otimes X, D \otimes id_X).$

THEN, WE GET AN EQUIV. OF CATS:

Rex_{Mod-e} (A-Mod(e), B-Mod(e))

(B, A)-Bimod(e).

RIGHTEXACT

MOD. CAT. FUNCTORS

LEMMA: TAKE A, B & Alg (&). THEN,

 $Q \otimes_{A} -: A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C})$

IS RIGHT EXACT,

YQ ∈ (B,A)-Bimod(E).

[F: A-Mod(&) -> B-Mod(&)]

F(A Area)

· HAVE F(A) & B-Mod(e).

ASSUME & ABELIAN MONOIDAL \$\\
\(\times \) \

GENERALIZED EW THEOREM

TAKE A, B & Alg(&).

RECALL A-Mod(2), B-Mod(2) & Mod-2

VIA $(M,D) \leq_{A \circ RB} X := (M \otimes X, D \otimes id_X).$

THEN, WE GET AN EQUIV. OF CATS:

Rex_{Mod-e} (A-Mod(e), B-Mod(e))

(B, A)-Bimod(e).

RIGHTEXACT

MOD. CAT. FUNCTORS

LEMMA: TAKE A, B \in Alg(\mathcal{C}). THEN, $Q \otimes_A - : A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C})$ IS RIGHT EXACT, $\forall Q \in (B,A) - Bimod(\mathcal{C}).$ [F: A-Mod(C) -> B-Mod(C)]

F(A Areg)

- · HAVE F(A) & B-Mod(e).
- · DEFINE

$$A_{F(A)}: F(A) \otimes A \longrightarrow F(A)$$

III DEF $f(A) \otimes A$
 $F(A) \otimes A \longrightarrow F(A)$
 $F(A) \otimes A \longrightarrow F(A)$

TO GET F(A) & Mod-A(C).

ASSUME & ABELIAN MONOIDAL \$\\\ \X\times -, -\times \times \text{ARE RIGHT EXACT \\X\times \text{Y}}

GENERALIZED EW THEOREM

TAKE A, B & Alg(&).

RECALL A-Mod(2), B-Mod(2) & Mod-2

VIA $(M, D) \underset{A \circ RB}{\triangleleft} X := (M \otimes X, D \otimes id_X).$

THEN, WE GET AN EQUIV. OF CATS:

Rex_{Mod-e} (A-Mod(e), B-Mod(e))

(B, A)-Bimod(e).

RIGHTEXACT

MOD. CAT. FUNCTORS

LEMMA: TAKE A, B \in Alg(\mathcal{C}). THEN, $Q \otimes_{A} -: A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C})$ IS RIGHT EXACT, $\forall Q \in (B, A) - Binod(\mathcal{C}).$ [F: A-Mod(&) -> B-Mod(&)] Φ F(A Areg)

- · HAVE F(A) & B-Mod(e).
- · DEFINE

TO GET F(A) & Mod-A(C).

· CHECK F(A) ∈ (B, A)—Bimod(e).

ASSUME & ABELIAN MONOIDAL \$ X&-, -&X ARE RIGHT EXACT YXEY

GENERALIZED EW THEOREM

TAKE A, B & Alg(&).

RECALL A-Mod(&), B-Mod(&) & Mod-&

THEN, WE GET AN EQUIV. OF CATS:

Rexmoder (A-Mod(&), B-Mod(&))

= (B, A)-Bimod(E).

RIGHTEXACT

MOD. CAT. FUNCTORS

LEMMA: TAKE A, BE Alg (C). THEN,

 $Q \otimes_{A} -: A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C})$

IS RIGHT EXACT,

YQ ∈ (B,A)-Bimod(e).

[F: A-Mod(&) -> B-Mod(&)]

VIA $(M,D) \triangleleft_{ABB} X := (M \otimes X,D \otimes id_X)$. $[Q \otimes_A - : A - Mod(C) \rightarrow B - Mod(C)] \xleftarrow{\Psi}_{R} Q_A$

ASSUME & ABELIAN MONOIDAL & X&-, -&X ARE RIGHT EXACT YXEY

GENERALIZED EW THEOREM

TAKE A, B & Alg(&).

RECALL A-Mod(2), B-Mod(2) & Mod-2

VIA $(M, D) \underset{A \circ RB}{\triangleleft} X := (M \otimes X, D \otimes id_X).$

THEN, WE GET AN EQUIV. OF CATS:

Rex_{Mod-e} (A-Mod(e), B-Mod(e))

= (B, A)-Bimod(e).

RIGHTEXACT

MOD. CAT. FUNCTORS

LEMMA: TAKE A, B & Alg (C). THEN,

 $Q \otimes_{A} -: A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C})$

IS RIGHT EXACT,

YQ ∈ (B,A)-Bimod(E).

 $\begin{bmatrix} F: A-Mod(\mathcal{C}) \longrightarrow B-Mod(\mathcal{C}) \end{bmatrix}$

[Q&_-: A-Mod(&) -> B-Mod(&)] (BQA

7 THIS IS RIGHT EXACT/

ASSUME & ABELIAN MONOIDAL \$\\\ \X\times -, -\times \times \text{ARE RIGHT EXACT \\X\times \text{Y}}

GENERALIZED EW THEOREM

TAKE A, B & Alg(&).

RECALL A-Mod(&), B-Mod(&) & Mod-&

VIA $(M, D) \underset{A \circ RB}{\triangleleft} X := (M \otimes X, D \otimes id_X).$

THEN, WE GET AN EQUIV. OF CATS:

Rex_{Mod-e} (A-Mod(e), B-Mod(e))

∠ (B, A) – Bimod(E).

RIGHTEXACT

MOD. CAT. FUNCTORS

LEMMA: TAKE A, B \in Alg(\mathcal{C}). THEN, $Q \otimes_{A} -: A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C})$ IS RIGHT EXACT, $\forall Q \in (B,A) - Bimod(\mathcal{C}).$ [F: A-Mod(&) -> B-Mod(&)]

F(A Area)

[QQ_-:A-Mod(E) → B-Mod(E)] ← BQA

THIS IS RIGHT EXACT

THIS IS A Y-MODINE FUNCTOR:

$$Q \otimes_{A} (M \triangleleft_{A} \times) = Q \otimes_{A} (M \otimes \times)$$

$$\cong (Q \otimes_A M) \otimes X$$

$$\equiv \left(Q \otimes_{A} M \right) \triangleleft_{B} X_{A}$$

YXEC, MEA-MODIC).

PF/

ASSUME & ABELIAN MONOIDAL \$\\\ \X\times - \, -\infty \text{ARE RIGHT EXACT YXEY}

GENERALIZED EW THEOREM

TAKE A, B & Alg(&).

RECALL A-Mod(2), B-Mod(2) & Mod-2

VIA $(M, D) \underset{A \text{ or } B}{\triangleleft} X := (M \otimes X, D \otimes id_X).$

THEN, WE GET AN EQUIV. OF CATS:

Rex_{Mod-e} (A-Mod(e), B-Mod(e))

(B,A)-Bimod(e).

RIGHTEXACT

MOD. CAT. FUNCTORS

LEMMA: TAKE A, B \in Alg(\mathcal{C}). THEN, $Q \otimes_{A} -: A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C})$ IS RIGHT EXACT, $\forall Q \in (B, A) - Binod(\mathcal{C}).$

[F: A-Mod(&) -> B-Mod(&)]

F(A Areg)

[Q&-:A-Mod(&) -> B-Mod(&)] (BQA

7 THIS IS RIGHT EXACT/

THIS IS A Y-MODINE FUNCTOR:

 $Q \otimes_{A} (M \triangleleft_{A} \times) = Q \otimes_{A} (M \otimes \times)$

 $\cong (Q \otimes_{A} M) \otimes X$

 $\equiv \left(Q \otimes_{A} M \right) \triangleleft_{B} X_{A}$

YXEC, MEA-MODIC).

: Y IS WELL-DEFINED.

PF/

ASSUME & ABELIAN MONOIDAL

GENERALIZED EW THEOREM

TAKE A, B & Alg(&).

