# MATH 466/566 SPRING 2024

CHELSEA WALTON RICE U.

# LAST TIME

LECTURE #21

- · ALG. & (BI) MOD. OPERATIONS
- · GEN'D EILENBERG-WATTS THM
- · GEN'D MORITA'S THM

# TOPICS:

I. INTERNAL END ALGEBRAS (§4.8.1)

II. OSTRIK'S THEOREM (54.8.2)

TAKE A MONOIDAL CATEGORY &:= (&, &, 1L, a, 1, r),
HAVE TWO TYPES OF MODULES:

TAKE A MONOIDAL CATEGORY &:= (&, &, 1L, a, l, r), HAVE TWO TYPES OF MODULES:

INTERNAL TO &

FOR A := (A,M,u) & Alg (&)

Mod-A(C)

RIGHT A-MODULES

M:= (M, A) IN &

TAKE A MONOIDAL CATEGORY &:= (&, &, 1L, a, l, r),
HAVE TWO TYPES OF MODULES:

# INTERNAL TO &

FOR A := (A,M,u) & Alg (a)

Mod-A(C)

RIGHT A-MODULES

M:= (M, A) IN &

TAKE A MONOIDAL CATEGORY &:= (&, &, 1L, a, l, r),
HAVE TWO TYPES OF MODULES:

 $Mod-A(C) \in C-Mod VIA$   $X \triangleright (M,d) := (X \otimes M, d = id_X \otimes d)$ 

INTERNAL TO &

FOR A := (A,M,u) & Alg (a)

Mod-A(C)

RIGHT A-MODULES M := (M, a) IN & EXTERNAL TO &

M:=(M,D,M,P)

E G-Mod

LEFT G-MODULE

CATEGORY

TAKE A MONOIDAL CATEGORY &:= (&, &, 1L, a, l, r),
HAVE TWO TYPES OF MODULES:

 $Mod-A(C) \in C-Mod VIA$  $X \triangleright (M, \Delta) := (X \otimes M, \Delta = id_X \otimes \Delta)$ 

INTERNAL TO &

FOR A := (A,M,u) & Alg (a)

Mod-A(C)

RIGHT A-MODULES M := (M, A) IN & EXTERNAL TO C

M := (M, D, M, P)

€ G-Mod

LEFT G-MODULE CATEGORY

WE WILL DISCUSS WHEN THIS OCCURS M & C-Mod IS REPRESENTED BY A & Alg(C) IF  $M \simeq Mod-A(C)$  AS LEFT C-MODULE CATEGORIES.

 $Mod-A(C) \in C-Mod VIA$   $X \triangleright (M, \Delta) := (X \otimes M, \Delta = id_X \otimes \Delta)$ 

INTERNAL TO &

FOR A := (A,M,u) & Alg (a)

Mod-A(C)

RIGHT A-MODULES M := (M,A) IN & EXTERNAL TO C

M := (M, D, M, P)

€ G-Mod

LEFT G-MODULE CATEGORY

WE WILL DISCUSS WHEN THIS OCCURS M & Mod-& IS REPRESENTED BY A & Alg(G) IF  $M \simeq A - Mod(G)$  AS RIGHT &-MODULE CATEGORIES.

 $Mod-A(C) \in C-Mod VIA$   $X \triangleright (M, \Delta) := (X \otimes M, \Delta = id_X \otimes \Delta)$ 

INTERNAL TO &

FOR A := (A,M,u) & Alg (a)

Mod-A(C)

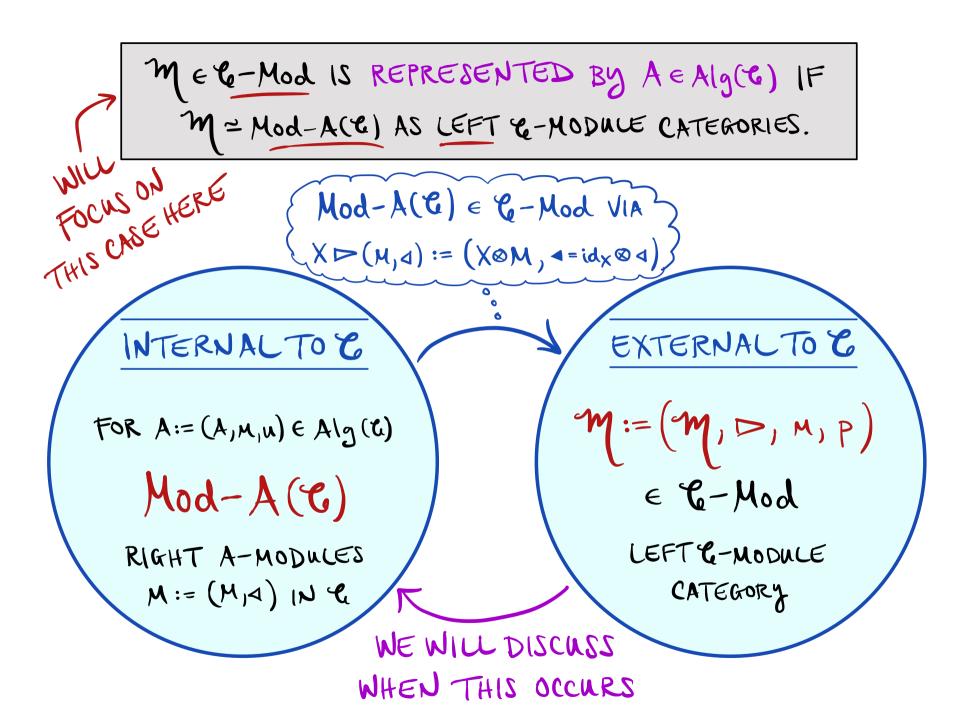
RIGHT A-MODULES M := (M,A) IN & EXTERNAL TO &

M := (M, D, M, P)

€ G-Mod

LEFT G-MODULE CATEGORY

WE WILL DISCUSS WHEN THIS OCCURS





= RECALL=

A LEFT &-MODULE CATEGORY

$$(M, D: e \times M \rightarrow M, e \times e \times M \rightarrow M, m \rightarrow M, m \rightarrow M)$$

IS CLOSED IF  $(-DM): e \rightarrow M$  HAS A

RIGHT ADJOINT  $(M, -): M \rightarrow C$ ,  $\forall M \in M$ .

= RECALL=

A LEFT &-MODINE CATEGORY  $(M, D: e \times M \rightarrow M, e \times e \times M \rightarrow M, m \rightarrow M, m \rightarrow M)$ IS CLOSED IF  $(-DM): e \rightarrow M$  HAS A

RIGHT ADJOINT  $(M, -): M \rightarrow e$ ,  $\forall M \in M$ .

HOM (M,N) E'C = INTERNAL HOM OF M,NEM End(M) E'C = INTERNAL END OF M EM

= RECALL=

A LEFT &-MODULE CATEGORY  $(M, D: e \times M \rightarrow M, e \times e \times M \rightarrow M, m \rightarrow PM)$ IS CLOSED IF  $(-DM): e \rightarrow M$  HAS A

RIGHT ADJOINT  $\underbrace{Hom}(M, -): M \rightarrow e, \forall M \in M$ .

HOM (M,N) & = INTERNAL HOM OF M,N&M

END (M) & = INTERNAL END OF M & M

WILL GIVE THESE ALGEBRAIC STRUCTURE IN &

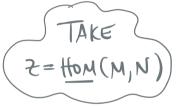
LEFT &-MODULE CATEGORY ( $^{M}$ ,  $^{D}$ ,  $^{M}$ ,  $^{P}$ ) IS CLOSED IF ( $^{-D}$   $^{M}$ ):  $^{E}$   $\rightarrow ^{M}$  HAS A RIGHT ADJOINT  $\overset{L}{\text{tom}}(M,-): ^{M}$   $\rightarrow ^{E}$ ,  $^{V}$   $^{E}$   $^{M}$ .  $^{S:=}$   $^{S:}$   $^{S:}$ 

LEFT &-MODULE CATEGORY ( $^{M}$ ,  $^{D}$ ,  $^{M}$ ,  $^{D}$ ) is CLOSED IF ( $^{-D}$ M): &  $\rightarrow$ M

HAS A RIGHT ADJOINT  $\underline{\text{Hom}}(M,-):_{M} \rightarrow \&$ ,  $\forall M \in M$ .  $3:=3_{2,N}:\underline{\text{Homm}}(ZDM,N) \xrightarrow{\sim} \underline{\text{Home}}(Z,\underline{\text{Hom}}(M,N))$ 

id HOM (M,N) Z= HOM (M,N)

LEFT &-MODULE CATEGORY (M,D,M,P) IS CLOSED IF (-DM): & -> M HAS A RIGHT ADJOINT HOM (M, -): m → e, VM € m. J := JZ,N: HOMM (ZDM,N) ~ HOME (Z, HOM (M,N))



LEFT &-MODULE CATEGORY (M,D,M,P) IS CLOSED IF (-DM): &  $\rightarrow m$ HAS A RIGHT ADJOINT  $\underline{\text{Hom}}(M,-): m \rightarrow e$ ,  $\forall M \in m$ .  $\exists := \exists_{z,N} : \underline{\text{Homm}}(zDM,N) \xrightarrow{\sim} \underline{\text{Home}}(z,\underline{\text{Hom}}(M,N))$ 

LEFT &-MODULE CATEGORY ( $^{M}$ ,  $^{D}$ ,  $^{M}$ ,  $^{D}$ ) is CLOSED IF ( $^{-D}$ M): &  $\rightarrow$ M

HAS A RIGHT ADJOINT  $\underline{\text{Hom}}(M,-):_{M} \rightarrow \&$ ,  $\forall M \in M$ .  $3:=J_{2,N}:\underline{\text{Hom}}(ZDM,N) \xrightarrow{\sim} \underline{\text{Hom}}(Z,\underline{\text{Hom}}(M,N))$ 

LEFT &-MODINE CATEGORY ( $^{M}$ ,  $^{D}$ ,  $^{M}$ ,  $^{P}$ ) IS CLOSED IF ( $^{-D}$ M):  $^{Q}$   $\rightarrow ^{M}$  HAS A RIGHT ADJOINT  $\underline{\text{Hom}}(M,-):_{M} \rightarrow ^{Q}$ ,  $\forall M \in ^{M}$ .  $3:=3_{2,N}:\underline{\text{Homm}}(ZDM,N) \xrightarrow{\sim} \underline{\text{Home}}(Z,\underline{\text{Hom}}(M,N))$ 

LEFT &-MODULE CATEGORY ( $^{M}$ ,  $^{D}$ ,  $^{M}$ ,  $^{D}$ ) is CLOSED IF ( $^{-D}$ M): &  $\rightarrow$ M

HAS A RIGHT ADJOINT  $\underline{\text{Hom}}(M,-):_{M} \rightarrow \&$ ,  $\forall M \in M$ .  $3:=J_{2,N}:\underline{\text{Hom}}(ZDM,N) \xrightarrow{\sim} \underline{\text{Hom}}(Z,\underline{\text{Hom}}(M,N))$ 

LEFT &-MODILE CATEGORY (M,D,M,P) IS CLOSED IF  $(-DM): \mathcal{C} \to \mathcal{M}$ HAS A RIGHT ADJOINT HOM  $(M,-): \mathcal{M} \to \mathcal{C}$ ,  $\forall M \in \mathcal{M}$ .  $\mathcal{J}:=\mathcal{J}_{\mathcal{Z},N}: Hom_{\mathcal{M}}(\mathcal{Z}DM,N) \xrightarrow{\sim} Hom_{\mathcal{C}}(\mathcal{Z}, Hom_{\mathcal{M}}(M,N))$ 

LEFT &-MODULE CATEGORY ( $^{m}$ ,  $^{d}$ ,  $^{m}$ ,  $^{p}$ ) IS CLOSED IF ( $^{-d}$ M):  $^{e}$   $\rightarrow$   $^{m}$ HAS A RIGHT ADJOINT  $\overset{1}{\text{Hom}}(M,-): m \rightarrow e$ ,  $\forall M \in m$ .  $3:=3_{e,N}: \overset{1}{\text{Hom}}(ZDM,N) \overset{\sim}{\rightarrow} \overset{1}{\text{Hom}}(Z,t)$ 

DEFINE:

$$\frac{\text{Hom}(N,P) \otimes \text{Hom}(M,N)}{\text{Mom}(N,P), \text{Hom}(N,N),M} \longrightarrow \frac{\text{EV}_{M,N},P}{\text{ev}_{M,N}} \xrightarrow{\text{id} \text{Pev}_{M,N}} \xrightarrow{\text{Hom}(N,P)} \text{DEF}$$

$$\frac{\text{Hom}(N,P), \text{Hom}(N,N),M}{\text{Hom}(N,P)} \longrightarrow \frac{\text{id} \text{Pev}_{M,N}}{\text{Hom}(N,P)} \xrightarrow{\text{Hom}(N,P)} \text{DN}$$

Z = HOM(N,P) & HOM(M,N)

J(eVM,N,P): HOM(N,P) & HOM(M,N) -> HOM(M,P)

LEFT &-MODULE CATEGORY (M,D,M,P) IS CLOSED IF (->M): & ->M

HAS A RIGHT ADJOINT  $\underline{\text{Hom}}(M,-): M \to \&$ ,  $\forall M \in M$ .  $3:=J_{E,N}: \underline{\text{Homm}}(ZDM,N) \xrightarrow{\sim} \underline{\text{Home}}(Z,\underline{\text{Hom}}(M,N))$ 

DEFINE:

OBTAIN:

CEFT &-MODULE CATEGORY (M,D,M,P) IS CLOSED IF (-DM) -1 HOM(M,-)

S:= Jz,N: HOMM(ZDM,N) -> HOMZ(Z, HOM(M,N))

\*\*M

$$\frac{\text{eV}_{M,N} := \int_{-1}^{-1} \left( \text{id}_{\text{Hom}(M,N)} \right) : \left( \frac{\text{Hom}(N,P) \otimes \text{Hom}(M,N)}{\text{Min}_{M,N}} \otimes \frac{\text{Hom}(M,N)}{\text{Mon}_{M,N}} \right) \Rightarrow M \xrightarrow{\text{eV}_{M,N}} P \in \mathcal{M} \\
\frac{\text{Hom}(M,N) DM}{\text{Mon}_{M,N}} := \int_{-1}^{-1} \left( \frac{\text{Hom}(N,P) \otimes \text{Hom}(M,N)}{\text{Mon}_{M,N}} \otimes \frac{\text{Hom}(M,N)}{\text{Mon}_{M,N}} \right) \Rightarrow M \xrightarrow{\text{eV}_{M,N}} P \in \mathcal{M} \\
\frac{\text{Hom}(N,P) DM}{\text{Mon}_{M,N}} := \int_{-1}^{-1} \left( \frac{\text{Hom}(N,P) \otimes \text{Hom}(M,N)}{\text{Mon}_{M,N}} \otimes \frac{\text{Hom}(M,N)}{\text{Mon}_{M,N}} \right) \Rightarrow M \xrightarrow{\text{eV}_{M,N}} P \in \mathcal{M} \\
\frac{\text{Hom}(N,P) DM}{\text{Mon}_{M,N}} := \int_{-1}^{-1} \left( \frac{\text{Hom}(N,P) \otimes \text{Hom}(M,N)}{\text{Mon}_{M,N}} \otimes \frac{\text{Hom}(M,N)}{\text{Mon}_{M,N}} \otimes \frac{\text{Hom}(M,N)}{\text{Mon}_{M,N}} \right) \Rightarrow M \xrightarrow{\text{eV}_{M,N}} P \in \mathcal{M} \\
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\frac{\text{Hom}(N,P) \otimes \text{Hom}(M,N)}{\text{Mon}_{M,N}} \otimes \frac{\text{Hom}(M,N)}{\text{Mon}_{M,N}} \otimes \frac{\text{Hom}(M,N)}{\text{Mon}_{M,$$

CEFT &-MODULE CATEGORY (M,D,M,P) IS CLOSED IF (-DM) -1 HOM(M,-)

S:= Jz,N: HOMM(ZDM,N) -> HOMZ(Z, HOM(M,N))

\*\*M

$$eV_{M,N} := \int_{-1}^{-1} (id_{\underline{HoM}(M,N)}) :$$
 $\underline{HoM}(M,N) \triangleright M \longrightarrow N \in \mathcal{M}$ 

$$(\underbrace{\text{Hom}}(N,P) \otimes \underbrace{\text{Hom}}(M,N)) \supset M \xrightarrow{\text{ev}_{M,N}} P \in \mathcal{M}$$

$$M_{\underbrace{\text{Hom}}(N,P),\underbrace{\text{Hom}}(M,N),M}) \xrightarrow{\text{DEF}} (\underbrace{\text{ev}_{M,N}}_{id \rightarrow ev_{M,N}} \underbrace{\text{Hom}}(N,P) \supset N$$

$$\underbrace{\text{Hom}}(N,P) \supset (\underbrace{\text{Hom}}(M,N) \supset M) \xrightarrow{\text{id} \rightarrow ev_{M,N}} \underbrace{\text{Hom}}(N,P) \supset N$$

PROP: LET (M,D,M,P) BE A CLOSED LEFT G-MODINE CATEGORY.

LEFT &-MODILE CATEGORY (M,D,M,P) IS CLOSED IF (-DM) -1 HOM (M,-) S:= Jz,N: HOMm (ZDM,N) ~ HOMz (Z, HOM (M,N))

 $eV_{M,N} := \int_{-1}^{-1} \left( id \overline{HoM}(M,N) \right)$ : HOM (M,N)DM -> N & m

(tom (N,P) @ tom (M,N)) DM €VM,N,P >P € M  $\frac{M_{\underline{\mathsf{Hom}}(\mathsf{N},\mathsf{P}),\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}),\mathsf{M}}{\mathsf{M}} \xrightarrow{\mathsf{DEF}} \frac{1}{\mathsf{M}} \xrightarrow{\mathsf{P}} \frac{1}{\mathsf{M}} \underbrace{\mathsf{M}}_{\mathsf{N}} \underbrace{\mathsf{N}}_{\mathsf{P}} + \underbrace{\mathsf{M}}_{\mathsf{N}} \underbrace{\mathsf{M}}_{\mathsf{N}} + \underbrace{\mathsf{M}}_{\mathsf$ 

COMPMNP := J(eVMN,P): HOM(N,P) & HOM(M,N) → HOM (M,P) € C

PROP: LET (M,D,M,P) BE A CLOSED LEFT G-MODINE CATEGORY. THEN (a) End(M) & Alg(e) WITH

YMEM [MULTIPLICATION] ??

[UNIT] ??

CEFT &-MODULE CATEGORY (M,D,M,P) IS CLOSED IF (-DM) -1 HOM(M,-)

S:= Jz,N: HOMM(ZDM,N) -> HOMZ(Z, HOM(M,N))

\*\*M

 $eV_{M,N} := \int_{-1}^{-1} (id_{HOM}(M,N)) :$   $form(M,N) > M \longrightarrow N \in M$ 

 $(\underbrace{tom}(N,P) \otimes \underbrace{tom}(M,N)) \supset M \xrightarrow{eV_{M,N},P} P \in \mathcal{M}$   $M_{\underbrace{tom}(N,P),\underbrace{tom}(M,N),M}) \xrightarrow{DEF} (\underbrace{eV_{M,N}}) \xrightarrow{id D eV_{M,N}} \underbrace{tom}(N,P) \supset N$   $\underbrace{tom}(N,P) \supset (\underbrace{tom}(M,N) \supset M) \xrightarrow{id D eV_{M,N}} \underbrace{tom}(N,P) \supset N$ 

COMPM,N,P := J(eVM,N,P): HOM(N,P) ⊗ HOM(M,N) -> HOM (M,P) ∈ C

PROP: LET (M,D,M,P) BE A CLOSED LEFT &-MODINE CATEGORY.

THEN (a) End(M) & Alg(E) WITH

¥M∈m

[MULTIPLICATION] COMPM,M,M:  $End(M) \otimes End(M) \longrightarrow End(M)$ ,

[UNIT]  $f(p_M): 11 \longrightarrow End(M)$ .  $f_M: 11 \nearrow M \xrightarrow{\sim} M$ 

LEFT &-MODULE CATEGORY (M,D,M,P) IS CLOSED IF (-DM) -1  $\underline{Hom}(M,-)$  $S:= J_{z,N}: \underline{Hom}_{m}(ZDM,N) \xrightarrow{\sim} \underline{Hom}_{z}(Z,\underline{Hom}_{m,N})$   $\stackrel{\forall M}{\rightarrow}$ 

 $ComP_{M,N,P} := \mathcal{J}^{-1}(id_{\underline{HoM}(M,N)}): \qquad \underbrace{(\underline{HoM}(N,P) \otimes \underline{HoM}(M,N))}_{\underline{M_{\underline{MM}(N,P),\underline{HoM}(M,N),M}}} \xrightarrow{\underline{DEF}} \underbrace{(\underline{HoM}(N,P) \otimes \underline{HoM}(M,N))}_{\underline{M_{\underline{MM}(N,P),\underline{HoM}(M,N),M}}} \xrightarrow{\underline{DEF}} \underbrace{(\underline{HoM}(N,P) \otimes \underline{HoM}(M,N))}_{\underline{M_{\underline{MM}(N,P),\underline{HoM}(M,N),M}}} \xrightarrow{\underline{HoM}(N,P) \otimes \underline{HoM}(M,N)}_{\underline{M_{\underline{MM}(N,P),\underline{MM}(M,N)}}} \xrightarrow{\underline{HoM}(M,P) \otimes \underline{HoM}(M,N)}_{\underline{M_{\underline{MM}(N,P),\underline{MM}(M,N),M}}} \xrightarrow{\underline{HoM}(M,P) \otimes \underline{HoM}(M,N)}_{\underline{M_{\underline{MM}(N,P),\underline{MM}(M,N),M}}} \xrightarrow{\underline{HoM}(M,P) \otimes \underline{HoM}(M,N)}_{\underline{M_{\underline{MM}(N,P),\underline{MM}(M,N),M}}} \xrightarrow{\underline{HoM}(M,P) \otimes \underline{HoM}(M,N)}_{\underline{M_{\underline{MM}(N,P),\underline{MM}(M,N),M}}} \xrightarrow{\underline{HoM}(M,P) \otimes \underline{HoM}(M,N)}_{\underline{M_{\underline{MM}(M,N,P),\underline{MM}(M,N),M}}} \xrightarrow{\underline{HoM}(M,P) \otimes \underline{HoM}(M,N)}_{\underline{M_{\underline{MM}(M,N,P),M}}} \xrightarrow{\underline{HoM}(M,P) \otimes \underline{HoM}(M,N)}_{\underline{M_{\underline{MM}(M,N,P),M}}}$ 

PROP: LET (M,D,M,P) BE A CLOSED LEFT G-MODILE CATEGORY.

THEN (a) End(M) & Alg(e) WITH

YMEM [MULTIPLICATION] COMPH,M,M: End(M) & End(M) -> End(M),

[UNIT] S(PM): 11 -> End(M).

(b) Hom(M,N) & Mod-End(M)(e) WITH

YM,NEM [RIGHT ACTION] ??

CEFT &-MODULE CATEGORY (M,D,M,P) IS CLOSED IF (-DM) -1 HOM(M,-)

S:= Jz,N: HOMM(ZDM,N) -> HOMZ(Z, HOM(M,N))

\*\*M

 $\begin{array}{ll} \text{ev}_{M,N} := \mathcal{G}^{-1} \big( \text{id}_{\underline{HoM}(M,N)} \big) : & & & & & & & & & & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{\underline{HoM}(M,N)} \big) : & & & & & & & & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{\underline{HoM}(M,N)} \big) : & & & & & & & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{\underline{HoM}(M,N)} \big) : & & & & & & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & & & & & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & & & & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & & & & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & & & & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & & & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & & & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & & & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1} \big( \text{id}_{M,N,P} \big) : & \\ \text{tom}_{M,N,P} := \mathcal{G}^{-1}$ 

PROP: LET (M, D, M, P) BE A CLOSED LEFT &-MODINE CATEGORY.

THEN (a) End(M) & Alg(E) WITH

YMEM [MULTIPLICATION] COMPM,M,M: End(M) & End(M) -> End(M),

[UNIT] & (PM): 11 -> End(M).

(b) Hom(M,N) & Mod-End(M)(E) WITH

YM,NEM [RIGHTACTION] COMPM,M,N: Hom(M,N) & End(M) -> Hom(M,N).

LEFT &-MODULE CATEGORY (M,D,M,P) IS CLOSED IF (-DM) -1 HOM(M,-)

S:= Jz,N: HOMM(ZDM,N) -> HOME(Z, HOM(M,N))

AM

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\frac{\text{ev}_{M,N} := \int_{-1}^{-1} \left( \text{id}_{\frac{\text{Hom}}{M}(M,N)} \right) : \qquad \left( \frac{\text{Hom}}{M}(N,P) \otimes \frac{\text{Hom}}{M}(M,N) \right) D M \xrightarrow{\text{ev}_{M,N}} P \in \mathcal{M} 
\frac{\text{Hom}}{M}(M,N) D M \longrightarrow N \in \mathcal{M} \qquad \frac{\text{Hom}}{M}(N,P) D \left( \frac{\text{Hom}}{M}(M,N) D M \right) \xrightarrow{\text{id} D \in V_{M,N}} \frac{\text{Hom}}{M}(N,P) D N
\frac{\text{Hom}}{M}(N,P) D \left( \frac{\text{Hom}}{M}(M,N) D M \right) \xrightarrow{\text{id} D \in V_{M,N}} \frac{\text{Hom}}{M}(N,P) D N
\frac{\text{Hom}}{M}(N,P) \otimes \frac{\text{Hom}}{M}(M,N) \longrightarrow \frac{\text{Hom}}{M}(M,P) \in \mathcal{L}
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PROP: LET (M, D,M,P) BE A CLOSED LEFT &-MODINE CATEGORY.

THEN (a) End(M) & Alg(e) WITH

YMEM [MULTIPLICATION] COMPM,M,M: End(M) & End(M) -> End(M),

(b) [UNIT] P(PM): 11 -> End(M).

(c) HOM(N,M) & End(M)-Mod(&) WITH

YM,NEM [LEFT ACTION] ??
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LEFT &-MODULE CATEGORY (M,D,M,P) IS CLOSED IF (-DM) -1 HOM(M,-)

S:= Jz,N: HOMM(ZDM,N) -> HOME(Z, HOM(M,N))

AM

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\frac{\text{ev}_{M,N} := \int_{-1}^{-1} \left( \text{id}_{\frac{\text{Hom}}{M}(M,N)} \right) : \qquad \left( \frac{\text{Hom}}{M}(N,P) \otimes \frac{\text{Hom}}{M}(M,N) \right) D M \xrightarrow{\text{ev}_{M,N}} P \in \mathcal{M} 
\frac{\text{Hom}}{M}(M,N) D M \longrightarrow M \in \mathcal{M} \qquad \frac{\text{Hom}}{M}(N,P) D \left( \frac{\text{Hom}}{M}(M,N) D M \right) \xrightarrow{\text{id} D \in V_{M,N}} \frac{\text{Hom}}{M}(N,P) D M 
\frac{\text{Hom}}{M}(N,P) D \left( \frac{\text{Hom}}{M}(M,N) D M \right) \xrightarrow{\text{id} D \in V_{M,N}} \frac{\text{Hom}}{M}(N,P) D M 
\frac{\text{Hom}}{M}(N,P) \otimes \frac{\text{Hom}}{M}(M,N) \longrightarrow \frac{\text{Hom}}{M}(M,P) \in \mathcal{L}
```

```
PROP: LET (M, D, M, P) BE A CLOSED LEFT &-MODINE CATEGORY.

THEN (a) End(M) \in Alg(E) with

\forall M \in M

[MULTIPLICATION] COMP_{M,M,M} : End(M) \otimes End(M) \longrightarrow End(M),

(b) [UNIT] f(p_M) : 11 \longrightarrow End(M).