RECALL A-Mod(&), B-Mod(&) & Mod-&

THEN, WE GET AN EQUIV. OF CATS:

Rex_{Mod-e} (A-Mod(&), B-Mod(&))

= (B, A)-Bimod(e).

RIGHTEXACT

MOD. CAT. FUNCTORS

[F: A-Mod(&) -> B-Mod(&)]

VIA $(M,D) \triangleleft_{A \bowtie B} X := (M \otimes X,D \otimes id_X)$. $[Q \otimes_A : A - Mod(X) \rightarrow B - Mod(X)] \xleftarrow{\Psi}_{B} Q_{A}$

ASSUME & ABELIAN MONOIDAL & XO-, -OX ARE RIGHT EXACT YXEY

GENERALIZED EW THEOREM

TAKE A, B & Alg(&).

RECALL A-Mod(2), B-Mod(2) & Mod-2

VIA $(M, D) \underset{A \text{ or } B}{\triangleleft} X := (M \otimes X, D \otimes id_X).$

THEN, WE GET AN EQUIV. OF CATO:

Rex_{Mod-e} (A-Mod(e), B-Mod(e))

(B,A)-Bimod(e).

RIGHTEXACT

MOD. CAT. FUNCTORS

F/

 $[F: A-Mod(\mathcal{C}) \longrightarrow B-Mod(\mathcal{C})]$

 $\xrightarrow{\Phi} F(AAreg)$

[Q&-:A-Mod(x) -> B-Mod(x)] (I) BQA

Now $\Phi \Psi(Q) = \Phi(Q \otimes_A -)$ = $Q \otimes_A A(A_{reg}) \cong Q$.

ASSUME & ABELIAN MONOIDAL \$\\\ \X\times -, -\times \times \text{ARE RIGHT EXACT \\X\times \text{Y}}

GENERALIZED EW THEOREM

TAKE A, B & Alg(&).

RECALL A-Mod(&), B-Mod(&) & Mod-&

VIA $(M, D) \underset{A \text{ or } B}{\triangleleft} X := (M \otimes X, D \otimes id_X).$

THEN, WE GET AN EQUIV. OF CATO:

Rex_{Mod-e} (A-Mod(e), B-Mod(e))

(B, A)-Bimod(e).

RIGHTEXACT

MOD. CAT. FUNCTORS

PF/

[F: A-Mod(&) -> B-Mod(&)]

\[\bullet = \bullet (\lambda \text{Acco}) \]

[Q&-:A-Mod(&) -> B-Mod(&)] (BQA

Now $\Phi + (Q) = \Phi (Q \otimes_A -)$ = $Q \otimes_A A (A_{reg}) \cong Q$.

ON THE OTHER HAND, TAKE:

F: A-Mod(E) -> B-Mod(E) & Mod-E RIGHTEXACT

CLAIM: F(A Areg) & M = F(M) IN B-Mod(&) JM & A-Mod(&)

4<u>∮(</u>F) = F

ASSUME & ABELIAN MONOIDAL \$\\\ \X\Times_, -\Times \times \text{ARE RIGHT EXACT \\X\times \times \\\

GENERALIZED EW THEOREM

TAKE A, B & Alg(&).

RECALL A-Mod(&), B-Mod(&) & Mod-&

VIA $(M, D) \underset{A \text{ or } B}{\triangleleft} X := (M \otimes X, D \otimes id_X).$

THEN, WE GET AN EQUIV. OF CATO:

Rex_{Mod-e} (A-Mod(e), B-Mod(e)) (B,A)-Bimod(e).

RIGHTEXACT

MOD. CAT. FUNCTORS

 $F: A-Mod(Y) \longrightarrow R-Mod(Y)$

[F: A-Mod(C) -> B-Mod(C)]

F(A Areg)

[Q&-: A-Mod(C) -> B-Mod(C)] CO BQA

Now $\Phi + (Q) = \Phi (Q \otimes_A -)$ = $Q \otimes_A A (A_{reg}) \cong Q$.

ON THE OTHER HAND, TAKE:

F: A-Mod(&) -> B-Mod(&) & Mod-&
RIGHTEXACT

CLAIM: F(A Areg) @ M = F(M) IN B-Mod(&) JM & A-Mod(&)

YPF/ GET: F(A) & A M -> F(M) WIA UNIV. PROP. OF COED., &

PF/

ΨΦ(F) = F F(M) = F(A⊗AM) -> F(A)⊗AM = FRIGHT EX. & PRES. COEQS.

... CHECK MUTUALLY INV. //

ASSUME & ABELIAN MONOIDAL & X&-, -&X ARE RIGHT EXACT YXER

MORITA'S THEOREM

TAKE IR-ALGEBRAS A & B. THEN:

A IS MORITA EQUIVALENT TO B

3 BIMODULES APB & BQA .7.

POBQ = Areg AS A-BIMODULES

\$ Q ØA P = Brey AS B-BIMODULES.

[FROM LECTURE #9

ASSUME & ABELIAN MONOIDAL & X&-, -&X ARE RIGHT EXACT YXEC)

GENERALIZED MORITA'S THEOREM

MORITA'S THEOREM

TAKE IR-ALGEBRAS A & B. THEN:

A IS MORITA EQUIVALENT TO B

3 BIMODULES APB & BQA .7.

POBQ = Areg AS A-BIMODULES

\$ Q ØA P = Brey AS B-BIMODULES.

[FROM LECTURE #9

WE WILL GENERALIZE
THIS IN THE
MONOIDAL SETTING

ASSUME & ABELIAN MONOIDAL & X&-, -&X ARE RIGHT EXACT YXEY

GENERALIZED MORITA'S THEOREM

TAKE A, B & Alg (C). THEN:

A-Mod(e) ~ B-Mod(e)

AS RIGHT G-MODULE CATEGORIES

1

3 BIMODULES APB & BQA . 7.

POBQ = Area AS A-BIMODULES IN &

\$ Q ØA P = Brey AS B-BIMODULES IN C.

MORITA'S THEOREM

TAKE IR-ALGEBRAS A & B. THEN:

A IS MORITA EQUIVALENT TO B

3 BIMODULES APB & BQA .7.

POBQ = Area AS A-BIMODULES

* Q @ P = Brey AS B-BIMODULES.

[FROM LECTURE #9

WE WILL GENERALIZE
THIS IN THE
MONOIDAL SETTING

GENERALIZED MORITA'S THEOREM

TAKE A, B & Alg (C). THEN:

 $A-Mod(e) \simeq B-Mod(e)$

AS RIGHT G-MODULE CATEGORIES

1

3 BIMODULES APB & BQA .7.

POBQ = Areg AS A-BIMODULES IN &

\$ Q @ P = Breg AS B-BIMODULES IN C.

SAY A,B ARE

MORITA EQUIVALENT IN &

GENERALIZED MORITA'S THEOREM

TAKE A, B & Alg (C). THEN:

A-Mod(E) = B-Mod(E)

AS RIGHT G-MODULE CATEGORIES

1

3 BIMODULES APB & BQA .7.

POBQ = Area AS A-BIMODULES IN &

\$ Q @ P = Breg AS B-BIMODULES IN 6.

PF/
(1) DEFINE:

 $F := Q \otimes_A - : A - Mod(\mathcal{C}) \rightarrow B - Mod(\mathcal{C})$

 $G := P \otimes_{B} - : B - Mod(\mathcal{C}) \longrightarrow A - Mod(\mathcal{C}).$

THEN YME A-Mod(&),

 $GF(M) = P \otimes_{\mathcal{B}} (Q \otimes_{A} M) \cong (P \otimes_{\mathcal{B}} Q) \otimes_{A} M$ $\cong A \otimes_{A} M \cong M.$

GENERALIZED MORITA'S THEOREM

TAKE A, B & Alg (C). THEN:

A-Mod(E) = B-Mod(E)

AS RIGHT G-MODULE CATEGORIES

1

3 BIMODULES APB & BQA . 7.

POBQ = Area AS A-BIMODULES IN &

\$ Q ØA P = Brey AS B-BIMODULES IN C.