(c) Hom(N, M) \in End(M) - Mod(E) with

\forall M, N \in M

[LEFT ACTION] COMP_{N,M,M} : End(M) \otimes Hom(N,M) \longrightarrow Hom(N,M).
```

CEFT &-MODULE CATEGORY (M,D,M,P) IS CLOSED IF (-DM) -1 HOM(M,-)

S:= Jz,N: HOMM(ZDM,N) -> HOMZ(Z, HOM(M,N))

\*\*M

 $\frac{\text{ev}_{M,N} := \int_{-1}^{-1} \left( \text{id}_{\text{Hom}(M,N)} \right) :}{\text{Hom}(M,N) \triangleright M} \xrightarrow{\text{fom}(M,N)} \text{pos}_{\text{fom}(M,N)} \Rightarrow \text{fom}(M,N) \triangleright M \xrightarrow{\text{ev}_{M,N}} \text{fom}(N,P) \triangleright N} \xrightarrow{\text{fom}(M,N)} \text{pos}_{\text{fom}(M,N)} \Rightarrow \text{fom}(M,P) \triangleright N$   $\frac{\text{fom}(M,P) \triangleright \left( \text{fom}(M,N) \triangleright M \right)}{\text{fom}(M,N) \triangleright M} \xrightarrow{\text{fom}(M,P) \triangleright N} \xrightarrow{\text{fom}(M,P) \triangleright N} \text{fom}(M,P) \triangleright N$   $\frac{\text{fom}(M,P) \triangleright \left( \text{fom}(M,N) \triangleright M \right)}{\text{fom}(M,N) \triangleright M} \xrightarrow{\text{fom}(M,P) \triangleright N} \text{fom}(M,P) \in \mathcal{C}$ 

PROP: LET (M, D, M, P) BE A CLOSED LEFT &-MODINE CATEGORY.

THEN (a) End(M) & Alg(E) WITH "INTERNAL END ALGEBRA"

YMEM [MULTIPLICATION] COMPH,M,M: End(M) & End(M) -> End(M),

[UNIT] & (PM): 11 -> End(M).

(b) Hom (M, N) & Mod-End (M) (E) WITH "INTERNAL HOM MODINE"

YM, NEM: [C] [RIGHTACTION] COMPM,MIN: HOM (M,N) & End(M) -> HOM (M,N).

CEFT &-MODULE CATEGORY (M,D,M,P) IS CLOSED IF (-DM) -1 HOM(M,-)

S:= Jz,N: HOMM(ZDM,N) -> HOMZ(Z, HOM(M,N))

\*\*M

 $\frac{\text{eV}_{M,N} := \int_{-1}^{-1} \left( \text{id}_{\frac{\text{Hom}}{M}(M,N)} \right) : \quad \left( \frac{\text{Hom}}{M}(N,P) \otimes \frac{\text{Hom}}{M}(M,N) \right) D M \xrightarrow{\text{eV}_{M,N}} P \in \mathcal{M} \\
\frac{\text{Hom}}{M}(M,N) D M \longrightarrow M \in \mathcal{M} \qquad \left( \frac{\text{Hom}}{M}(N,P) D \left( \frac{\text{Hom}}{M}(M,N) D M \right) \xrightarrow{\text{id}_{\frac{\text{ev}_{M,N}}{M}}} \frac{\text{Hom}}{M}(N,P) D M \right) \xrightarrow{\text{ev}_{M,N}} \frac{\text{Hom}}{M}(N,P) D M$   $\frac{\text{Comp}_{M,N,P} := \int_{-1}^{\infty} (\text{ev}_{M,N,P}) : \frac{\text{Hom}}{M}(N,P) \otimes \frac{\text{Hom}}{M}(M,N) \longrightarrow \frac{\text{Hom}}{M}(M,N) \longrightarrow \frac{\text{Hom}}{M}(M,P) \in \mathcal{C}$ 

PROP: LET (M,D,M,P) BE A CLOSED LEFT &-MODINE CATEGORY PROOF = EXER.4.51

THEN (a) End(M) & Alg(e) WITH "INTERNAL END ALGEBRA"

THEN (a) End(M) & End(M) -> End(M),

[UNIT] J(PM): 11 -> End(M).

[UNIT] J(PM): 11 -> End(M).

(b) Hom(M,N) & Mod-End(M)(E) WITH

THEN (A) INTERNAL HOM MODINE"

THOM (M,N) & End(M) -> Hom (M,N).

TAKE  $(M, D, M, P) \in \mathcal{C}-Mod$  CLOSED  $f: Hom_{\mathcal{C}}(ZDM, N) \xrightarrow{\sim} Hom_{\mathcal{C}}(Z, Hom_{\mathcal{C}}(M, N))$ 

 $ev_{M,N}: \underline{\mathsf{Hom}}(M,N) \triangleright M \longrightarrow N \in \mathcal{M}$   $ev_{M,N,P}: (\underline{\mathsf{Hom}}(N,P) \otimes \underline{\mathsf{Hom}}(M,N)) \triangleright M \longrightarrow P \in \mathcal{M}$   $\mathsf{Comp}_{M,N,P}: \underline{\mathsf{Hom}}(N,P) \otimes \underline{\mathsf{Hom}}(M,N) \longrightarrow \underline{\mathsf{Hom}}(M,P) \in \mathcal{C}$ 

(End(M), COMPM,M,M, &(PM)) & Alg(C)
"INTERNAL END ALGEBRA"

(Hom(M,N), compannin) & Mod-End(M)(&)
"INTERNAL HOM MODILE"

# EXAMPLE

TAKE (M, D, M, P) & G-Mod CLOSED

9: Homm(ZDM, N) ~ Home(Z, Hom(M, N))

evm, N: Hom (M, N) DM ~ N & M

evm, N, P: (Hom(N, P) & Hom(M, N)) DM ~ P & M

Compm, N, P: Hom(N, P) & Hom(M, N) ~ Hom(M, P) & G

(End(M), Compm, M, N, P(PM)) & Alg(E)

"INTERNAL END ALGEBRA"

(Hom(M, N), Compm, M, N) & Mod-End(M)(E)

"INTERNAL HOM MODILE"

EXAMPLE & RIGID 
$$\Rightarrow$$

(Creg,  $D := \otimes$ )  $\in C - Mod$  is closed:

Hom  $(Y, z) := z \otimes Y^*$ .

TAKE (M, D, M, P) & G-Mod CLOSED

S: Homm(ZDM, N) ~ Home(Z, Hom(M, N))

eVM, N: Hom (M, N) DM ~ N & M

eVM, N: (Hom(N, P) & Hom(M, N)) DM ~ P & M

COMPM, N, P: (Hom(N, P) & Hom(M, N)) ~ Hom(M, P) & C

(END(M), COMPM, M, N, P(PM)) & Alg(C)

"INTERNAL END ALGEBRA"

(Hom(M, N), COMPM, M, N) & Mod-End(M)(C)

"INTERNAL HOM MODILE"

EXAMPLE & RIGID  $\Rightarrow$ (Creg,  $D := \otimes$ )  $\in \mathcal{C}$ -Mod is closed:  $\text{Hom}(Y, Z) := Z \otimes Y^{*}.$ TNDEED,  $(-\otimes Y) + (-\otimes Y^{*})$ , i.e.  $\text{Home}(X \otimes Y, Z) \cong \text{Home}(X, Z \otimes Y^{*}).$ 

TAKE (M, D, M, P) & G-Mod CLOSED

9: Homy(ZDM, N) ~> Homy(Z, Hom(M, N))

eVM, N: Hom (M, N) DM ~> N & M

eVM, N: (Hom(N, P) & Hom(M, N)) DM ~> P & M

COMPM, N, P: (Hom(N, P) & Hom(M, N)) ~> Hom(M, P) & G

(END(M), COMPM, M, N, P(PM)) & Alg(E)

"INTERNAL END ALGEBRA"

(Hom(M, N), COMPM, M, N) & Mod-End(M)(E)

"INTERNAL HOM MODILE"

EXAMPLE & RIGID 
$$\Rightarrow$$

(Creg,  $D := \emptyset$ )  $\in \mathcal{C}$ -Mod is closed:

 $\underbrace{\text{Hom}(Y, Z)} := Z \otimes Y^*.$ 

TNDEED,  $(-\otimes Y) + (-\otimes Y^*)$ , i.e.

 $\underbrace{\text{Hom}_{\mathcal{C}}(X \otimes Y, Z)} \cong \underbrace{\text{Hom}_{\mathcal{C}}(X, Z \otimes Y^*)}.$ 
 $f \mapsto [X \xrightarrow{\text{id} \otimes \text{over} Y} X \otimes Y \otimes Y^* \xrightarrow{\text{f} \otimes \text{id}} Z \otimes Y^*]$ 

EXAMPLE & RIGID 
$$\Longrightarrow$$

(Creg,  $D := \emptyset$ )  $\in \mathcal{C}$ —Mod is closed:

 $ttom(Y, Z) := Z \otimes Y^*$ .

INDEED,  $(-\otimes Y) - (-\otimes Y^*)$ , i.e.

 $ttom_{\mathcal{C}}(X \otimes Y, Z) \cong ttom_{\mathcal{C}}(X, Z \otimes Y^*)$ .

 $f \longmapsto [X \xrightarrow{id \otimes \alpha evY} X \otimes Y \otimes Y^* \xrightarrow{f \otimes id} Z \otimes Y^*]$ 
 $[X \otimes Y \xrightarrow{g \otimes id} Z \otimes Y^* \otimes Y \xrightarrow{id \otimes evY} Z] \longleftarrow g$ 

EXAMPLE & RIGID 
$$\Rightarrow$$

(Creg,  $D := \emptyset$ )  $\in \mathcal{C}$ -Mod is closed:

 $thom(Y, z) := z \otimes Y^*$ .

INDEED,  $(-\otimes Y) + (-\otimes Y^*)$ , i.e.

 $thom_{\mathcal{C}}(X \otimes Y, z) \xrightarrow{s} thom_{\mathcal{C}}(X, z \otimes Y^*)$ .

 $f \mapsto [X \xrightarrow{id \otimes coevY} X \otimes Y \otimes Y^* \xrightarrow{f \otimes id} z \otimes Y^*]$ 
 $[X \otimes Y \xrightarrow{g \otimes id} z \otimes Y^* \otimes Y \xrightarrow{id \otimes cevY} z] \leftarrow 1$ 

TAKE (M, D, M, P) & G-Mod CLOSED T: Homm(ZDM, N) ~ Home(Z, Hom(M, N))

 $ev_{M,N}: \underline{\mathsf{Hom}}(M,N) \supset M \longrightarrow N \in \mathcal{M}$   $ev_{M,N,P}: (\underline{\mathsf{Hom}}(N,P) \otimes \underline{\mathsf{Hom}}(M,N)) \supset M \longrightarrow P \in \mathcal{M}$   $\mathsf{Comp}_{M,N,P}: \underline{\mathsf{Hom}}(N,P) \otimes \underline{\mathsf{Hom}}(M,N) \longrightarrow \underline{\mathsf{Hom}}(M,P) \in \mathcal{C}$ 

(End(M), COMPH,M,M, &(PM)) & Alg(E)
"INTERNAL END ALGEBRA"

(Hom(M,N), compminIN) & Mod-End(M)(&)
"INTERNAL HOM MODINE"

EXAMPLE & RIGID  $\Rightarrow$ (Creg,  $D := \otimes$ )  $\in \mathcal{C}$ -Mod is closed:  $\text{Hom}(Y, Z) := Z \otimes Y^*.$ 

JUSES COEY J-1 USES EV

TAKE (M, D, M, P) & G-MOD CLOSED S: HOMM(ZDM, N) ~ HOME(Z, HOM(M, N))

 $ev_{M,N}: \underline{\mathsf{Hom}}(M,N) \triangleright M \longrightarrow N \in \mathcal{M}$   $ev_{M,N,P}: (\underline{\mathsf{Hom}}(N,P) \otimes \underline{\mathsf{Hom}}(M,N)) \triangleright M \longrightarrow P \in \mathcal{M}$   $\mathsf{Comp}_{M,N,P}: \underline{\mathsf{Hom}}(N,P) \otimes \underline{\mathsf{Hom}}(M,N) \longrightarrow \underline{\mathsf{Hom}}(M,P) \in \mathcal{C}$ 

(End(M), compu, m, m, g(pm)) & Alg(C)
"INTERNAL END ALGEBRA"

(Hom(M,N), compminIN) & Mod-End(M)(&)
"INTERNAL HOM MODING"

EXAMPLE & RIGID >>

(Creg, D:= ⊗) ∈ C-Mod 15 CLOSED:

HOM (Y, Z) := ZOY\*.

GET End(y) := Y & Y\* & Alg(E) VIA:

guses coer g-1 uses ev

TAKE (M, D, M, P) & G-Mod CLOSED

9: Homm(ZDM, N) ~ Home(Z, Hom(M, N))

eVM, N: Hom (M, N) DM ~ N & M

eVM, N: (Hom(N, P) & Hom(M, N)) DM ~ P & M

COMPM, N, P: (Hom(N, P) & Hom(M, N) ~ Hom(M, P) & C

(End(M), COMPM, M, M, P(PM)) & Alg(C)

"INTERNAL END ALGEBRA"

(Hom(M, N), COMPM, M, N) & Mod-End(M)(C)

"INTERNAL HOM MODILE"

EXAMPLE & RIGID  $\Rightarrow$ (Creg,  $D := \emptyset$ )  $\in C-Mod$  is closed:  $\underbrace{Hom}(Y, z) := Z \otimes Y^*.$ 

GET End(y) := Y & Y \* & Alg(&) VIA:

MEND(Y) := COMPYIYIY = S(evyIYIY):

 $\gamma \otimes \gamma^* \otimes \gamma \otimes \gamma^* \longrightarrow \gamma \otimes \gamma^*$ 

 $\frac{1}{2} \text{ NSES COEV} \qquad \frac{1}{2} \text{ NSES EV}$   $\frac{1}{2} \text{ NSES COEV} \qquad \frac{1}{2} \text{ NSES EV}$   $\frac{1}{2} \text{ NSES COEV} \qquad \frac{1}{2} \text{ NSES EV}$   $\frac{1}{2} \text{ NSES COEV} \qquad \frac{1}{2} \text{ NSES EV}$   $\frac{1}{2} \text{ NSES COEV} \qquad \frac{1}{2} \text{ NSES EV}$   $\frac{1}{2} \text{$ 

EXAMPLE & RIGID 
$$\Rightarrow$$

(Creg,  $D:=\otimes$ )  $\in C-Mod$  is closed:

 $tom(Y, z) := z \otimes Y^*.$ 

GET  $End(Y) := Y \otimes Y^* \in Alg(C)$  VIA:

 $Mend(Y) := comp_{Y|Y|Y} = S(ev_{Y|Y|Y}):$ 
 $Y \otimes Y^* \otimes Y \otimes Y^* \longrightarrow Y \otimes Y^*$ 
 $((y \otimes Y^*) \otimes (y \otimes Y^*) \otimes Y \longrightarrow Y \otimes Y^*) \otimes Y \otimes Y^*$ 
 $(y \otimes Y^*) \otimes ((y \otimes Y^*) \otimes Y)$ 
 $(y \otimes Y^*) \otimes ((y \otimes Y^*) \otimes Y)$ 
 $(y \otimes Y^*) \otimes ((y \otimes Y^*) \otimes Y)$ 
 $(y \otimes Y^*) \otimes ((y \otimes Y^*) \otimes Y)$ 

EXAMPLE & RIGID 
$$\Rightarrow$$

(Creg,  $D:=\emptyset$ )  $\in$  &-Mod is closed:

 $\text{Hom}(Y, z) := Z \otimes Y^*.$ 

GET End(Y) :=  $Y \otimes Y^* \in Alg(C)$  VIA:

 $M_{\text{End}(Y)} := \text{comp}_{Y_1Y_1Y} = \mathcal{G}(\text{ev}_{Y_1Y_1Y}):$ 
 $Y \otimes Y^* \otimes Y \otimes Y^* \longrightarrow Y \otimes Y^*$ 
 $(Y \otimes Y^*) \otimes (Y \otimes Y^*) \otimes Y \longrightarrow Y \otimes Y^* \otimes Y$ 
 $(Y \otimes Y^*) \otimes (Y \otimes Y^*) \otimes Y \longrightarrow Y \otimes Y^* \otimes Y$ 
 $(Y \otimes Y^*) \otimes (Y \otimes Y^*) \otimes Y \longrightarrow Y \otimes Y^* \otimes Y$ 
 $= \text{STRICT FOR EASE} =$ 

EXAMPLE & RIGID 
$$\Rightarrow$$

(Creg,  $D:=\otimes$ )  $\in \mathcal{C}$ -Mod is closed:

 $\text{Hom}(Y, \mathcal{E}) := \mathcal{E} \otimes Y^*.$ 

GET  $\text{End}(Y) := Y \otimes Y^* \in \text{Alg}(\mathcal{C}) \text{ VIA:}$ 
 $\text{Mend}(Y) := \text{compy,y,y} = \mathcal{G}(\text{evy,y,y}):$ 
 $\text{Y} \otimes Y^* \otimes Y \otimes Y^* \longrightarrow \text{Y} \otimes Y^*$ 
 $\text{(Y} \otimes Y^*) \otimes (Y \otimes Y^*) \otimes Y \longrightarrow \text{Y} \otimes Y^* \otimes Y^*$ 
 $\text{(Y} \otimes Y^*) \otimes (Y \otimes Y^*) \otimes Y \longrightarrow \text{Y} \otimes Y^* \otimes Y^*$ 
 $\text{Y} \otimes Y^* \otimes Y \otimes Y^* \longrightarrow \text{Y} \otimes Y^* \otimes Y^*$ 
 $\text{Y} \otimes Y^* \otimes Y \otimes Y^* \longrightarrow \text{Y} \otimes Y^* \otimes Y^* \otimes Y^*$ 
 $\text{Y} \otimes Y^* \otimes (Y \otimes Y^*) \otimes Y \longrightarrow \text{Y} \otimes Y^* \otimes Y^* \otimes Y^*$ 
 $\text{N} = \text{STRICT FOR EASE} =$ 

$$\frac{1}{2} \text{ USES CoeV} \qquad \frac{1}{2} \text{ USES eV}$$

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$$\frac{1}{2} \text{ USES CoeV} \qquad \frac{1$$

EXAMPLE & RIGID 
$$\Rightarrow$$

(Creg,  $D:=\otimes$ )  $\in$  &-Mod is closed:

 $\text{Hom}(Y, Z) := Z \otimes Y^{*}.$ 

GET  $\text{End}(Y) := Y \otimes Y^{*} \in \text{Alg}(Z) \text{ VIA:}$ 
 $\text{Mend}(Y) := \text{comp}_{Y_{1}Y_{1}Y} = \text{S}(\text{ev}_{Y_{1}Y_{1}Y}):$ 
 $\text{V} \otimes \text{V}^{*} \otimes \text{V} \otimes \text{V}^{*} \xrightarrow{\text{id}_{Y} \otimes \text{ev}_{Y}^{*} \otimes \text{id}_{Y}^{*}} \text{V} \otimes \text{V}^{*}$ 
 $\text{(youth of (Y of Y^{*}) of$ 

TAKE (M, D, M, P) & G-MOD CLOSED S: HOMM(ZDM, N) ~ HOME(Z, HOM(M, N))

 $eV_{M,N}: \underline{ttom}(M,N) \supset M \longrightarrow N \in \mathcal{M}$   $eV_{M,N,P}: (\underline{ttom}(N,P) \otimes \underline{ttom}(M,N)) \supset M \longrightarrow P \in \mathcal{M}$   $ComP_{M,N,P}: \underline{ttom}(N,P) \otimes \underline{ttom}(M,N) \longrightarrow \underline{ttom}(M,P) \in \mathcal{C}$ 

(End(M), compu, m, m, g(pm)) & Alg(C)
"INTERNAL END ALGEBRA"

(Hom(M,N), compmIMIN) & Mod-End(M)(&)
"INTERNAL HOM MODILE"

guses coer g-1 uses ev

EXAMPLE & RIGID  $\Rightarrow$ (Creg,  $D := \emptyset$ )  $\in C - Mod$  is closed:  $\underbrace{Hom}(Y, Z) := Z \otimes Y^*.$ 

GET End(y) := Y & Y \* & Alg(E) VIA:

MEND(Y) := COMPYIYIY = g(evyiyiy):

Y > Y & Y & Y & Y & idy & evy & idy\*

UEnd(y) := J(Py): 11->> Y&Y\*

TAKE (M,D,M,P) & G-MOD CLOSED S: Homm(ZDM,N) ~> Home(Z, Hom(M,N))

 $eV_{M,N}: \underline{ttom}(M,N) \supset M \longrightarrow N \in \mathcal{M}$   $eV_{M,N,P}: (\underline{ttom}(N,P) \otimes \underline{ttom}(M,N)) \supset M \longrightarrow P \in \mathcal{M}$   $ComP_{M,N,P}: \underline{ttom}(N,P) \otimes \underline{ttom}(M,N) \longrightarrow \underline{ttom}(M,P) \in \mathcal{C}$ 

(End(M), COMPH,M,M, &(PM)) & Alg(C)
"INTERNAL END ALGEBRA"

(Hom(M,N), compM,M,N) & Mod-End(M)(&)
"INTERNAL HOM MODILE"

JUSES COEY J-1 USES EV

Py: 1184 - 24

EXAMPLE & RIGID  $\Rightarrow$ (Creg,  $D := \emptyset$ )  $\in \mathcal{C}$ -Mod is closed:  $\text{Hom}(Y, Z) := Z \otimes Y^*.$ 

GET End(y) := Y & Y\* & Alg(E) VIA:

MEND(Y) := COMPYIYIY = g(evyiyiy):

UEnd(y) := J(Py): 11->> Y&Y\*

TAKE  $(M, D, M, P) \in \mathcal{C}-Mod CLOSED$  $f: Hom_{\mathcal{C}}(ZDM, N) \xrightarrow{\sim} Hom_{\mathcal{C}}(Z, Hom_{\mathcal{C}}(M, N))$ 

 $eV_{M,N}: \underline{tom}(M,N) \supset M \longrightarrow N \in \mathcal{M}$   $eV_{M,N,P}: (\underline{tom}(N,P) \otimes \underline{tom}(M,N)) \supset M \longrightarrow P \in \mathcal{M}$   $ComP_{M,N,P}: \underline{tom}(N,P) \otimes \underline{tom}(M,N) \longrightarrow \underline{tom}(M,P) \in \mathcal{C}$ 

(End(M), COMPH,M,M, &(PM)) & Alg(C)
"INTERNAL END ALGEBRA"

(Hom(M,N), compM,M,N) & Mod-End(M)(&)
"INTERNAL HOM MODILE"

guses coer g-1 uses ev

Py: 1184 - DY

EXAMPLE & RIGID >>

(Creg, D:= &) & &-Mod is closed:

HOM (Y, Z) := Z & Y\*.

GET End(y) := Y & Y\* & Alg(E) VIA:

MEND(Y) := COMPYIYIY = S(evyIYIY):

Y > Y & Y & Y & Y & idy & evy & idy\*

UENd(Y) := J(PY): 11 COEVY > YOUY\*

TAKE  $(M, D, M, P) \in \mathcal{C}-Mod$  CLOSED  $f: Hom_{M}(ZDM, N) \xrightarrow{\sim} Hom_{\mathcal{C}}(Z, Hom_{M}, N))$   $ev_{M,N}: \underline{Hom}(M, N) DM \longrightarrow N \in \mathcal{M}$  $ev_{M,N,P}: (\underline{Hom}(N,P) \otimes \underline{Hom}(M,N)) DM \longrightarrow P \in \mathcal{M}$ 

(End(M), COMPH,M,M, &(PM)) & Alg(C)
"INTERNAL END ALGEBRA"

COMPM,N,P: HOM (N,P) ⊗ HOM (M,N) → HOM (M,P) € C

(Hom(M,N), compMMN) & Mod-End(M)(&)
"INTERNAL HOM MODINE"

EXAMPLE & RIGID => (Creg, D:=⊗) ∈ C-Mod 15 CLOSED: HOM (Y, Z) := Z⊗Y\*. GET End(y) := Y & Y \* & Alg(C) VIA: MEND(Y) := COMPYIYIY = J(evyIYIY): UEnd(y) := J(Py): 11 coevy > Y@Y\* EXERCISE 4.52 CHECK THE DETAILS

TAKE (M, D, M, P) & G-Mod CLOSED

9: Homy(ZDM, N) ~ Homy(Z, Hom(M, N))

evm, N: Hom (M, N) DM ~ N & M

evm, N, P: (Hom(N, P) & Hom(M, N)) DM ~ P & M

COMPM, N, P: Hom(N, P) & Hom(M, N) ~ Hom(M, P) & C

(End(M), COMPM, M, M, P(PM)) & Alg(E)

"INTERNAL END ALGEBRA"

(Hom(M, N), COMPM, M, N) & Mod-End(M)(E)

"INTERNAL HOM MODILE"

EXERCISE 4.54: G = GROUP

Forg: G-Mod -> Vec is monoidal

>> Vec \( \) (G-Mod) - Mod

EXAMPLE & RIGID => (Creg, D:= ⊗) ∈ C-Mod 15 CLOSED: HOM (Y, Z) := Z⊗Y\*. GET End(y) := Y & Y \* & Alg(C) VIA: MENDCY) := COMPYIYIY = J(evyIYIY): UENd(Y) := J(PY): 11 - COEVY > Y@Y\* EXERCISE 4.52 CHECK THE DETAILS

TAKE (M, D, M, P) & G-Mod CLOSED

S: Homy(ZDM, N) ~ Homy(Z, Hom(M, N))

evm, N: Hom (M, N) DM ~ N & M

evm, N: Hom(N, P) & Hom(M, N) DM ~ P & M

Compm, N, P: (Hom(N, P) & Hom(M, N) ~ Hom(M, P) & C

(End(M), Compm, M, M, P (Pm)) & Alg(C)

"INTERNAL END ALGEBRA"

(Hom(M, N), Compm, M, N) & Mod-End(M)(C)

"INTERNAL HOM MODILE"

EXERCISE 4.54: G=GROUP

Forg: G-Mod -> Vec IS MONOIDAL

-> Vec \( \) (G-Mod) - Mod

For  $lk \in Vec$ , GET:

End (lk)

\( \) Alg(G-Mod).

EXAMPLE & RIGID => (Creg, D:=⊗) ∈ C-Mod IS CLOSED: HOM (Y, Z) := Z⊗Y\*. GET End(y) := Y & Y \* & Alg(C) VIA: MEND(Y) := COMPYIYIY = J(evyIYIY): UEnd(y) := J(py): 11 coevy > Y@Y\* EXERCISE 4.52 CHECK THE DETAILS

TAKE (M, D, M, P) & G-Mod CLOSED

9: Homy(ZDM, N) ~ Homy(Z, Hom(M, N))

evm, N: Hom (M, N) DM ~ N & M

evm, N: (Hom(N, P) & Hom(M, N)) DM ~ P & M

Compan, P: (Hom(N, P) & Hom(M, N) ~ Hom(M, P) & C

(End(M), Compan, M, M, P(PM)) & Alg(E)

"INTERNAL END ALGEBRA"

(Hom(M, N), Compan, M, N) & Mod-End(M)(E)

"INTERNAL HOM MODILE"

EXERCISE 4.54: G=GROUP

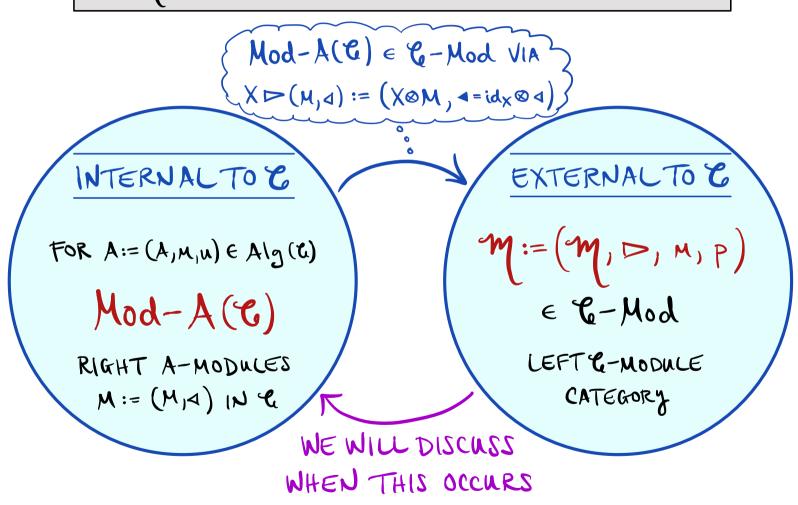
Forg: G-Mod -> Vec IS MONOIDAL

-> Vec \( \) (G-Mod) - Mod

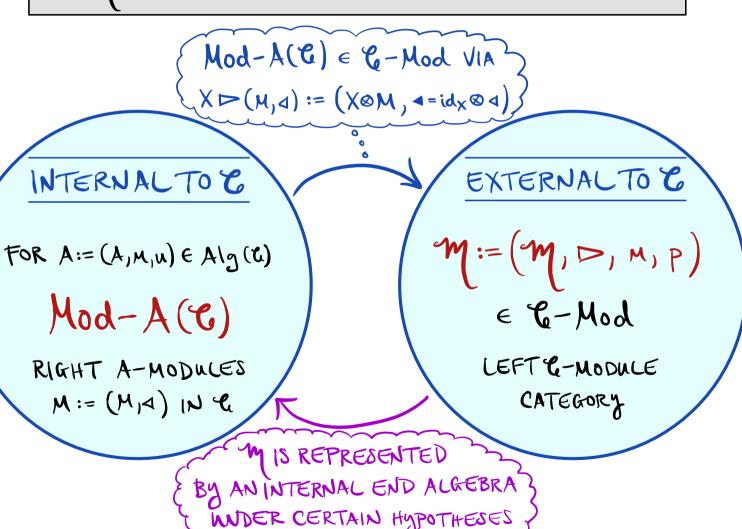
For  $|k \in Vec, GET:$ End  $(|k|) \cong (|kG|)^*$ \( \) Alg (G-Mod).

EXAMPLE & RIGID => (Creg, D:=⊗) ∈ C-Mod IS CLOSED: HOM (Y, Z) := Z⊗Y\*. GET End(y) := Y & Y \* & Alg(C) VIA: MEND(Y) := COMPYIYIY = J(evyIYIY): YOY\* WYO WYO WY WILLY YOY \* UEnd(y) := J(py): 11 coevy > Y@Y\* EXERCISE 4.52 CHECK THE DETAILS

Mee-Mod is represented by Aealg(C) IF  $M \simeq Mod-A(C)$  As left c-Module Categories.



Mee-Mod is represented by Aealg(c) IF  $M \simeq Mod-A(c)$  as left e-module categories.



(m, D) ∈ E-Mod CLOSED ( Homm(ZDM, N))

≥ Home(Z, Hom(M, N))

End(M) ∈ Alg(E)

Hom(M, N) ∈ Mod-End(M)(E)

LET'S COLLECT SOME FACTS -

(m, D) ∈ &-Mod CLOSED ( Homm(ZDM, N))

= Home(Z, Hom(M, N))

End(M) ∈ Alg(Z)

Hom(M, N) ∈ Mod-End(M)(Z)

LET'S COLLECT SOME FACTS -

LEMMA: WHEN (M,D) & C-Mod IS CLOSED,

GET FOR ALL XE & AND M, NEM:

HOM (M, XDN) = X & HOM (M,N).

(M, D) ∈ E-Mod CLOSED ( HOMM(ZDM, N))

= Home(Z, Hom(M, N))

End(M) ∈ Alg(Z)

Hom(M, N) ∈ Mod-End(M)(Z)

LET'S COLLECT SOME FACTS -

LEMMA: WHEN (M,D) & G-Mod IS CLOSED, GET FOR ALL X&G AND M, N&M: HOM (M, XDN) = X® HOM (M,N).

PF/ Home (Y, Hom (M, XDN))

Homm (YDM, XDN)

(m, D) ∈ &-Mod CLOSED ( Homm(ZDM, N)

= Home(Z, Hom(M, N))

End(M) ∈ Alg(E)

Hom(M,N) ∈ Mod-End(M)(E)

LET'S COLLECT SOME FACTS -

LEMMA: WHEN (M,D) & G-Mod IS CLOSED, GET FOR ALL XEG AND M, NEM: HOM (M, XDN) = X & HOM (M,N).

PF/ Home (Y, Hom (M, XDN))

Homm (YDM, XDN)

Homm (X\*D(YDM), N)

 $(X^* \triangleright -) \rightarrow (X \triangleright -) \sim$ 

(M, D) ∈ &-Mod CLOSED ( Homm(ZDM, N))

≥ Home(Z, Hom(M, N))

End(M) ∈ Alg(E)

Hom(M,N) ∈ Mod-End(M)(E)

LET'S COLLECT SOME FACTS -

LEMMA: WHEN (M,D) & G-Mod IS CLOSED, GET FOR ALL XEG AND M, NEM: HOM (M, XDN) = X & HOM (M,N).

PF/ Home (Y, Hom (M, XDN))

Homm (YDM, XDN)

Homm (X\*D(YDM), N)

MOD CAT

ASSOC.
Homm ((X\*&Y)DM, N)

 $(X \times D -) - (XD -) -$ 