(1) DEFINE:

PF/

F:= Q & -: A-Mod(C) -> B-Mod(C),

 $G := P \otimes_{B} - : B - Mod(\mathcal{C}) \longrightarrow A - Mod(\mathcal{C}).$

THEN YME A-Mod(&),

 $GF(M) = P \otimes_{\mathcal{B}} (Q \otimes_{A} M) \cong (P \otimes_{\mathcal{B}} Q) \otimes_{A} M$ $\cong A \otimes_{A} M \cong M.$

: GF = Id_-Mod(e)

LIKEWISE, FG= Id B-Mod(e)

GENERALIZED MORITA'S THEOREM

TAKE A, B & Alg (&). THEN:

A-Mod(e) ~ B-Mod(e)

AS RIGHT G-MODULE CATEGORIES

1

3 BIMODULES APB & BQA .7.

POBQ = Area AS A-BIMODULES IN &

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PF/ (1) DEFINE:

 $F := Q \otimes_{A} - : A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C}),$

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THEN YME A-Mod(&),

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: GF = Id_-Mod(e)

LIKEWISE, FG= Id B-Mod(e)

:. F IS AN EQUIVALENCE OF CATEGORIES.

ASSUME & ABELIAN MONOIDAL \$\\
\(\bigk \times - \, -\infty \times \text{ARE RIGHT EXACT \(\bigk \text{XEV} \)

GENERALIZED MORITA'S THEOREM

TAKE A, B & Alg (C). THEN:

A-Mod(e) ~ B-Mod(e)

AS RIGHT G-MODULE CATEGORIES

1

3 BIMODULES APB & BQA . 7.

POBQ = Area AS A-BIMODULES IN &

\$ Q @ P = Breg AS B-BIMODULES IN C.

ALSO AME A-MOD(G), XEG:

 $F(M \triangleleft_A X) = F(M \otimes X)$

= Q O (M O X) = (Q O M) O X

 $= F(M) \otimes X = F(M) \triangleleft_{B} X$

PF/ (1) DEFINE:

 $F := Q \otimes_{A} - : A - Mod(\mathcal{C}) \longrightarrow B - Mod(\mathcal{C}),$

 $G := P \otimes_{B} - : B - Mod(\mathcal{C}) \rightarrow A - Mod(\mathcal{C}).$

THEN YME A-Mod(&),

 $GF(M) = P \otimes_{\mathcal{B}} (Q \otimes_{A} M) \cong (P \otimes_{\mathcal{B}} Q) \otimes_{A} M$ $\cong A \otimes_{A} M \cong M.$

: GF = Id_Mod(e)

LIKEWISE, FG= IdB-Mod(e)

:. F IS AN EQUIVALENCE OF CATEGORIES.

ASSUME & ABELIAN MONOIDAL } \$\times X\omega-, -\omega X ARE RIGHT EXACT \times X\omega'\omega'

GENERALIZED MORITA'S THEOREM

TAKE A, B & Alg (C). THEN:

A-Mod(E) ~ B-Mod(E)

AS RIGHT G-MODULE CATEGORIES

1

3 BIMODULES APB & BQA . 7.

POBQ = Area AS A-BIMODULES IN &

\$ Q @ P = Breg AS B-BIMODULES IN 6.

ALSO YMEA-Mod(C), XEC:

 $F(M \triangleleft_A X) = F(M \otimes X)$

= Q OA (M OX) = (QOA M) OX

 $= F(M) \otimes X = F(M) \triangleleft_{B} X$

PF/ (1) DEFINE:

 $F := Q \otimes_A - : A - Mod(\mathcal{C}) \rightarrow B - Mod(\mathcal{C}),$

 $G := P \otimes_{B} - : B - Mod(\mathcal{C}) \rightarrow A - Mod(\mathcal{C}).$

THEN YME A-Mod(&),

 $GF(M) = P \otimes_{\mathcal{B}} (Q \otimes_{A} M) \cong (P \otimes_{\mathcal{B}} Q) \otimes_{A} M$ $\cong A \otimes_{A} M \cong M.$

: GF = Id_-Mod(e)

LIKEWISE, FG= IdB-Mod(e)

.. F IS AN EQUIVALENCE

OF, CATEGORIES.

MODULE

ASSUME & ABELIAN MONOIDAL & X&-, -&X ARE RIGHT EXACT YXE'S

GENERALIZED MORITA'S THEOREM

TAKE A, B & Alg (C). THEN:

A-Mod(e) ~ B-Mod(e)

AS RIGHT G-MODULE CATEGORIES

1

3 BIMODULES APB & BQA .7.

POBQ = Area AS A-BIMODULES IN &

\$ Q ØA P = Brey AS B-BIMODULES IN C.

PF/

(V) TAKE QUASI-INVERSES:

F: A-Mod(4) -> B-Mod(4)

 $G: B-Mod(C) \rightarrow A-Mod(C).$

ASSUME & ABELIAN MONOIDAL & X&-, -&X ARE RIGHT EXACT YXEC

GENERALIZED MORITA'S THEOREM

TAKE A, B & Alg (C). THEN:

A-Mod(e) ~ B-Mod(e)

AS RIGHT G-MODULE CATEGORIES

1

3 BIMODULES APB & BQA .7.

POBQ = Area AS A-BIMODULES IN &

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PF/

(V) TAKE QUASI-INVERSES:

F: A-Mod(4) -> B-Mod(4),

 $G: B-Mod(C) \rightarrow A-Mod(C).$

THEY'RE EQUIVALENCES,

GENERALIZED MORITA'S THEOREM

TAKE A, B & Alg (C). THEN:

A-Mod(e) ~ B-Mod(e)

AS RIGHT G-MODULE CATEGORIES

1

3 BIMODULES APB & BQA . 7.

POBQ = Area AS A-BIMODULES IN &

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PF/

() TAKE QUASI-INVERSES:

F: A-Mod(4) -> B-Mod(4),

 $G: B-Mod(C) \rightarrow A-Mod(C).$

THEY'RE EQUIVALENCES,
SO HAVE RIGHT ADJOINTS

GENERALIZED MORITA'S THEOREM

TAKE A, B & Alg (C). THEN:

A-Mod(e) ~ B-Mod(e)

AS RIGHT G-MODULE CATEGORIES

1

3 BIMODULES APB & BQA . 7.

POBQ = Area AS A-BIMODULES IN &

\$ Q ØA P = Brey AS B-BIMODULES IN C.

PF/

() TAKE QUASI-INVERSES:

 $F: A-Mod(\mathcal{C}) \rightarrow B-Mod(\mathcal{C})$

 $G: B-Mod(C) \rightarrow A-Mod(C).$

THEY'RE EQUIVALENCES,

80 HAVE RIGHT ADJOINTS

80 ARE RIGHT EXACT.

ASSUME & ABELIAN MONOIDAL & X&-, -&X ARE RIGHT EXACT YXEX

GENERALIZED MORITA'S THEOREM

TAKE A, B & Alg (C). THEN:

A-Mod(e) ~ B-Mod(e)

AS RIGHT G-MODULE CATEGORIES

1

3 BIMODULES APB & BQA . 7.

POBQ = Area AS A-BIMODULES IN &

\$ Q ØA P = Brey AS B-BIMODULES IN C.

PF/
(IL) TAKE QUASI-INVERSES:

F: A-Mod(4) -> B-Mod(4),

G: B-Mod(4) -> A-Mod(4).

THEY'RE EQUIVALENCES,

80 HAVE RIGHT ADJOINTS

80 ARE RIGHT EXACT.

BQA.3. F=QQA-APB.3. G=PQB-

GENERALIZED EW THEOREM

TAKE A, B \in Alg($^{\circ}$). THEN,

Rex_{Mod-e} (A-Mod($^{\circ}$), B-Mod($^{\circ}$)) \cong (B, A)—Bimod($^{\circ}$).

ASSUME & ABELIAN MONOIDAL XXEX

IN A-Bimod(4)

GENERALIZED MORITA'S THEOREM

TAKE A, B & Alg (C). THEN:

A-Mod(e) ~ B-Mod(e)

AS RIGHT G-MODULE CATEGORIES

1

3 BIMODULES APB & BQA . 7.

POBQ = Area AS A-BIMODULES IN &

\$ Q @ P = Bren AS B-BIMODULES IN 6.

PF/
(IL) TAKE QUASI-INVERSES:

 $F: A-Mod(\mathcal{C}) \rightarrow B-Mod(\mathcal{C})$

 $G: B-Mod(C) \rightarrow A-Mod(C).$

THEY'RE EQUIVALENCES,
SO HAVE RIGHT ADJOINTS
SO ARE RIGHT EXACT.

BQA.F. F=Q&A- CLAIM
APB.F. G=P&B- P&BQ=A

GENERALIZED EW THEOREM

TAKE A, B & Alg(&). THEN,

Rex_{Mod-e} (A-Mod(e), B-Mod(e))

= (B, A)-Bimod(&).

ASSUME & ABELIAN MONOIDAL & X&-, -&X ARE RIGHT EXACT YXEX

GENERALIZED MORITA'S THEOREM

TAKE A, B & Alg (C). THEN:

A-Mod(e) ~ B-Mod(e)

AS RIGHT G-MODULE CATEGORIES

1

3 BIMODULES APB & BQA . 7.

POBQ = Area AS A-BIMODULES IN &

\$ Q ØA P = Brey AS B-BIMODULES IN C.

PF/HAVE GF = Id_A-Mod(e)
VIA NATURAL ISOMORPHISM \$\Darpsilon\$.

(I) TAKE QUASI-INVERSES:

F: A-Mod(4) -> B-Mod(4),

 $G: B-Mod(\mathcal{C}) \longrightarrow A-Mod(\mathcal{C}).$

THEY'RE EQUIVALENCES,
SO HAVE RIGHT ADJOINTS
SO ARE RIGHT EXACT.

 $\exists \begin{cases} \mathbb{R}^{\mathbb{Q}_{A}} . \exists \mathbb{R}^{\mathbb{Q}_{A}} & \mathbb{R}^{\mathbb{Q}_{A}} & \mathbb{R}^{\mathbb{Q}_{A}} \\ \mathbb{R}^{\mathbb{Q}_{A}} . \exists \mathbb{R}^{\mathbb{Q}_{A}} & \mathbb{R}^{\mathbb{Q}_{A}} & \mathbb{R}^{\mathbb{Q}_{A}} & \mathbb{R}^{\mathbb{Q}_{A}} \\ \mathbb{R}^{\mathbb{Q}_{A}} . \exists \mathbb{R}^{\mathbb{Q}_{A}} & \mathbb{R}^{\mathbb{Q}_{A}} & \mathbb{R}^{\mathbb{Q}_{A}} & \mathbb{R}^{\mathbb{Q}_{A}} \\ \mathbb{R}^{\mathbb{Q}_{A}} & \mathbb{R}^{\mathbb{Q}_{A}} & \mathbb{R}^{\mathbb{Q}_{A}} & \mathbb{R}^{\mathbb{Q}_{A}} & \mathbb{R}^{\mathbb{Q}_{A}} \\ \mathbb{R}^{\mathbb{Q}_{A}} & \mathbb{R}^{\mathbb{Q}_{A}} & \mathbb{R}^{\mathbb{Q}_{A}} & \mathbb{R}^{\mathbb{Q}_{A}} & \mathbb{R}^{\mathbb{Q}_{A}} & \mathbb{R}^{\mathbb{Q}_{A}} \\ \mathbb{R}^{\mathbb{Q}_{A}} & \mathbb{Q}^{\mathbb{Q}_{A}} & \mathbb{Q}^{\mathbb{Q}} & \mathbb{Q}$

PF/

GENERALIZED MORITA'S THEOREM

TAKE A, B & Alg (C). THEN:

A-Mod(E) ~ B-Mod(E)

AS RIGHT G-MODULE CATEGORIES

1

3 BIMODULES APB & BQA .7.

POBQ = Area AS A-BIMODULES IN &

\$ Q @ P = Bren AS B-BIMODULES IN C.

PF/HAVE GF = Id_A-Mod(e)
VIA NATURAL 180 MORPHISM .

() TAKE QUASI-INVERSES:

F: A-Mod(()) > B-Mod(())

 $G: B-Mod(\mathcal{C}) \longrightarrow A-Mod(\mathcal{C}).$

THEY'RE EQUIVALENCES,

80 HAVE RIGHT ADJOINTS

80 ARE RIGHT EXACT.

 $\exists \begin{cases} \mathbb{R} \mathbb{Q} A . \exists . \exists \mathbb{Q} \mathbb{Q}_A - \mathbb{C} LAIM \\ A \mathbb{Q}_B . \exists . \mathbb{Q} \mathbb{Q}_B - \mathbb{Q} \mathbb{Q}_B - \mathbb{Q} \mathbb{Q} = A \end{cases}$ IN A-Bimod(%)

THE COMPONENT $\Phi_A: GF(A) \xrightarrow{\sim} A$ DOES THE TRICK ... // $P \otimes_B Q \cong P \otimes_B (Q \otimes_A A)$

PF/

GENERALIZED MORITA'S THEOREM

TAKE A, B & Alg (C). THEN:

A-Mod(e) ~ B-Mod(e)

AS RIGHT G-MODULE CATEGORIES

1

3 BIMODULES APB & BQA . 7.

POBQ = Area AS A-BIMODULES IN &

\$ Q @ P = Bren AS B-BIMODULES IN 6.

PF/HAVE GF = Id_A-Mod(e)
VIA NATURAL 180 MORPHISM \$\Darpsilon\$.

() TAKE QUASI-INVERSES:

 $F: A-Mod(\mathcal{C}) \rightarrow B-Mod(\mathcal{C})$

 $G: B-Mod(C) \rightarrow A-Mod(C).$

THEY'RE EQUIVALENCES,

80 HAVE RIGHT ADJOINTS

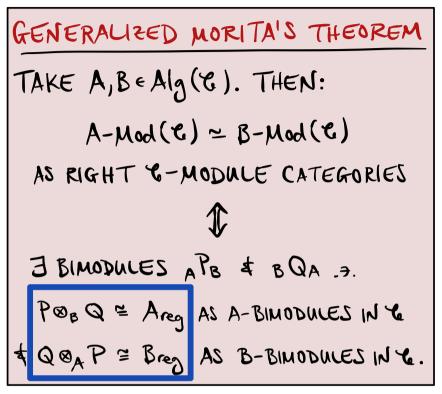
80 ARE RIGHT EXACT.

 $\exists \begin{cases} \mathbb{R} \mathbb{Q} A . \exists . F \cong \mathbb{Q} \otimes_{A} - & \text{CLAIM} \\ \mathbb{Q} P_{B} . \exists . G \cong \mathbb{P} \otimes_{B} - & \mathbb{P} \otimes_{B} \mathbb{Q} \cong \mathbb{A} \end{cases}$ IN $A - \text{Bimod}(\mathcal{C}_{A})$

THE COMPONENT $\Phi_A: GF(A) \xrightarrow{\sim} A$ DOES THE TRICK ... // $P \otimes_B Q \cong P \otimes_B (Q \otimes_A A)$ UKEWISE, $Q \otimes_A P \cong B$ IN B-BIMOD(4)//

PF/

ASSUME & ABELIAN MONOIDAL \$\\ \X\&-, -\&\X\\ ARE RIGHT EXACT \\X\ce{\X}\\



= SHOWING THIS IN PRACTICE =

USE UNIVERSAL PROPERTY OF COEQUALIZERS TO GET:

 $P \otimes_{\mathcal{B}} Q \xrightarrow{\not A_{A}} A \notin Q \otimes_{A} P \xrightarrow{\not A_{\mathcal{B}}} B$ $\in A \text{-Bimod}(\mathcal{C}) \in B \text{-Bimod}(\mathcal{C})$

THEN

ASSUME & ABELIAN MONOIDAL * X8-, - 8X ARE RIGHT EXACT YXE'C

GENERALIZED MORITA'S THEOREM

TAKE A, B & Alg (&). THEN:

A-Mod(C) ~ B-Mod(C)

AS RIGHT G-MODULE CATEGORIES

3 BIMODULES APB & BQA .7.

POBQ = Areg AS A-BIMODULES IN &

\$ Q ØA P = Bred AS B-BIMODULES IN C.