LET'S COLLECT SOME FACTS -(m, D) ∈ &-Mod CLOSED ( HOMM(ZDM,N) LEMMA: WHEN (M,D) & G-Mod IS CLOSED, = Home(Z, Hom(M,N)) GET FOR ALL XE'S AND M, NE'M: End(M) & Alg(E) HOM (M, XDN) = X & HOM (M,N) Hom (M,N) & Mod-End(M)(Ce) PF/ Home (Y, Hom (M, XDN)) = HOMM (YDM, XDN) Houm (X\*D(YDM), N)

MOD CAT

ASSOC.

Houm ((X\*&Y)DM, N) Home (X\*@Y, Hom (M,N))  $(X^* \triangleright -) \rightarrow (X \triangleright -)$ 

(M, D) ∈ G-Mod CLOSED ( HOMM(ZDM, N))

= Home(Z, Hom(M, N))

End(M) ∈ Alg(C)

Hom(M, N) ∈ Mod-End(M)(C)

LET'S COLLECT SOME FACTS -

LEMMA: WHEN (M,D) & G-Mod IS CLOSED, GET FOR ALL XE'S AND M, NEM: HOM (M, XDN) = X® HOM (M,N).

PF Home (Y, Hom (M, XDN))

How (Y, Hom (M, XDN))

How (X\*D(YDM), N)

MOD CAT

ASSTOC. How ((X\*@Y)DM, N)

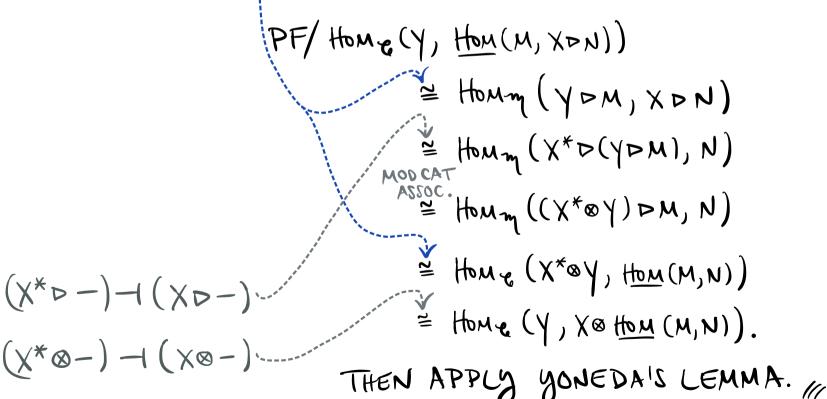
Home (X\*@Y, Hom (M, N))

Home (Y, X@ Hom (M, N)).

# (M,D) ∈ C-Mod CLOSED ( HOMM(ZDM,N)) Home(Z, Hom(M,N)) End(M) ∈ Alg(E) Hom(M,N) ∈ Mod-End(M)(E)

# LET'S COLLECT SOME FACTS -

LEMMA: WHEN (M,D) & G-Mod IS CLOSED, GET FOR ALL XEG AND M, NEM: HOM (M, XDN) = X & HOM (M,N).



LET'S COLLECT SOME FACTS -(m, D) ∈ &-Mod CLOSED ( HOMM(ZDM,N) LEMMA: WHEN (M, D) & G-Mod IS CLOSED, = Home(Z, Hom(M,N)) GET FOR ALL XEE AND M, NEM: End(M) & Alg(E) HOM (M, XDN) = X & HOM (M,N) Hom (M, N) & Mod-End(M)(C) Hom (M, XDN) = X & Hom (M, N) YXEC; M, NEM PF/ Home (Y, Hom (M, XDN)) = HOMM (YDM, XDN) = HOMM (X\*D(YDM), N) ASSOC. HOMM ((X\*&Y) DM, N) Home (X\*&Y, Hom (M,N)) (X\*p-)-(Xp-= Home (Y, X & Hom (M, N)) YYEC.  $(\chi^* \otimes -) \rightarrow (\chi \otimes -)$ THEN APPLY YONEDA'S LEMMA.

(M, D) ∈ G-Mod CLOSED ( Homm(ZDM, N))

= Home(Z, Hom(M, N))

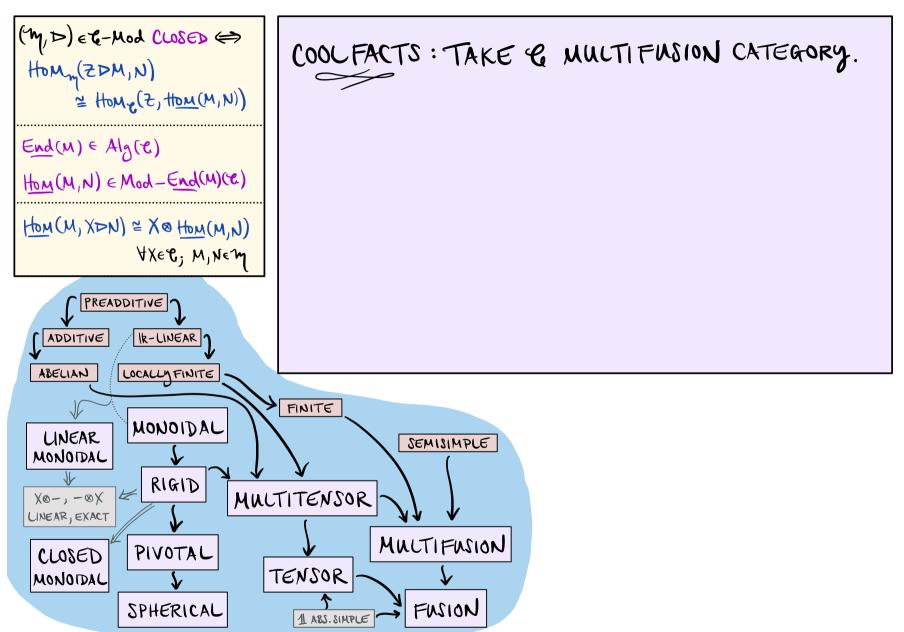
End(M) ∈ Alg(Z)

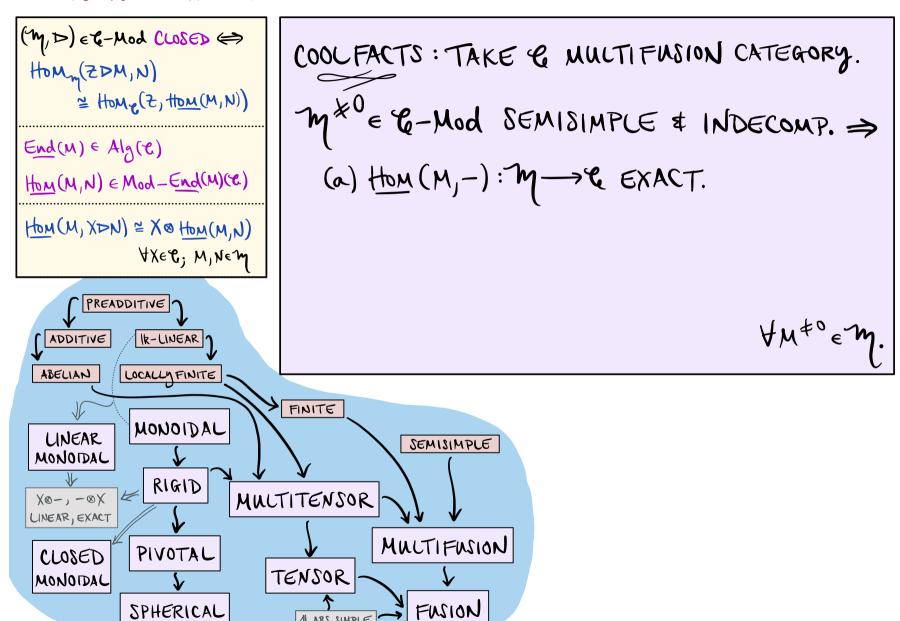
Hom(M, N) ∈ Mod-End(M)(Z)

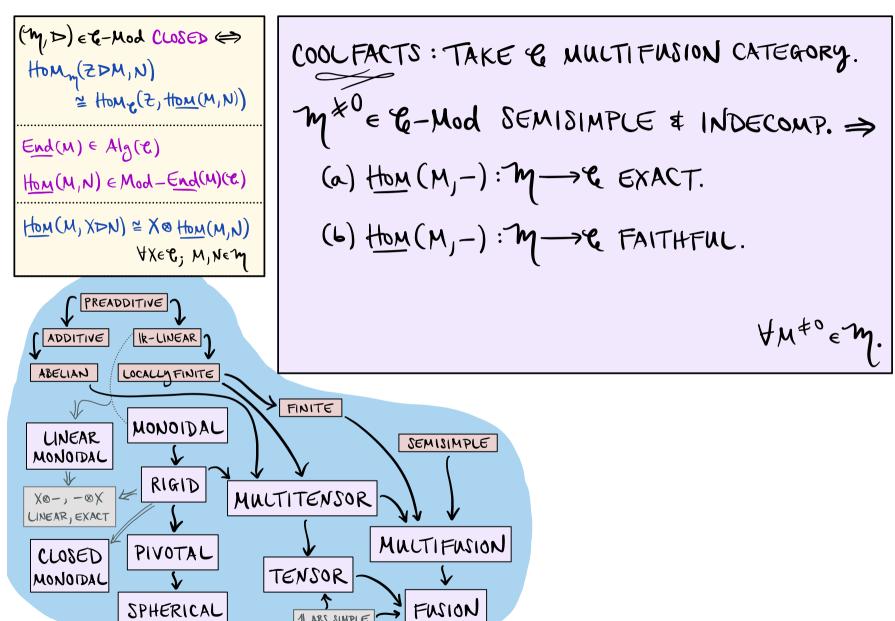
Hom(M, XDN) = X ⊗ Hom(M, N)

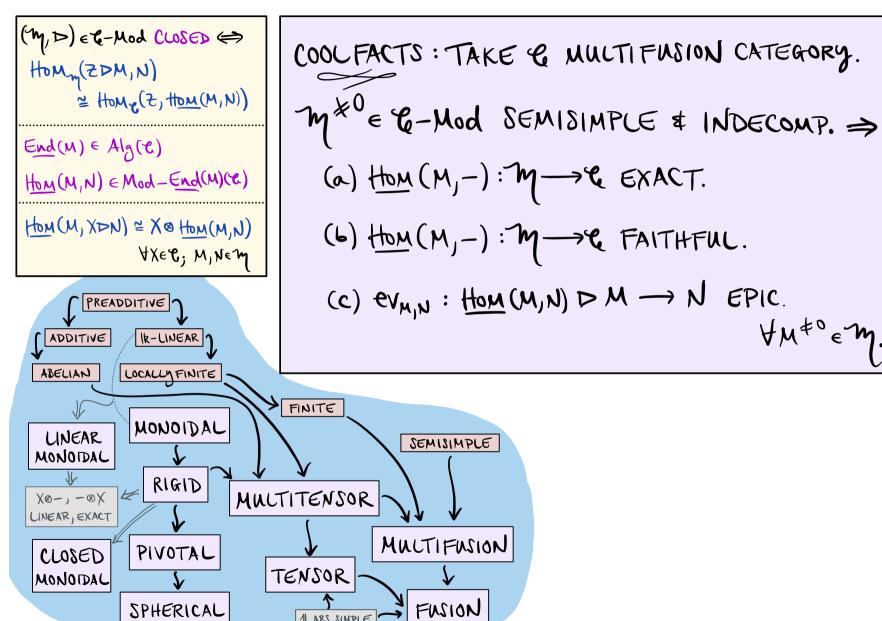
YXEC; M, NEM

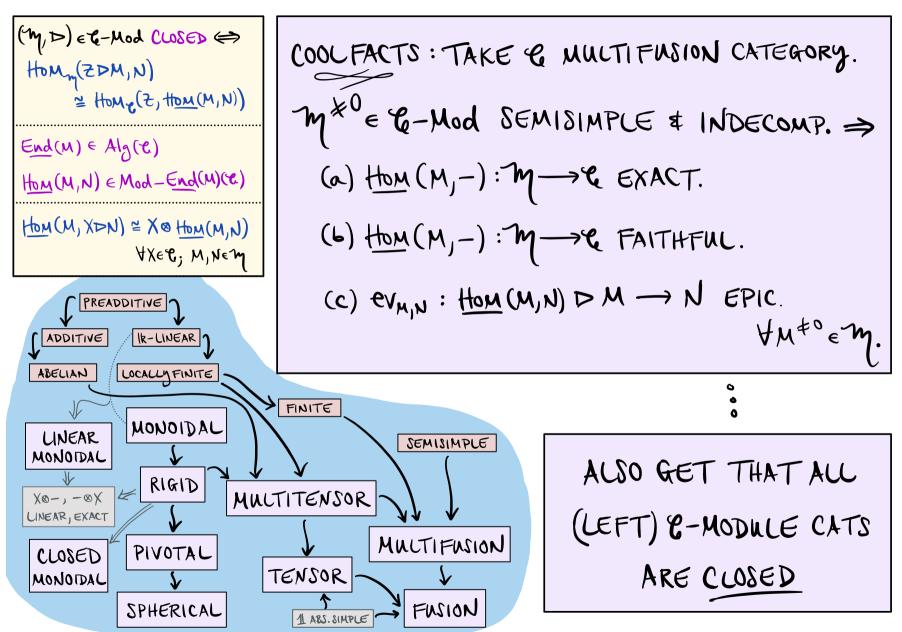
COOLFACTS: TAKE & MULTIFUSION CATEGORY.











(m, >) ∈ G-Mod CLOSED ⇒ Homm(ZDM,N) = Home(Z, Hom(M,N)) End(M) € Alg(E) Hom (M,N) & Mod-End(M)(C) Hom (M, XDN) = X @ Hom (M,N) YXEC; M, NEM Ty\*0 SEMISIMPLE

\$ INDECOMPOSABLE ⇒ (a) thom(M,-):7y→& EXACT (b) ttom(M,-): M→& FAITHFUL (c) eVMN: HOM(M,N)DM→N EPIC



ALSO GET THAT ALL
(LEFT) &-MODILE CATS
ARE CLOSED



(M, D) ∈ C-Mod CLOSED ( HOMM(ZDM, N))

≥ HOME(Z, HOM(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) ≈ X® Hom (M,N) ∀X6°C; M,N6°M

M\*0 SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a) thom(M,-):7y→& EXACT

(b) thom(M,-):7y→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC YM<sup>\$0</sup>€M. THEOREM COSTRIK] MERCHOOL THEN

M = Mod - End(M)(C)

AS LEFT C-MODULE CATEGORIES YMERCY.

( MULTIFUSION)

(m, D) e &-Mod CLOSED ( Homm(ZDM, N))

≥ Home(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) = X & Hom (M,N)

YXET; M, NEM

Ty\*0 SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a) thom(M,-):7y→& EXACT

(b) ttom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>¢0</sup>∈M.

PF/
TAKE A:= End(M),
F:  $M \longrightarrow Mod-A(C)$   $N \mapsto Hom(M,N)$ .

THEOREM COSTRIK] M = C-Mod. THEN

M = Mod-End(M)(C)

AS LEFT C-MODULE CATEGORIES YM\* Em.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

{ MULTIFUSION}

(M,D) ∈ C-Mod CLOSED ( HOMM(ZDM,N))

= HOMZ(Z, HOM(M,N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) ≈ X® Hom (M,N) ∀Xee; M, Nem

M<sup>\*0</sup> SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a)  $\underline{\text{Hom}}(M,-):M \longrightarrow C \in XACT$ 

(b) thom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>¢o</sup>∈M.

PF/
TAKE A:= End(M),
F:  $M \longrightarrow Mod-A(C)$   $N \mapsto Hom(M,N)$ .

THEOREM COSTRIK] MEGE G-Mod. THEN

M = Mod-End(M)(C)

AS LEFT G-MODULE CATEGORIES YMEGM.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS A LEFT &-MODULE FUNCTOR -

· FUNCTOR /

{ MULTIFUSION}

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ Home(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) ≈ X® Hom (M,N) ∀XEC; M,NEM

Ty\*0 SEMISIMPLE

\$ INDECOMPOSABLE =>

(a)  $\underline{t_{\text{DM}}}(M,-):M \longrightarrow \mathcal{E}$  EXACT

(b) thom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>¢0</sup>∈M.

PF/
TAKE A:=  $\underline{End}(M)$ ,

F:  $\underline{M} \longrightarrow Mod - A(C)$   $N \mapsto \underline{Hom}(M,N)$ .

THEOREM COSTRIK] Mto & C-Mod. THEN

M = Mod - End(M)(C)

AS LEFT C-MODULE CATEGORIES YMEM.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS A LEFT &-MODULE FUNCTOR -

- · FUNCTOR /
- ... OF C-MODULE CATEGORIES:

F(XDN) = HOM (M, XDN)

YXet, Nem.

{ MULTIFUSION }

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ Home(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) = X@ Hom (M,N).

Ty\*0 SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a) thom(M,-):7m→& EXACT

(b) thom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>\$0</sup> € M.

PF/
TAKE A:= End(M),
F:  $M \longrightarrow Mod-A(C)$   $N \mapsto Hom(M,N)$ .

THEOREM COSTRIK] MER & C-Mod. THEN

M = Mod- End(M)(C)

AS LEFT C-MODULE CATEGORIES YMEM.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS A LEFT &-MODULE FUNCTOR -

- · FUNCTOR /
- ... OF G-MODULE CATEGORIES:

YXeC, Nem.

( MULTIFUSION)

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ Home(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) = X & Hom (M,N).

Ty\*0 SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a) thom(M,-):7m→& EXACT

(b) thom(M,-): M→& FAITHFUL

(c) eV<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>\$0</sup> € M.

PF/
TAKE A:=  $\underline{End}(M)$ ,

F:  $\underline{M} \longrightarrow Mod - A(C)$   $N \mapsto \underline{Hom}(M,N)$ .

THEOREM COSTRIK] MEGE G-Mod. THEN

M = Mod- End(M)(C)

AS LEFT G-MODULE CATEGORIES YMEGM.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS A LEFT &-MODULE FUNCTOR -

- · FUNCTOR V
- ... OF G-MODULE CATEGORIES:

~ X⊗ Hom (M,N)

= X PA F(N)

YXEC, NEM.

( & MULTIFUSION)

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ Home(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) = X & Hom (M, N)

YXEC; M, NEM

Ty\*0 SEMISIMPLE

\$ INDECOMPOSABLE =>

(a) thom(M,-):7y→& EXACT

(b) thom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>¢0</sup> ∈ M.

PF/
TAKE A:= End(M),

F:  $M \longrightarrow Mod-A(C)$   $N \mapsto Hom(M,N)$ .

THEOREM COSTRIK] MER & C-Mod. THEN

M = Mod- End(M)(C)

AS LEFT C-MODULE CATEGORIES YMEM.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS FULLY FAITHFUL

& MULTIFUSION)

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N)

≥ Homy(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) ≈ X® Hom (M,N) ∀XEE; M,NEM

Ty\*0 SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a) tom(M,-):7y→& EXACT

(b) thom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>¢0</sup>∈M.

PF/
TAKE A:= End(M),

F:  $M \rightarrow Mod-A(C)$   $N \mapsto Hom(M,N)$ .

THEOREM COSTRIK] MER & G-Mod. THEN

M = Mod- End(M)(C)

AS LEFT G-MODULE CATEGORIES YMER.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS FULLY FAITHFUL IF N=XDM FOR SOME XE'S

{ & MULTIFUSION }

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ Home(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) ≈ X® Hom (M,N) ∀Xe&; M, Nem

7y<sup>\*0</sup> SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a) thom(M,-):7m→& EXACT

(b) thom(M,-):7y→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>¢0</sup>∈M.

PF/
TAKE A:=  $\underline{End}(M)$ ,

F:  $\underline{M} \longrightarrow Mod-A(C)$   $N \mapsto \underline{Hou}(M,N)$ .

THEOREM COSTRIK] M = C-Mod. THEN

M = Mod-End(M)(C)

AS LEFT C-MODULE CATEGORIES YM = M.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

FIS FULLY FAITHFUL IF N=XDM FOR SOME XEE, HOMMOD-A(E) (F(N), F(N')) = HOMMOD-A(E) (F(XDM), F(N'))

: = Homm (N, N')

( MULTIFUSION)

(m, D) ∈ C-Mod CLOSED ( HOMM(ZDM, N))

≥ HOMZ(Z, HOM(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) = X & Hom (M,N) - A

 $m^{*0}$  SEMISIMPLE  $\Rightarrow$ 

(a)  $\underline{t_{lom}}(M,-): \mathcal{M} \longrightarrow \mathcal{C} \in XACT$ 

(b) thom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC YM\*0∈M.

PF/
TAKE A:= End(M),
F:  $M \longrightarrow Mod-A(C)$   $N \longmapsto HoM(M,N)$ .

THEOREM COSTRIK] MEGE G-Mod. THEN

M = Mod- End(M)(C)

AS LEFT G-MODULE CATEGORIES YMEGM.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

FIS FULLY FAITHFUL IF N=XDM FOR SOME XEC.)

HOMMOD-A(C) (F(N), F(N')) = HOMMOD-A(C) (F(XDM), F(N'))

HOMMOD-A(C) (X@A, F(N'))

: = Homm(N, N')

& MULTIFUSION

(m, D) ∈ C-Mod CLOSED ( HOMM(ZDM, N))

≥ HOMZ(Z, HOM(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) = X@ Hom (M,N)~

7y<sup>\*0</sup> SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a) thom(M,-):7m→& EXACT

(b) thom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>¢o</sup>∈M.

PF/
TAKE A:= End(M),
F:  $M \rightarrow Mod-A(C)$   $N \mapsto Hom(M,N)$ .

THEOREM COSTRIK] MEGE G-Mod. THEN

M = Mod-End(M)(C)

AS LEFT G-MODULE CATEGORIES YMEGM.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

FIS FULLY FAITHFUL IF N=XDM FOR SOME XEG

 $Hom_{Mod-A(C)}(F(N),F(N')) \stackrel{\checkmark}{=} Hom_{Mod-A(C)}(F(XDM),F(N'))$ 