= SHOWING THIS IN PRACTICE =

USE UNIVERSAL PROPERTY OF COEQUALIZERS TO GET:

 $P \otimes_{R} Q \xrightarrow{\varphi_{A}} A \neq Q \otimes_{A} P \xrightarrow{\varphi_{B}} B$

€ A-Bimod(E) € B-Bimod(E)

PROP TAKE APB, BQA. IF JEPIS $P \otimes_{R} Q \xrightarrow{\phi_{A}} A \neq Q \otimes_{A} P \xrightarrow{\phi_{B}} B$ € A-Bimod(E) € B-Bimod(E) ·3.

THEN ØA, ØB ARE 1808.

THEN

ASSUME & ABELIAN MONOIDAL { \$ X8-, -8X ARE RIGHT EXACT YXE'C

GENERALIZED MORITA'S THEOREM

TAKE A, B & Alg (&). THEN:

A-Mod(C) ~ B-Mod(C)

AS RIGHT G-MODULE CATEGORIES

3 BIMODULES APB & BQA .7.

POBQ = Areg AS A-BIMODULES IN &

\$ Q ØA P = Brey AS B-BIMODULES IN C.

= SHOWING THIS IN PRACTICE =

USE UNIVERSAL PROPERTY OF COEQUALIZERS TO GET:

 $P \otimes_{\mathbb{R}} \mathbb{Q} \xrightarrow{\phi_{\mathbb{A}}} \mathbb{A} \not\subset \mathbb{Q} \otimes_{\mathbb{A}} \mathbb{P} \xrightarrow{\phi_{\mathbb{B}}} \mathbb{B}$

€ A-Bimod(E) € B-Bimod(E)

PROP TAKE APB, BQA. IF JEPIS $P \otimes_{R} Q \xrightarrow{\phi_{A}} A \neq Q \otimes_{A} P \xrightarrow{\phi_{B}} B$ € A-Bimod(E) € B-Bimod(E) $(P \otimes_B Q) \otimes_A P \xrightarrow{\sim} P \otimes_B (Q \otimes_A P)$ ASAP POBBB $(Q \otimes_{A} P) \otimes_{B} Q \xrightarrow{\sim} Q \otimes_{A} (P \otimes_{B} Q)$ THEN ØA, ØB ARE 1808.

ASSUME & ABELIAN MONOIDAL \$ X&-, -&X ARE RIGHT EXACT YXEY

GEN'D MORITA'S THM

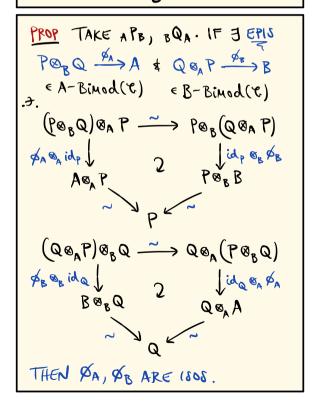
TAKE A, B & Alg (&). THEN:

A-Mod (&) ~ B-Mod (&) IN Mod-4

BIMODULES APB & BQA . F.

POBQ = Areg IN A-Bimod (&)

Q QAP = Brey IN B-Bimod (&)



ASSUME & ABELIAN MONOIDAL & W SIMPLE 11 RIGID

GEN'D MORITA'S THM

TAKE A, B & Alg (C). THEN:

A-Mod (C) = B-Mod (C) IN Mod-Y.

B BIMODULES APB & BQA ...

POBQ = Areg IN A-Bimod (C)

Q QAP = Brey IN B-Bimod (C)

PROP TAKE APB, BQA. IF $\exists EPIS$ $P \otimes_{B} Q \xrightarrow{\varphi_{A}} A \neq Q \otimes_{A} P \xrightarrow{\varphi_{E}} B$ $A = Bimod(\mathcal{C}) \in B - Bimod(\mathcal{C})$ $A \otimes_{A} P \xrightarrow{Q} P \otimes_{B} Q \otimes_{A} P$ $A \otimes_{A} P \xrightarrow{Q} P \otimes_{B} B$ $A \otimes_{A} P \xrightarrow{Q} P \otimes_{B} B$ $A \otimes_{A} P \xrightarrow{Q} Q \otimes_{A} P \otimes_{B} Q$ $A \otimes_{A} P \xrightarrow{Q} Q \otimes_{A} P \otimes_{B} Q$ $A \otimes_{A} P \xrightarrow{Q} Q \otimes_{A} P \otimes_{B} Q$ $A \otimes_{A} P \xrightarrow{Q} Q \otimes_{A} P \otimes_{B} Q$ $A \otimes_{A} P \xrightarrow{Q} Q \otimes_{A} P \otimes_{B} Q$ $A \otimes_{A} P \xrightarrow{Q} Q \otimes_{A} P \otimes_{B} Q$ $A \otimes_{A} P \xrightarrow{Q} Q \otimes_{A} P \otimes_{B} Q$ $A \otimes_{A} P \xrightarrow{Q} Q \otimes_{A} P \otimes_{B} Q$ $A \otimes_{A} P \xrightarrow{Q} Q \otimes_{A} P \otimes_{B} Q$ $A \otimes_{A} P \xrightarrow{Q} Q \otimes_{A} P \otimes_{B} Q$ $A \otimes_{A} P \otimes_{B} Q \xrightarrow{Q} Q \otimes_{A} Q \otimes_{A} Q$

THEN ØA, ØB ARE 1808.

EXER.4.48 SHOW (1, Ju=ru, idu) AND

(X®X*, idx@evx@idx*, coevx)

ARE MORITA EQUIVALENT YX & C.

ASSUME & ABELIAN MONOIDAL 3

GEN'D MORITA'S THM

TAKE A, B & Alg (C). THEN:

A-Mod (C) = B-Mod (C) IN Mod-Y.

B BIMODULES APB & BQA ...

POBQ = Areg IN A-Bimod (C)

Q QAP = Brey IN B-Bimod (C)

PROP TAKE APB, BQA. IF 3 EPIS

POBO AA & QOAP BB

EA-BIMOD(C) EB-BIMOD(C)

(POBQ)OAP P POB(QOAP)

AOAP POBB

(QOAP)OBQ POBB

(QOAP)OBQ POBB

BOBQ

DIDGE SABA

THEN OA, OB ARE 1505.

EXER.4.48 SHOW (1, lu=ru, idu) AND

(X®X*, idx®evx®idx*, coevx)

ARE MORITA EQUIVALENT YX & C.

- ASSUME & ABELIAN MONOIDAL 3 W SIMPLE 11 RIGID

GEN'D MORITA'S THM

TAKE A,B & Alg (C). THEN:

A-Mod (C) = B-Mod (C) IN Mod-Y.

BIMODULES APB & BQA ...

POBQ = Areg IN A-Bimod (C)

QOAP = Brey IN B-Bimod (C)

EXER.4.48 SHOW (1, Ju=ru, idu) AND

(X&X*, idx&evx&idx*, coevx)

ARE MORITA EQUIVALENT YX & C.

EX. C = FdVec

HAVE IR IS MORITA EQUIV. TO Math(IR)

II

Let the the theorem the tensor of the tenso

ASSUME & ABELIAN MONOIDAL }
W SIMPLE 1 RIGID

GEN'D MORITA'S THM

TAKE A, B & Alg (C). THEN:

A-Mod (C) = B-Mod (C) IN Mod-Y.

BIMODULES APB & BQA . F.