HOMMON-A(T) (XOA, F(N'))

Free - Torg - Home (X, Forg (F(N1))

: = Homm(N, N')

& MULTIFUSION

(m, D) ∈ C-Mod CLOSED ( Homy(ZDM, N))

≥ Homy(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) = X@ Hom (M,N) - A

7y<sup>\*0</sup> SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a)  $\underline{tom}(M,-): M \longrightarrow C \in XACT$ 

(b) thom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>‡0</sup> € M.

PF/
TAKE A:=  $\underline{End}(M)$ ,
F:  $\underline{M} \longrightarrow Mod-A(C)$   $N \mapsto \underline{Hou}(M,N)$ .

THEOREM COSTRIK] Mto & G-Mod. THEN

M = Mod- End(M)(G)

AS LEFT G-MODULE CATEGORIES YME M.

WANT: F IS AN EQUIV. OF LEFT &-MOD. CATS.

F IS FULLY FAITHFUL IF N=XDM FOR SOME XEG

 $Hom_{Mod-A(C)}(F(N),F(N')) \stackrel{\checkmark}{=} Hom_{Mod-A(C)}(F(XDM),F(N'))$ 

HOMMON-A(Z) (XOA, F(N'))

Free - Torg Home (X, Forg (F(N1))

DEF HOME (X, HOM (M, N'))

: = Homm (N, N')

& MULTIFUSION }

(M,D) ∈ G-Mod CLOSED ( HOMm(ZDM,N)) ~= HOMe(Z, Hom(M,N)) ~=

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) = X@ Hom (M,N) -- VXEC; M, NEM

 $70^{*0}$  SEMISIMPLE \* INDECOMPOSABLE  $\Rightarrow$ 

(a) thom(M,-):7y→& EXACT

(b) thom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>¢0</sup>∈M.

PF/
TAKE A:= End(M),
F:  $M \longrightarrow Mod-A(C)$   $N \longmapsto HoM(M,N)$ .

THEOREM COSTRIK] ME & G-Mod. THEN

M = Mod- End(M)(C)

AS LEFT G-MODULE CATEGORIES YMEM.

WANT: F IS AN EQUIV. OF LEFT &-MOD. CATS.

F IS FULLY FAITHFUL IF N=XDM FOR SOME XEG

 $Hom_{Mod-A(C)}(F(N),F(N')) \stackrel{\checkmark}{=} Hom_{Mod-A(C)}(F(XDM),F(N'))$ 

Hommod-A(2) (X&A, F(N'))

Free - Torg Home (X, Forg (F(N1))

DEF HOME (X, HOM (M, N'))

HOMm (XDM, N')

= Homm (N, N')

& MULTIFUSION

(M, D) ∈ G-Mod CLOSED ( HOMm(ZDM, N)) ~ HOMe(Z, HOM(M, N)) ~

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$   $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) = X & Hom (M,N) =

 $10^{*0}$  SEMISIMPLE \$ INDECOMPOSABLE  $\Rightarrow$ 

(a) thom(M,-):7m→& EXACT

(b) thom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>¢o</sup>∈M.

PF/
TAKE A:= End(M),

F:  $M \longrightarrow Mod-A(C)$   $N \mapsto HoM(M,N)$ .

THEOREM COSTRIK] ME C-Mod. THEN

M = Mod- End(M)(C)

AS LEFT C-MODULE CATEGORIES YMEM.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS FULLY FAITHFUL IF N=XDM FOR SOME XEG,

HOMMON-A(E) (F(N), F(N')) = HOMMON-A(E) (F(XDM), F(N'))

Hommod-A(2) (X&A, F(N'))

Free - Torg = Home (X, Forg (F(N1))

DEF HOME (X, HOM (M, N'))

HOMm (XDM, N')

\* Homm (N, N')

( MULTIFUSION)

(M, D) ∈ C-Mod CLOSED ( HOMM(ZDM, N))

≥ HOMZ(Z, HOM(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) = X & Hom (M,N) VXEC; M, NEM

 $10^{*0}$  SEMISIMPLE  $\Rightarrow$ 

(a) thom(M,-):7y→& EXACT

(b) thom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>¢0</sup> €~M.

PF/
TAKE A:= End(M),
F:  $M \rightarrow Mod-A(C)$   $N \mapsto Hom(M,N)$ .

THEOREM COSTRIK] MER & G-Mod. THEN

M = Mod- End(M)(C)

AS LEFT G-MODULE CATEGORIES YMEM.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS FULLY FAITHFUL IF N=XDM FOR SOME XEG

( & MULTIFUSION)

(m, D) ∈ C-Mod CLOSED ( Homm(ZDM, N))

≥ Home(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) ≈ X® Hom (M,N) ∀Xe &; M, N∈ m

7y<sup>\*0</sup> SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a) thom(M,-):7y→& EXACT

(b) thom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>\$0</sup> € M.

PF/
TAKE A:= End(M),
F:  $M \longrightarrow Mod-A(C)$   $N \mapsto Hom(M,N)$ .

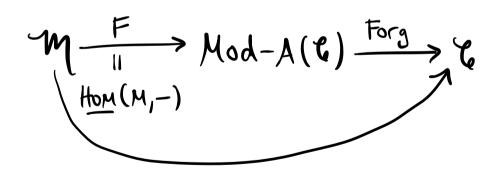
THEOREM COSTRIK] MER & G-Mod. THEN

M = Mod-End(M)(C)

AS LEFT G-MODULE CATEGORIES YMEM.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS FULLY FAITHFUL IF N=XDM FOR SOME XEG



{ & MULTIFUSION }

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ Home(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) ≈ X® Hom (M,N) ∀XE°C; M, N∈m

 $10^{*0}$  SEMISIMPLE  $\Rightarrow$ 

(a)  $\underline{tom}(M,-): \mathcal{M} \longrightarrow \mathcal{E} \in XACT$ 

(b) thom(M,-):7y→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>¢o</sup> e<sup>m</sup>.

PF/
TAKE A:= End(M),

F:  $M \rightarrow Mod-A(C)$   $N \mapsto Hom(M,N)$ .

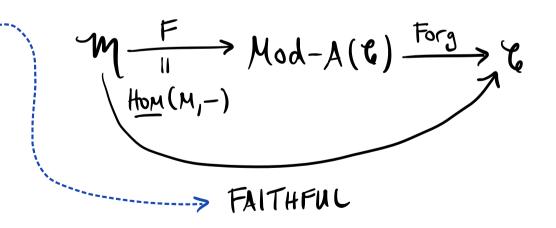
THEOREM COSTRIK] MFG &-Mod. THEN

M = Mod- End(M)(C)

AS LEFT &-MODULE CATEGORIES YMFGM.

WANT: F IS AN EQUIV. OF LEFT &-MOD. CATS.

F IS FULLY FAITHFUL IF N=XDM FOR SOME XEG



& MULTIFUSION

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ Homy(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) = X @ Hom (M,N) ∀X ∈ E; M, N ∈ m

 $10^{*0}$  SEMISIMPLE  $\Rightarrow$ 

(a)  $\underline{tom}(M,-): \mathcal{M} \longrightarrow \mathcal{E} \in XACT$ 

(b) thom(M,-):7y→& FAITHFUL

(c)  $ev_{M,N}$ :  $\underline{Hom}(M,N) \supset M \longrightarrow N \in PIC$   $\forall M^{\neq 0} \in \mathcal{M}.$ 

PF/
TAKE A:= End(M),

F:  $M \longrightarrow Mod-A(C)$   $N \mapsto Hou(M,N)$ .

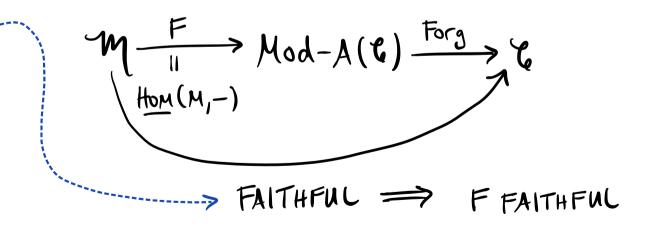
THEOREM COSTRIK] MERCE &-Mod. THEN

M = Mod - End(M)(C)

AS LEFT &-MODULE CATEGORIES YMERCEM.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

FIS FULLY FAITHFUL IF N=XDM FOR SOME XEG



( MULTIFUSION)

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ Homy(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) ≈ X® Hom (M,N) ∀XEC; M, NEM

 $10^{*0}$  SEMISIMPLE  $\Rightarrow$ 

(a) tom(M,-):7y→& EXACT

(b) thom(M,-):7y→& FAITHFUL

(c)  $ev_{M,N}$ :  $\underline{Hom}(M,N) \supset M \longrightarrow N \in PIC$   $\forall M^{\neq 0} \in \mathcal{M}.$ 

PF/
TAKE A:= End(M),

F:  $M \longrightarrow Mod-A(C)$   $N \mapsto Hou(M,N)$ .

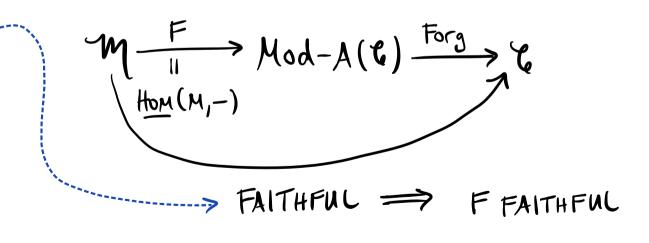
THEOREM COSTRIK] ME C-Mod. THEN

M = Mod - End(M)(C)

AS LEFT C-MODULE CATEGORIES YMEM.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

FIS FULLY FAITHFUL IF N=XDM FOR SOME XEG



& MULTIFUSION

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ Home(Z, Hom(M, N))

 $E_{nd}(M) \in Alg(C)$  $H_{om}(M,N) \in Mod - E_{nd}(M)(C)$ 

Hom (M, XDN) ≈ X® Hom (M,N) ∀XEC; M, NEM

7y<sup>\*0</sup> SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a)  $\underline{tom}(M,-): \mathcal{M} \longrightarrow \mathcal{E} \in XACT$ 

(b) thom(M,-): M→& FAITHFUL

(c)  $ev_{M,N}$ :  $\underline{Hom}(M,N) \supset M \longrightarrow N \in PIC$   $\forall M^{\neq 0} \in \mathcal{M}.$ 

PF/
TAKE A:=  $\underline{End}(M)$ ,
F:  $\underline{M} \longrightarrow Mod-A(C)$   $N \mapsto \underline{Hou}(M,N)$ .

THEOREM COSTRIK] ME C-Mod. THEN

M = Mod - End(M)(C)

AS LEFT C-MODULE CATEGORIES YMEM.

WANT: F IS AN EQUIV. OF LEFT &-MOD. CATS.

FIS FULLY FAITHFUL IF N=XDM FOR SOME XEG)

· FULL IN GENERAL:

& MULTIFUSION

(m, D) ∈ C-Mod CLOSED ( Homm(ZDM, N))

≥ Home(Z, Hom(M, N))

 $E_{Nd}(M) \in Alg(E)$  $H_{OM}(M,N) \in Mod - E_{Nd}(M)(E)$ 

Hom (M, XDN) ≈ X® Hom (M,N) ∀XE°C; M, N∈m

 $10^{*0}$  SEMISIMPLE  $\Rightarrow$ 

(a) <u>Hom</u>(M,-): M→& EXACT

(b) thom(M,-): M→& FAITHFUL

(c) eV<sub>M,N</sub>: Hom(M,N)DM→N EPIC-YM<sup>\$0</sup>€M.

PF/
TAKE A:= End(M),
F:  $M \rightarrow Mod-A(C)$ 

N +> Hom(M,N).

THEOREM COSTRIK] ME C-Mod. THEN

M = Mod-End(M)(C)

AS LEFT C-MODULE CATEGORIES YMEM.

WANT: F IS AN EQUIV. OF LEFT &-MOD. CATS.

F IS FULLY FAITHFUL IF N=XDM FOR SOME XEG

· FULL IN GENERAL: ker(evm, N)

->GET S.E.S.: O→ K-> Hom (M,N) DM -N N-O

& MULTIFUSION

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ Homy(Z, Hom(M, N))

 $E_{nd}(M) \in Alg(e)$  $H_{om}(M,N) \in Mod - E_{nd}(M)(e)$ 

Hom (M, XDN) ≈ X® Hom (M,N) ∀XEE; M,NEM

7y<sup>\*0</sup> SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a) thom(M,-): M→& EXACT

(b) thom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC-YM<sup>\$0</sup>€M.

PF/
TAKE A:= End(M),
F:  $M \longrightarrow Mod-A(C)$   $N \longmapsto HoM(M,N)$ .

THEOREM COSTRIK] ME C-Mod. THEN

M = Mod - End(M)(C)

AS LEFT C-MODULE CATEGORIES YMEM.

WANT: F IS AN EQUIV. OF LEFT &-MOD. CATS.

F IS FULLY FAITHFUL IF N=XDM FOR SOME XEG

· FULL IN GENERAL: ker(evm, N)

GET S.E.S.: O > K -> Hom (M,N) DM -N N -> 0

GET S.E.S.:  $0 \rightarrow F(K) \rightarrow F(Hom(M,N) \triangleright M) \longrightarrow F(N) \longrightarrow 0$ .

& MULTIFUSION

(m, D) ∈ G-Mod CLOSED ( HOMM(ZDM,N) > Home (2, Hom (M, N))

End(M) ∈ Alg(E) Hom (M, N) & Mod-End(M)(C)

Hom (M, XDN) = X & Hom (M, N) YXEC; M, NEM

7y<sup>\*0</sup> SEMISIMPLE \$ INDECOMPOSABLE ⇒

(b) thom(M,-): M→& FAITHFUL

(c) evmin: Hom (M,N)DM -N EPIC

PF/ TAKE A := End(M), F: m -> Mod-A(C) N +> Hom(M,N). THEOREM COSTRIK] MER &-Mod. THEN M ~ Mod- End (M) (C) AS LEFT G-MODULE CATEGORIES YMEGM.

FAITHFULV WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS FULLY FAITHFUL IF N=XDM FOR SOME XEG

· FULL IN GENERAL: ker(evm, N)

(a) thom(M,-): M - & EXACT - > GET S.E.S.: O -> K -> Hom(M,N) DM - N -> 0

GET S.E.S.: 0 > F(K) -> F(HOM(M,N) > M) -> F(N) -> O.

APPLY LEFT EXACT, CONTRAVARIANT FUNCTORS TO GET

0 -> HOMM(N,N') -> HOMM(XDM, N') ----> HOMM(K,N')

& MULTIFUSION

(m, D) ∈ G-Mod CLOSED ( HOMM(ZDM,N) = Home(Z, Hom(M,N))

End(M) ∈ Alg(E) Hom (M,N) & Mod-End(M)(C)

Hom (M, XDN) = X & Hom (M, N) YXEC; M, NEM

M<sup>\*0</sup> SEMISIMPLE \$ INDECOMPOSABLE ⇒

(b) thom(M,-): M→& FAITHFUL

(c) eVMN: HOM (M,N)DM -N EPIC