POBQ = Areg IN A-Bimod(Y)

QOAP = Brey IN B-Bimod(Y)

PROP TAKE APB, BQA. IF JEPIS

POBO AA & QOAP BB

EA-BIMOD(Y) EB-BIMOD(Y)

J. (POBQ)OAP P POB(QOAP)

AOAP POBB

QOAPOBBQ POBBB

QOAPOBBQ POBBQ

AOAP QOA(POBQ)

ABOBQ POBBQ

ABOBQ POBBQ POBBQ

ABOBQ POBBQ POBBQ

ABOBQ POBBQ PO

GUESS FOR APB AND BQA ??

$$1 P_{X \otimes X^{*}} := \left(?? , P: 1 \otimes P \xrightarrow{??} P, \right)$$

$$1 P_{X \otimes X^{*}} := \left(?? , P \otimes X \otimes X^{*} \xrightarrow{??} P \right)$$

$$\chi \otimes \chi^* Q_{1} := \left(\begin{array}{c} ?? \\ ?? \\ A : Q \otimes 1 \xrightarrow{??} Q \end{array} \right)$$

ASSUME & ABELIAN MONOIDAL 3 W SIMPLE 11 RIGID

GEN'D MORITA'S THM

TAKE A, B & Alg (C). THEN:

A-Mod (C) = B-Mod (C) IN Mod-Y.

BIMODULES APB & BQA . F.

POBQ = Areg IN A-Bimod(Y)

QOAP = Brey IN B-Bimod(Y)

PROP TAKE APB, BQA. IF JEPIS

P@BQ \$\frac{\psi_A}{A} \times Q\times_A P \$\frac{\psi_B}{B} B\$

\$\int A - Bimod(\psi) \times B - Bimod(\psi)

\$\frac{(P\times_B Q)\times_A P}{A\times_A idp} \$\frac{1}{2} \quad \frac{1}{2} \quad \fra

GUESS FOR APB AND BQA ??

$$\mathbb{P}_{X \otimes X^*} := \left(X^*, \quad D: \mathbb{I} \otimes X^* \longrightarrow X^*, \\ d: X^* \otimes X \otimes X^* \longrightarrow X^* \right)$$

ASSUME & ABELIAN MONOIDAL 3 W SIMPLE 11 RIGID

GEN'D MORITA'S THM

TAKE A,B & Alg (C). THEN:

A-Mod (C) = B-Mod (C) IN Mod-Y.

BIMODULES APB & BQA . F.

POBQ = Areg IN A-Bimod (C)

QOAP = Brey IN B-Bimod (C)

GUESS FOR APB AND BQA ??

$$\mathbb{P}_{X \otimes X^*} := \left(X^*, \quad D: \mathbb{I} \otimes X^* \xrightarrow{\mathbb{I}_{X^*}} X^*, \\ d: X^* \otimes X \otimes X^* \xrightarrow{\text{ev}_{X} \otimes \text{id}} X^* \right)$$

ASSUME & ABELIAN MONOIDAL & W SIMPLE 11 RIGID

GEN'D MORITA'S THM

TAKE A,B & Alg (C). THEN:

A-Mod (C) = B-Mod (C) IN Mod-Y.

BIMODULES APB & BQA ...

POBQ = Areg IN A-Bimod (C)

QOAP = Brey IN B-Bimod (C)

EXER. 4.48 SHOW (1, Ju=ru, idu) AND B=(X\otimes X*, idx\otimes evx\otimes idx*, coevx)

ARE MORITA EQUIVALENT YXEC.

ARE THESE REALLY BIMODILES ??

$$P_{X \otimes X^*} := \left(X^*, \quad D: 1 \otimes X^* \xrightarrow{Q_{X^*}} X^*, \\ A: X^* \otimes X \otimes X^* \xrightarrow{ev_X^* \otimes id} X^* \right)$$

ASSUME & ABELIAN MONOIDAL W SIMPLE 1 RIGID

GEN'D MORITA'S THM

TAKE A, B & Alg (&). THEN: A-Mod(c) ~ B-Mod(c) IN Mod-4. 3 BIMODULES APB & BQA . F. PopQ = Area IN A-Binod(Y)

EXER.4.48 SHOW (1, lu=ru, idu) AND $R = (X \otimes X^*, id_X \otimes ev_X \otimes id_{X^*}, coev_X)$

ARE MORITA EQUIVALENT YXEC.

ARE THESE REALLY BIMODILES ?? Youdo!

$$\mathbb{P}_{X \otimes X^*} := \left(X^*, \quad D: \mathbb{I} \otimes X^* \xrightarrow{\mathbb{Q}_{X^*}} X^*, \\ A: X^* \otimes X \otimes X^* \xrightarrow{ev_X^* \otimes id} X^* \right)$$

$$\chi \otimes \chi^* Q_{1} := \left(\begin{array}{c} \chi & p: \chi \otimes \chi^* \otimes \chi \xrightarrow{id \otimes ev_{\chi}^{i}} \chi, \\ \downarrow & \downarrow & \chi \otimes \chi \xrightarrow{r_{\chi}} \chi \end{array} \right)$$

ASSUME & ABELIAN MONOIDAL 3 W SIMPLE 1 RIGID

GEN'D MORITA'S THM

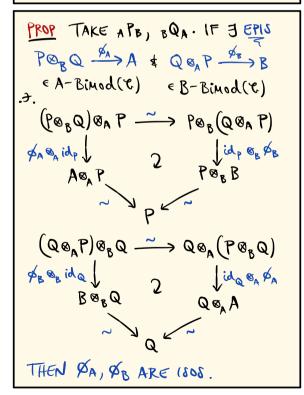
TAKE A, B & Alg (C). THEN:

A-Mod (C) = B-Mod (C) IN Mod-Y.

BIMODULES APB & BQA . F.

POBQ = Areg IN A-Bimod(Y)

QOAP = Brey IN B-Bimod(Y)



$${}_{\mathcal{X} \otimes \mathcal{X}^{*}} = (\mathcal{X}^{*}, \mathcal{D} = \mathcal{J}_{\mathcal{X}^{*}}, \mathcal{A} = ev_{\mathcal{X}}^{\mathcal{L}} \otimes id_{\mathcal{X}^{*}})$$

$${}_{\mathcal{X} \otimes \mathcal{X}^{*}} \mathcal{Q}_{\mathcal{L}} = (\mathcal{X}, \mathcal{D} = id_{\mathcal{X}} \otimes ev_{\mathcal{X}}^{\mathcal{L}}, \mathcal{A} = r_{\mathcal{X}})$$

ASSUME & ABELIAN MONOIDAL 3 W SIMPLE 11 RIGID

GEN'D MORITA'S THM

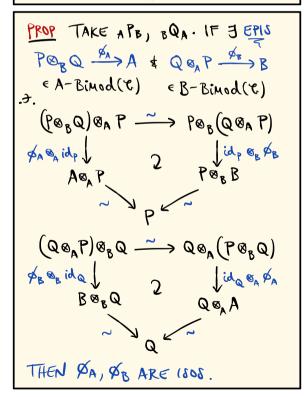
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ASSUME & ABELIAN MONOIDAL & W SIMPLE 1 RIGID

GEN'D MORITA'S THM

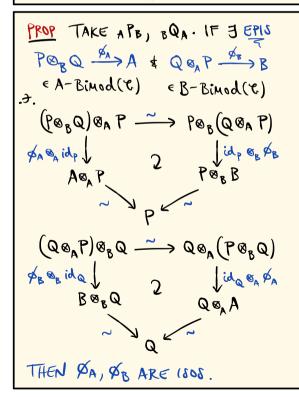
TAKE A, B & Alg (C). THEN:

A-Mod (C) = B-Mod (C) IN Mod-Y.

B BIMODULES APB & BQA . F.

POBQ = Areg IN A-Bimod (C)

Q QAP = Brey IN B-Bimod (C)



ARE MORITA EQUIVALENT YXEC.

NEED
$$\phi_{1}: \chi^* \otimes_{\chi \otimes \chi^*} \chi \longrightarrow 1$$
 (EPIC)

ASSUME & ABELIAN MONOIDAL & W SIMPLE 11 RIGID

GEN'D MORITA'S THM

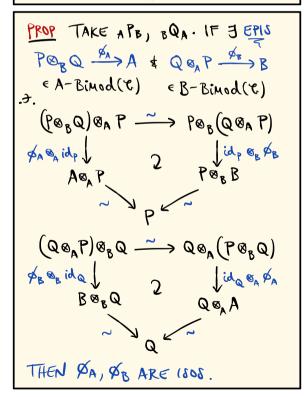
TAKE A, B & Alg (C). THEN:

A-Mod (C) = B-Mod (C) IN Mod-Y.