PF/ TAKE A := End(M), F: M -> Mod-A(C) THEOREM COSTRIK] MER &-Mod. THEN M = Mod- End(M)(C) AS LEFT G-MODULE CATEGORIES YMEGM.

FAITHFULV WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS FULLY FAITHFUL IF N=XDM FOR SOME XEG

· FULL IN GENERAL: ker(evm, N)

(a) thom(M,-): M - & EXACT - RET S.E.S.: O -> K -> Hom(M,N) DM -N N -> 0

GET S.E.S.:  $0 \rightarrow F(K) \rightarrow F(\underbrace{Hou}(H,N) \triangleright M) \longrightarrow F(N) \longrightarrow 0$ .

APPLY LEFT EXACT, CONTRAVARIANT FUNCTORS TO GET

0 -> HOMM(N,N') -> HOMM(XDM, N') ----- HOMM(K,N')

 $N \mapsto \underbrace{\text{Hom}(M,N)}.$  0  $\rightarrow$  How  $\underbrace{(F(N),F(N'))}$   $\rightarrow$  How  $\underbrace{(F(XDM),F(N'))}$   $\rightarrow$  How  $\underbrace{(F(K),F(N'))}$  Mod-A(8)

& MULTIFUSION

(m, D) ∈ G-Mod CLOSED ( HOMM(ZDM,N) > Home (Z, Hom (M, N))

End(M) ∈ Alg(E) Hom (M,N) & Mod-End(M)(C)

Hom (M, XDN) = X & Hom (M, N) YXEC; M, NEM

7y<sup>\*0</sup> SEMISIMPLE \$ INDECOMPOSABLE ⇒

(b) thom(M,-): M→& FAITHFUL

(c) evmin: Hom (M,N)DM -N EPIC

PF/ TAKE A := End(M),  $F: \mathcal{M} \longrightarrow Mod-A(\mathcal{C})$   $N \mapsto \underline{HoM}(M,N).$  THEOREM COSTRIK] MER &-Mod. THEN M = Mod- End(M)(C) AS LEFT G-MODULE CATEGORIES YMEGM.

FAITHFULV WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS FULLY FAITHFUL IF N=XDM FOR SOME XEG

· FULL IN GENERAL: ker(evm, N)

(a) thom(M,-): M - & EXACT - O GET S.E.S.: O -> K -> Hom (M,N) DM -N N -> O

GET S.E.S.:  $0 \rightarrow F(K) \rightarrow F(Hou(M,N) \rightarrow M) \longrightarrow F(N) \longrightarrow 0$ .

APPLY LEFT EXACT, CONTRAVARIANT FUNCTORS TO GET

$$0 \longrightarrow \text{Hom}_{\mathfrak{N}}(N,N') \longrightarrow \text{Hom}_{\mathfrak{N}}(X \triangleright M,N') \longrightarrow \text{Hom}_{\mathfrak{N}}(K,N')$$

$$\downarrow^{F_{0,0}} 2 \qquad \downarrow^{F_{N,N'}} \qquad 2 \qquad \downarrow^{F_{K,N'}} \qquad$$

& MULTIFUSION

(m, D) ∈ &-Mod CLOSED ( HOMM(ZDM,N) = Home(Z, Hom(M,N))

End(M) ∈ Alg(E) Hom (M,N) & Mod-End(M)(C)

Hom (M, XDN) = X & Hom (M, N) YXEC; M, NEM

Ty\* O SEMISIMPLE \* INDECOMPOSABLE >

(a)  $\underline{\text{tom}}(M_j-): M \longrightarrow C \in XACT$ 

PF/ TAKE A := End(M),  $F: \mathcal{M} \longrightarrow Mod-A(\mathcal{C})$   $N \mapsto \underline{Hou}(M,N).$  THEOREM COSTRIK] MER &-Mod. THEN M = Mod- End(M)(C)

AS LEFT G-MODULE CATEGORIES YME'M.

FAITHFULV WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS FULLY FAITHFUL IF N=XDM FOR SOME XE'S

· FULL IN GENERAL:

(b) 
$$\underline{tom}(M,-): M \rightarrow C$$
 FAITHFUL

(c)  $\underline{ev_{M,N}}: \underline{tom}(M,N) \supset M \rightarrow N$  EPIC

 $\underline{VM^{\sharp \circ}} \in M$ .

 $\underline{VM$ 

& MULTIFUSION

(m, D) ∈ &-Mod CLOSED ( HOMM(ZDM,N) = Home(Z, Hom(M,N))

End(M) ∈ Alg(E) Hom (M,N) & Mod-End(M)(C)

Hom (M, XDN) = X & Hom (M, N) YXEC; M, NEM

7y<sup>\*0</sup> SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a)  $t\underline{tom}(M,-): M \longrightarrow C \in XACT$ 

PF/ TAKE A := End(M),  $F: \mathcal{M} \longrightarrow Mod-A(C)$ N -> Hom(M,N). THEOREM COSTRIK] MER &-Mod. THEN M ~ Mod- End (M) (C) AS LEFT G-MODULE CATEGORIES YME'M.

FAITHFULV WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS FULLY FAITHFUL IF N=XDM FOR SOME XE'S

· FULL IN GENERAL:

T FULLY FAITHFUL IN THIS CASE

(b) 
$$\underline{tom}(M,-): M \rightarrow C$$
 FAITHFUL

(c)  $\underline{ev_{M,N}}: \underline{tom}(M,N) \supset M \rightarrow N$  EPIC

 $\underline{\forall M^{\sharp 0} \in M}.$ 

PF/

 $\underbrace{bom_{M,N}: \underline{bom_{M,N}} \supset M \rightarrow N}_{Mod-A(E)} = \underbrace{bom_{M,N}}_{Mod-A(E)} = \underbrace{bom_{M,N}}_{Mod-$ 

& MULTIFUSION

(m, D) ∈ G-Mod CLOSED ( HOMM(ZDM,N) = Home(Z, Hom(M,N))

End(M) ∈ Alg(E) Hom (M,N) & Mod-End(M)(C)

Hom (M, XDN) = X & Hom (M, N) YXEC; M, NEM

7y<sup>\*0</sup> SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a)  $t\underline{tom}(M,-): M \longrightarrow C \in XACT$ 

PF/ TAKE A := End(M),  $F: \mathcal{M} \longrightarrow Mod-A(C)$ N -> Hom(M,N). THEOREM COSTRIK] MER &-Mod. THEN M = Mod- End(M)(C) AS LEFT G-MODULE CATEGORIES YME'M.

FAITHFULL WANT: FISAN EQUIV. OF LEFT &-MOD. CATS.

F IS FULLY FAITHFUL IF N=XDM FOR SOME XE'S

· FULL IN GENERAL:

T FULLY FAITHFUL IN THIS CASE

(b) 
$$\underline{tom}(M,-): M \longrightarrow C$$
 FAITHFUL

(c)  $\underline{ev_{M,N}}: \underline{tom}(M,N) \supset M \longrightarrow N$  EPIC

 $\underline{\forall M^{\sharp 0} \in M}.$ 

PF/

 $\underline{O} \longrightarrow \underline{Hom}(N,N') \longrightarrow \underline{Hom}(X \supset M, N') \longrightarrow \underline{Hom}(K,N')$ 
 $\underline{O} \longrightarrow \underline{Hom}(N,N') \longrightarrow \underline{Hom}(X \supset M, N') \longrightarrow \underline{Hom}(K,N')$ 
 $\underline{\forall M^{\sharp 0} \in M}.$ 
 $\underline{O} \longrightarrow \underline{Hom}(N,N') \longrightarrow \underline{Hom}(X \supset M, N') \longrightarrow \underline{Hom}(K,N')$ 
 $\underline{\forall M^{\sharp 0} \in M}.$ 
 $\underline{O} \longrightarrow \underline{Hom}(N,N') \longrightarrow \underline{Hom}(X \supset M, N') \longrightarrow \underline{Hom}(K,N')$ 
 $\underline{\forall M^{\sharp 0} \in M}.$ 
 $\underline{O} \longrightarrow \underline{Hom}(N,N') \longrightarrow \underline{Hom}(X \supset M, N') \longrightarrow \underline{Hom}(K,N')$ 
 $\underline{\forall M^{\sharp 0} \in M}.$ 
 $\underline{O} \longrightarrow \underline{Hom}(F(N),F(N')) \longrightarrow \underline{Hom}(F(X \supset M),F(N')) \longrightarrow \underline{Hom}(F(K),F(N'))$ 
 $\underline{Mod-A(\mathfrak{C})}$ 

& MULTIFUSION

(m, D) ∈ G-Mod CLOSED ( HOMM(ZDM,N) = Home(Z, Hom(M,N))

End(M) ∈ Alg(E) Hom (M,N) & Mod-End(M)(C)

Hom (M, XDN) = X @ Hom (M, N) YXEC; M, NEM

7y<sup>\*0</sup> SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a)  $t\underline{tom}(M,-): M \longrightarrow C \in XACT$ 

PF/ TAKE A := End(M),  $F: \mathcal{M} \longrightarrow Mod-A(e)$   $N \mapsto \underline{HoM}(M,N).$  THEOREM COSTRIK] MER &-Mod. THEN M = Mod- End(M)(C) AS LEFT G-MODULE CATEGORIES YMEGM.

FAITHFULL WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS FULLY FAITHFUL IF N=XDM FOR SOME XEG

· FULL IN GENERAL:

TFULLY FAITHFUL IN THIS CASE

(b) 
$$\underbrace{\text{Hom}(M,-):M} \rightarrow \text{C} \ \text{FAITHFUL}$$
(c)  $\underbrace{\text{ev}_{M,N}: \underbrace{\text{Hom}(M,N)DM} \rightarrow N}_{\text{MM}} \in \mathcal{M}$ .

PF/

TAKE  $A := \underbrace{\text{End}(M)}_{\text{Mod-A(e)}}$ 

Tour-(EMMA)

Tour-(EMMA)

Tour-(EMMA)

Tour-(EMMA)

Tour-(EMMA)

Tour-(EMMA)

& MULTIFUSION

(m, D) ∈ &-Mod CLOSED ( HOMM(ZDM,N) > Home (2, Hom (M, N))

End(M) ∈ Alg(E) Hom (M,N) & Mod-End(M)(C)

Hom (M, XDN) = X & Hom (M, N) YXEC; M, NEM

7y<sup>\*0</sup> SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a) tom(M,-): m→ & EXACT

PF/ TAKE A := End(M), F: M -> Mod-A(C) N -> Hom(M,N).

THEOREM COSTRIK] MERC-Mod. THEN M ~ Mod- End (M) (C) AS LEFT G-MODULE CATEGORIES YME'M. FAITHFULL WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS. F IS FULLY FAITHFUL IF N=XDM FOR SOME XEG

PULLY FAITHFUL . FULL IN GENERAL: IN THIS CASE (b)  $\underline{\text{Hom}}(M,-): M \longrightarrow \mathbb{C}$  FAITHFUL

(c)  $\underline{\text{ev}}_{N,N}: \underline{\text{Hom}}(M,N)DM \longrightarrow N$  EPIC  $\underline{\text{Fo,o}}$   $\underline{\text{ZEPIC}}$   $\underline{\text{Fn,n'}}$   $\underline{\text{Fxom,n'}}$   $\underline{\text{Amic}}$   $\underline{\text{Fxom,n'}}$   $\underline{\text{Fxom,n'}}$   $\underline{\text{Fxom,n'}}$   $\underline{\text{Constant}}$ 0 > Hom (F(N), F(N')) -> Hom (F(XDM), F(N')) -> Hom (F(K), F(N'))

Mod-A(E)

Mod-A(E) (: FN,N' IS EPIC YH,N'

( & MULTIFUSION)

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ Home(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) ≈ X® Hom (M,N) ∀XEE; M,NEM

Ty\*0 SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a)  $+ \underline{tom}(M, -): M \longrightarrow C \in XACT$ 

(b) thom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC YM<sup>\$0</sup>€M.

PF/
TAKE A:= End(M),

F:  $M \longrightarrow Mod-A(C)$   $N \mapsto Hom(M,N)$ .

THEOREM COSTRIK] Mto C-Mod. THEN

M ~ Mod- End(M)(C)

AS LEFT C-MODULE CATEGORIES YME M.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS ESSENTIALLY SURJECTIVE

TAKE ZEMOD-A(C).

& MULTIFUSION }

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ HOMz(Z, Hom(M, N))

 $E_{Nd}(M) \in Alg(E)$  $H_{OM}(M,N) \in Mod - E_{Nd}(M)(E)$ 

Hom (M, XDN) = X & Hom (M,N) VXEC; M, NEM

 $10^{*0}$  SEMISIMPLE  $\Rightarrow$ 

(a)  $\underline{t_{lom}}(M,-): \mathcal{M} \longrightarrow \mathcal{C} \in XACT$ 

(b) thom(M,-): M→& FAITHFUL

(c) eV<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>\$0</sup> € M.

PF/
TAKE A:= End(M),
F:  $M \longrightarrow Mod-A(C)$   $N \mapsto Hom(M,N)$ .

THEOREM COSTRIK] Mto C-Mod. THEN

M = Mod - End(M)(C)

AS LEFT C-MODULE CATEGORIES YME M.

WANT: FISAN EQUIV. OF LEFT &-MOD. CATS.

F IS ESSENTIALLY SURJECTIVE

TAKE ZEMOD-A(C).

CONSIDER ADJUNCTION:

HOM Mod-A(E) (ZOA, Z) + HOME (Z, Forg (Z))

[EPI: ZOA -> Z] - idz

& MULTIFUSION }

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ Home(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) = X & Hom (M,N) VXEC; M, NEM