B BIMODULES APB & BQA . F.

POBQ = Areg IN A-Bimod (C)

Q QAP = Brey IN B-Bimod (C)



EXER.4.48 SHOW (1, lu=ru, idu) AND

B=(X\otin X*, idx\otin evx\otid idx*, coevx)

ARE MORITA EQUIVALENT \(\forall X\in \cdot \cdot \).

$$\begin{array}{ll}
 & P_{X\otimes X^*} = (X^*, P = J_{X^*}, \Delta = ev_X^L \otimes id_{X^*}) \\
 & \chi_{\otimes X^*} Q_{\mathcal{L}} = (X, P = id_X \otimes ev_X, \Delta = r_X)
\end{array}$$

$$\begin{array}{ll}
 & \varphi_{X\otimes X^*} : X \otimes_{\mathcal{L}} X^* \xrightarrow{\sim} X \otimes X^*
\end{array}$$

$$\begin{array}{ll}
 & \chi_{\otimes X^*} : X \otimes_{\mathcal{L}} X^* \xrightarrow{\sim} X \otimes X^*
\end{array}$$

$$\begin{array}{ll}
 & \chi_{\otimes X^*} : X \otimes_{\mathcal{L}} X^* & \xrightarrow{\sim} X \otimes X^*
\end{array}$$

$$\begin{array}{ll}
 & \chi_{\otimes X^*} : X \otimes_{\mathcal{L}} X^* \otimes_{X^*} X \xrightarrow{\sim} X \otimes X^*
\end{array}$$

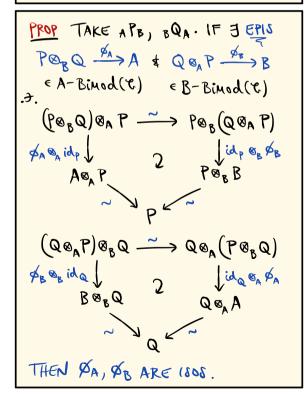
$$\begin{array}{ll}
 & \chi_{\otimes X^*} : X \otimes_{\mathcal{L}} X^* \otimes_{X^*} X \xrightarrow{\sim} X \otimes X^*
\end{array}$$

REVIEW DEFN OF COEQUALIZER
TO DERIVE THIS

ASSUME & ABELIAN, MONOIDAL W SIMPLE 1 RIGID

GEN'D MORITA'S THM

TAKE A, B & Alg (&). THEN: A-Mod(c) ~ B-Mod(c) IN Mod-4. 3 BIMODULES APB & BQA . 7. PosQ = Area IN A-Bimod(Y)



PROP TAKE APB, 5QA. IF
$$\exists \text{ EPIS}$$

$$P \otimes_{B} Q \xrightarrow{A_{A}} A \neq Q \otimes_{A} P \xrightarrow{A_{B}} B$$

$$= A - Bimod(x) = B - Bimod(x)$$

$$P \otimes_{B} Q \otimes_{A} P \xrightarrow{A_{B}} P \otimes_{B} Q \otimes_{A} P$$

$$A \otimes_{A} P \xrightarrow{A_{B}} P \otimes_{B} B$$

$$Q \otimes_{A} P \otimes_{B} B$$

$$Q \otimes_{A} P \otimes_{B} Q \xrightarrow{A_{B}} Q \otimes_{A} P$$

$$Q \otimes_{A} P \otimes_{B} Q \xrightarrow{A_{B}} Q \otimes_{A} P \otimes_{B} Q \otimes_{A} P$$

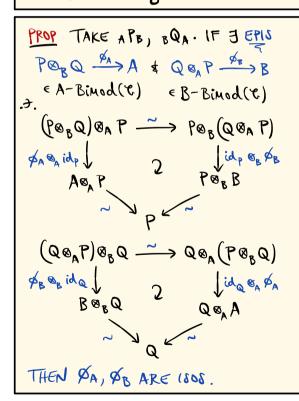
$$Q \otimes_{A} P \otimes_{B} Q \xrightarrow{A_{B}} Q \otimes_{A} P \otimes_{B} Q \otimes_{A} P$$

$$Q \otimes_{A} P \otimes_{B} Q \xrightarrow{A_{B}} Q \otimes_{A} P \otimes_{A} Q \otimes_{A}$$

ASSUME & ABELIAN, MONOIDAL W SIMPLE 1 RIGID

GEN'D MORITA'S THM

TAKE A, B & Alg (&). THEN: A-Mod(c) ~ B-Mod(c) IN Mod-4. 3 BIMODULES APB & BQA . 7. PosQ = Area IN A-Bimod(Y) + Q ØAP = Brey IN B-Bimod(4)

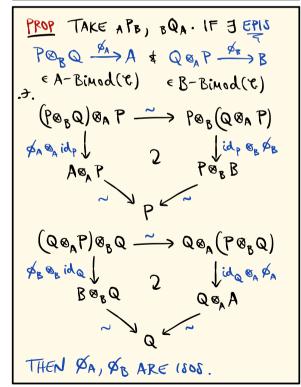


EXER.4.48 SHOW (1, lu=ru, idu) AND $R=(X\otimes X^*, id_X\otimes ev_X^L\otimes id_{X^*}, coev_X^L)$ ARE MORITA EQUIVALENT YXEV.

ASSUME & ABELIAN, MONOIDAL W SIMPLE 1 RIGID

GEN'D MORITA'S THM

TAKE A, B & Alg (&). THEN: A-Mod(c) ~ B-Mod(c) IN Mod-4. 3 BIMODULES APB & BQA . 7. Pop Q = Areg IN A-Bimod(2)



EXER.4.48 SHOW (1, lu=ru, idu) AND $R=(X\otimes X^*, id_X\otimes ev_X^L\otimes id_{X^*}, coev_X^L)$ ARE MORITA EQUIVALENT YXEV.

PROF TAKE APB,
$$\epsilon Q_A \cdot IF \exists \epsilon PIS$$
 $P \otimes_B Q \xrightarrow{A_A} A \notin Q \otimes_A P \xrightarrow{A_E} B$
 $\epsilon A - Bimod(\epsilon) = \epsilon B - Bimod(\epsilon)$
 $P \otimes_B Q \otimes_A P \xrightarrow{A_E} P \otimes_B Q \otimes_A P$
 $A \otimes_A P \xrightarrow{A_E} P \otimes_B B$
 $A \otimes_A P \xrightarrow{A_E} Q \otimes_A P \otimes_B Q \otimes_A P$
 $A \otimes_A P \xrightarrow{A_E} Q \otimes_A P \otimes_B Q \otimes_A P$
 $A \otimes_A P \xrightarrow{A_E} Q \otimes_A P \otimes_B Q \xrightarrow{A_E} Q \otimes_A P \otimes_B Q \otimes_A P$
 $A \otimes_A P \xrightarrow{A_E} Q \otimes_A P \otimes_B Q \xrightarrow{A_E} Q \otimes_A P \otimes_B Q \otimes_A P \otimes_A$

ASSUME & ABELIAN MONOIDAL 3 W SIMPLE 1 RIGID

GEN'D MORITA'S THM

TAKE A, B & Alg (C). THEN:

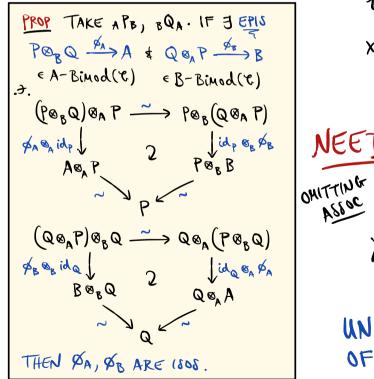
A-Mod (C) ~ B-Mod (C) IN Mod-Y.

I

BIMODULES APB & BQA . F.

POBQ = Areg IN A-Bimod (Y)

Q QAP = Brey IN B-Bimod (Y)



EXER.4.48 SHOW (1), Ju=ru, idu) AND

B=(X\otin X*, idx\otin ev_x\otid idx*, coev_x)

ARE MORITA EQUIVALENT \(\forall X \in \cdot \cdot \cdot \).

ASSUME & ABELIAN MONOIDAL 3 W SIMPLE 1 RIGID

GEN'D MORITA'S THM

TAKE A, B & Alg (C). THEN:

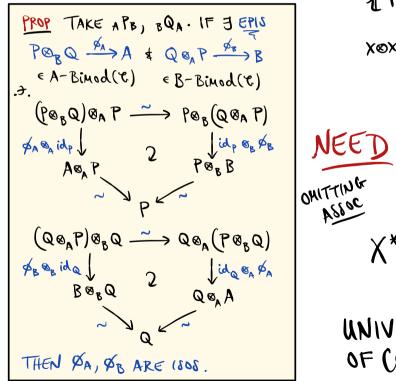
A-Mod (C) ~ B-Mod (C) IN Mod-Y.

I

BIMODULES APB & BQA . F.

POBQ = Areg IN A-Bimod (Y)

Q QAP = Brey IN B-Bimod (Y)



EXER.4.48 SHOW (1, lu=ru, idu) AND

B=(X®X*, idx@evx@idx*, coevx)

ARE MORITA EQUIVALENT YXEC.

ASSUME & ABELIAN MONOIDAL 3 W SIMPLE 11 RIGID

GEN'D MORITA'S THM

TAKE A, B & Alg (C). THEN:

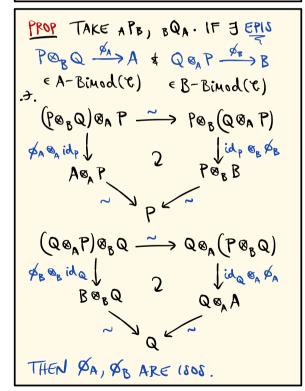
A-Mod (C) ~ B-Mod (C) IN Mod-Y.

I

BIMODULES APB & BQA . F.

POBQ = Areg IN A-Bimod (Y)

Q QAP = Brey IN B-Bimod (Y)



EXER. 4.48 SHOW (1, Ju=ru, idu) AND

B=(X®X*, idx®evx®idx*, coevx)

ARE MORITA EQUIVALENT YXEY.

ASSUME & ABELIAN MONOIDAL & W SIMPLE 11 RIGID

GEN'D MORITA'S THM

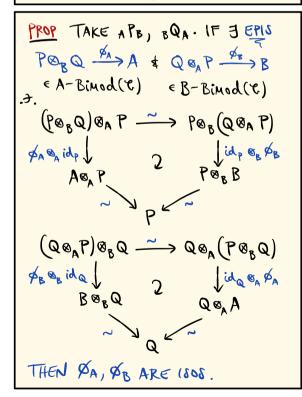
TAKE A, B & Alg (C). THEN:

A-Mod (C) = B-Mod (C) IN Mod-Y.

B BIMODULES APB & BQA ...

POBQ = Areg IN A-Bimod (C)

Q QAP = Brey IN B-Bimod (C)



$$P_{X \otimes X^{*}} = (X^{*}, P = I_{X^{*}}, d = eV_{X} \otimes id_{X^{*}})$$

$$X \otimes X^{*} Q_{\mathcal{L}} = (X, P = id_{X} \otimes eV_{X}, d = r_{X})$$

$$Q_{X \otimes X^{*}} : X \otimes_{\mathcal{L}} X^{*} \xrightarrow{\sim} X \otimes X^{*} \in PIC$$

ASSUME & ABELIAN MONOIDAL }
W SIMPLE 1 RIGID

GEN'D MORITA'S THM

TAKE A, B & Alg (&). THEN:

PROP TAKE APB, BQA. IF 3 EPIS

POBBO A \$ QOAP B

A-BIMOD(C) & B-BIMOD(C)

(POBQ)OAP POB(QOAP)

AOAP POBB

(QOAP)OBQ POBB

(QOAP)OBQ QOA(POBQ)

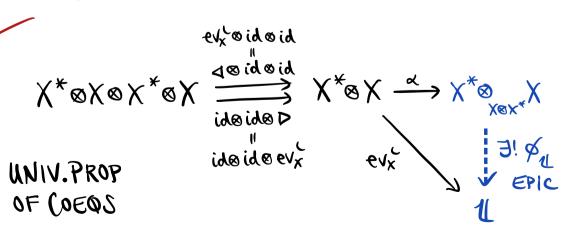
ABOBIDO QOAA

BOBO QOAA

THEN OA, OB ARE 1805.

ARE MORITA EQUIVALENT YXEV.

DO THESE DIAGRAMS COMMUTE?



ASSUME & ABELIAN MONOIDAL & W SIMPLE 11 RIGID

GEN'D MORITA'S THM

TAKE A,B & Alg (&). THEN: A-Mod(e) = B-Mod(e) IN Mod-4.

EXER. 4.48 SHOW (1, lu=ru, idu) AND

B=(X®X*, idx@evx@idx*, coevx)

ARE MORITA EQUIVALENT YXEV.

$$P_{X\otimes X^{*}} = (X^{*}, D = I_{X^{*}}, \Delta = ev_{X}^{L} \otimes id_{X^{*}})$$

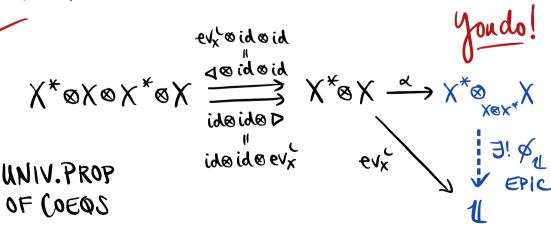
$$P_{X\otimes X^{*}} = (X, D = id_{X} \otimes ev_{X}^{L}, \Delta = r_{X}^{L})$$

$$P_{X\otimes X^{*}} = (X, D = id_{X} \otimes ev_{X}^{L}, \Delta = r_{X}^{L})$$

$$P_{X\otimes X^{*}} = (X^{*}, D = id_{X} \otimes ev_{X}^{L}, \Delta = r_{X}^{L})$$

$$P_{X\otimes X^{*}} = (X^{*}, D = id_{X} \otimes ev_{X}^{L}, \Delta = r_{X}^{L})$$

DO THESE DIAGRAMS COMMUTE?



ASSUME & ABELIAN MONOIDAL } \$\DEX X\infty - \infty \texact \forall \f

GENERALIZED MORITA'S THEOREM

TAKE A, B & Alg (C). THEN:

A-Mod(E) ~ B-Mod(E)

AS RIGHT G-MODULE CATEGORIES

1

3 BIMODULES APB & BQA . 7.

POBQ = Area AS A-BIMODULES IN &

\$ Q @ P = Brey AS B-BIMODULES IN 6.

GENERALIZED EW THEOREM

TAKE A, B & Alg(&).

THEN, WE GET AN EQUIV. OF CATO:

Rex_{Mod-e} (A-Mod(e), B-Mod(e))

 \simeq (B, A)-Bimod(\mathscr{C}).

RIGHT EXACT MOD. CAT. FUNCTORS MATH 466/566 SPRING 2024

CHELSEA WALTON RICE U.

NEXT TIME
OSTRIK'S THEOREM

LECTURE #20

TOPICS:

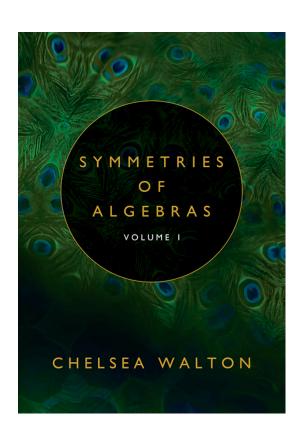
I. OPERATIONS ON ALGEBRAS & (BI) MODULES (§ 4.5)

I. GENERALIZED EILENBERG-WATTS THEOREM (54.7.1)

III. GENERALIZED MORITA'S THEOREM (§4.7.2)

Enjoy this lecture? You'll enjoy the textbook!

C. Walton's "Symmetries of Algebras, Volume 1" (2024)



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619 Wreath (at a discount)

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Also on Amazon & Google Play

<u>Lecture #20 keywords</u>: algebraic operations in monoidal categories, Generalized Eilenberg-Watts Theorem, Generalized Morita's Theorem