 $10^{*0}$  SEMISIMPLE  $\Rightarrow$ 

(a)  $\underline{tom}(M,-): \mathcal{M} \longrightarrow \mathcal{C} \in XACT$ 

(b) thom(M,-):7y→& FAITHFUL

(c) eV<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>\$0</sup> € M.

PF/
TAKE A:=  $\underline{End}(M)$ ,
F:  $\underline{M} \longrightarrow \underline{Mod} - \underline{A}(C)$   $\underline{N} \mapsto \underline{Hou}(M,N)$ .

THEOREM COSTRIK] ME C-Mod. THEN

M = Mod - End(M)(C)

AS LEFT C-MODULE CATEGORIES YMEM.

WANT: FISAN EQUIV. OF LEFT &-MOD. CATS.

F IS ESSENTIALLY SURJECTIVE

TAKE ZEMOD-A(C).

CONSIDER ADJUNCTION:

HOM Mod-A(E) (Z&A,Z) -> Home (Z, Forg (Z))

GET:

 $\begin{array}{c}
0 \\
X = \ker(\mathfrak{F}^{-1}(id_{\mathfrak{k}})) \\
2 \otimes A \longrightarrow 2 \longrightarrow 0
\end{array}$ 

& MULTIFUSION }

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ Home(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) = X & Hom (M,N) VXEC; M, NEM

 $10^{*0}$  SEMISIMPLE  $\Rightarrow$ 

(a)  $\underline{tom}(M,-): M \longrightarrow C \in XACT$ 

(b) thom(M,-):7y→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>\$0</sup> € M.

PF/
TAKE A:= End(M),
F: $M \longrightarrow Mod-A(C)$   $N \mapsto Hom(M,N)$ .

THEOREM COSTRIK] MERC-Mod. THEN

M = Mod-End(M)(C)

AS LEFT C-MODULE CATEGORIES YMERCY.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS ESSENTIALLY SURJECTIVE

TAKE ZEMON-A(C).

CONSIDER ADJUNCTION:

HOM Mod-A(e) (Z&A,Z) & Home (Z, Forg (Z))

GET:  $\begin{cases}
s^{-1}(id_{x}) \nearrow X = ker(s^{-1}(id_{t})) \\
X \otimes A \longrightarrow Z \otimes A \longrightarrow Z
\end{cases}$ 

( & MULTIFUSION)

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ Homy(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) = X & Hom (M,N) VXEC; M, NEM

 $10^{*0}$  SEMISIMPLE  $\Rightarrow$ 

(a)  $\underline{thom}(M,-): \mathcal{M} \longrightarrow \mathcal{C} \in XACT$ 

(b) thom(M,-):7y→& FAITHFUL

(c) eVM,N: HOM (M,N)DM → N EPIC YM+0 € M.

PF/
TAKE A:= End(M),
F: $M \longrightarrow Mod-A(C)$   $N \mapsto Hom(M,N)$ .

THEOREM COSTRIK] MERCE &-Mod. THEN

M = Mod = End(M)(C)

AS LEFT &-MODULE CATEGORIES YMERCEM.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS ESSENTIALLY SURJECTIVE

TAKE ZEMOD-A(C).

CONSIDER ADJUNCTION:

Hom Mod-A(e) ( $Z\otimes A, Z$ )  $\xrightarrow{g}$  Hom  $_{g}(Z, Forg(Z))$  $\begin{bmatrix} EPI: Z\otimes A \longrightarrow Z \end{bmatrix} \longleftrightarrow id_{Z}$ 

GET:  $\begin{array}{c}
S^{-1}(id_{x}) \neq X = \ker(S^{-1}(id_{t})) \\
X \otimes A \xrightarrow{CALLTHIS} \not \otimes X \longrightarrow Z \longrightarrow C
\end{array}$ 

& MULTIFUSION

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ Homy(Z, Hom(M, N))

 $E_{nd}(M) \in Alg(e)$  $H_{om}(M,N) \in Mod - E_{nd}(M)(e)$ 

Hom (M, XDN) ≈ X® Hom (M,N) ∀XEE; M,NEM

 $10^{*0}$  SEMISIMPLE  $\Rightarrow$ 

(a)  $\underline{tom}(M,-): M \longrightarrow C \in XACT$ 

(b) thom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>¢0</sup>∈M.

PF/
TAKE A:= End(M),
F: $M \longrightarrow Mod-A(C)$   $N \mapsto Hon(M,N)$ .

THEOREM COSTRIK] ME C-Mod. THEN

M = Mod - End(M)(C)

AS LEFT C-MODULE CATEGORIES YME.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS ESSENTIALLY SURJECTIVE

TAKE ZEMOD-A(C).

HOM: HOM MOD-A(E) (XOL, ZOL)

GET:  $\begin{array}{c}
S^{-1}(id_{x}) \nearrow X = \ker(S^{-1}(id_{t})) \\
X \otimes A \longrightarrow Z \otimes A \longrightarrow Z
\end{array}$ CALLTHIS  $\not \otimes$ 

& MULTIFUSION

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ Home(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) ≈ X® Hom (M,N).

YX&C; M, N&M

Ty\*0 SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a)  $+ \underline{tom}(M, -): M \longrightarrow C \in XACT$ 

(b) ttom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>¢0</sup>∈M.

PF/
TAKE A:= End(M),
F:  $M \longrightarrow Mod-A(C)$   $N \mapsto Hom(M,N)$ .

THEOREM COSTRIK] M = C-Mod. THEN

M = Mod-End(M)(C)

AS LEFT C-MODULE CATEGORIES YMEM.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS ESSENTIALLY SURJECTIVE

TAKE ZEMOD-A(C).

How Mod-A(E) (XOA, ZOA) = How mod-A(E) (F(XDM), F(ZDM))

GET:  $\begin{cases}
S^{-1}(id_{x}) & X = \ker(S^{-1}(id_{x})) \\
X \otimes A & \longrightarrow Z & \longrightarrow O
\end{cases}$ CALLTHIS  $\not S$ 

& MULTIFUSION }

(m, D) ∈ C-Mod CLOSED ( HOMM(ZDM, N))

≥ HOMZ(Z, HOM(M, N))

End(M) ∈ Alg(C) Hom(M,N) ∈ Mod-End(M)(C)

Hom (M, XDN) = X& Hom (M,N).

Ty\*0 SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a)  $+ \underline{tom}(M, -): M \longrightarrow C \in XACT$ 

(b) thom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>¢0</sup>∈M.

PF/
TAKE A:= End(M),
F:  $M \longrightarrow Mod-A(C)$   $N \mapsto Hom(M,N)$ .

THEOREM COSTRIK] M = C-Mod. THEN

M = Mod-End(M)(C)

AS LEFT C-MODULE CATEGORIES YMEM.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS ESSENTIALLY SURJECTIVE

TAKE ZEMOD-A(C).

MOW:

HOM

Mod-A(E)

Wood-A(E)

HOM

Mod-A(E)

HOM

Mod-A(E)

FEULY FAITHFUL

 $\begin{array}{c} \begin{array}{c} & & \\ & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ &$ 

& MULTIFUSION

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ Home(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) ≈ X® Hom (M,N).

YX&C; M, N&M

M<sup>\*0</sup> SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a)  $+ \underline{tom}(M, -): M \longrightarrow C \in XACT$ 

(b) thom(M,-):7y→& FAITHFUL

(c) eV<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>¢o</sup>e<sup>M</sup>.

PF/
TAKE A:= End(M),
F:  $M \longrightarrow Mod-A(C)$   $N \longmapsto Hom(M,N)$ .

THEOREM COSTRIK] MER & C-Mod. THEN

M = Mod-End(M)(C)

AS LEFT C-MODULE CATEGORIES YMER.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS ESSENTIALLY SURJECTIVE

TAKE ZEMOD-A(C).

HOM:

Hom Mod-A(E)

We have the mod-A(E) (F(XDM), F(ZDM))

Homm (XDM, ZDM)

CALLTHIS &

FEULLY FAITHFUL]

Y=ker(3-1(idz))

 $\begin{array}{c} S^{-1}(id_{x}) \xrightarrow{X} X = \ker(S^{-1}(id_{t})) \\ X \otimes A \xrightarrow{CALLTHIS} \not A \xrightarrow{CALLTHIS} \not \otimes A \xrightarrow{CALLTHIS}$ 

{ & MULTIFUSION }

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ Home(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) = X @ Hom (M,N) ∀X e &; M, N ∈ m

7y<sup>\*0</sup> SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a) tom(M,-):7y→& EXACT

(b) thom(M,-):7y→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>¢0</sup> ∈ M.

PF/
TAKE A:= End(M),
F:  $M \longrightarrow Mod-A(C)$   $N \mapsto Hom(M,N)$ .

THEOREM COSTRIK] ME C-Mod. THEN

M = Mod - End(M)(C)

AS LEFT C-MODULE CATEGORIES YMEM.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS ESSENTIALLY SURJECTIVE

TAKE ZEMOD-A(C).

Now:

HOM Mod-A(E) (XOA, ZOA) = HOM Mod-A(E) (F(XDM), F(ZDM))

= HOMM (XDM, ZDM)

CALLTHIS Ø'

FINALLY:  $F(\operatorname{coker} \emptyset') \cong \operatorname{coker} (F(\emptyset')) \cong Z.$ 

& MULTIFUSION

(m, D) ∈ C-Mod CLOSED ( HOMm(ZDM, N))

≥ Home(Z, Hom(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$  $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) = X @ Hom (M,N) ∀X ∈ E; M, N ∈ m

 $10^{*0}$  SEMISIMPLE  $\Rightarrow$ 

(a) <u>Hom</u>(M,-): M→& EXACT \_\_

(b) thom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>¢0</sup> € M.

PF/
TAKE A:= End(M),
F:  $M \rightarrow Mod-A(C)$ 

N → Hom(M,N).

THEOREM COSTRIK] M = C-Mod. THEN

M = Mod-End(M)(C)

AS LEFT G-MODULE CATEGORIES YME'M.

WANT: FIS AN EQUIV. OF LEFT &-MOD. CATS.

F IS ESSENTIALLY SURJECTIVE

TAKE Ze Mod-A(C).

Now:

HOM Mod-A(E) (XOA, ZOA) = HOM Mod-A(E) (F(XDM), F(ZDM))

= HOMM (XDM, ZDM)

CALLTHIS Ø'

FINALLY:

 $F(\operatorname{coker} \emptyset') \stackrel{\sim}{=} \operatorname{coker} (F(\emptyset')) \stackrel{\sim}{=} Z.$ 

& MULTIFUSION

(m, D) ∈ C-Mod CLOSED ( HOMM(ZDM, N))

≥ HOMZ(Z, HOM(M, N))

 $\underline{\mathsf{End}}(\mathsf{M}) \in \mathsf{Alg}(\mathsf{C})$   $\underline{\mathsf{Hom}}(\mathsf{M},\mathsf{N}) \in \mathsf{Mod} - \underline{\mathsf{End}}(\mathsf{M})(\mathsf{C})$ 

Hom (M, XDN) = X & Hom (M,N) VXEC; M, NEM

Ty\*0 SEMISIMPLE \$ INDECOMPOSABLE ⇒

(a) <u>Hom</u>(M,-): M→& EXACT \_\_

(b) thom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>\$0</sup> € M.

PF/
TAKE A:= End(M),
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THEOREM COSTRIK] MER & C-Mod. THEN

M = Mod-End(M)(C)

AS LEFT C-MODULE CATEGORIES YMER.

WANT: FISAN EQUIV. OF LEFT &-MOD. CATS.

F IS ESSENTIALLY SURJECTIVE

TAKE ZEMOD-A(C).

Now:

& MULTIFUSION }

(m, D) ∈ C-Mod CLOSED ( HOMM(ZDM, N))

≥ HOMZ(Z, HOM(M, N))

 $\underline{\operatorname{End}}(M) \in \operatorname{Alg}(\mathcal{C})$   $\underline{\operatorname{Hom}}(M,N) \in \operatorname{Mod} - \underline{\operatorname{End}}(M)(\mathcal{C})$ 

Hom (M, XDN) = X & Hom (M,N) VXEC; M, NEM

 $10^{*0}$  SEMISIMPLE  $\Rightarrow$ 

(a) <u>Hom</u>(M,-): M→& EXACT \_\_

(b) thom(M,-): M→& FAITHFUL

(c) ev<sub>M,N</sub>: Hom(M,N)DM→N EPIC ∀M<sup>\$0</sup> € M.

PF/
TAKE A:= End(M),
F:  $M \longrightarrow Mod-A(C)$   $N \longmapsto Hom(M,N)$ .

THEOREM COSTRIK] MER & C-Mod. THEN

M = Mod-End(M)(C)

AS LEFT C-MODULE CATEGORIES YMER.

WANT: FISAN EQUIV. OF LEFT &-MOD. CATS.

F IS ESSENTIALLY SURJECTIVE

TAKE ZEMOD-A(C).

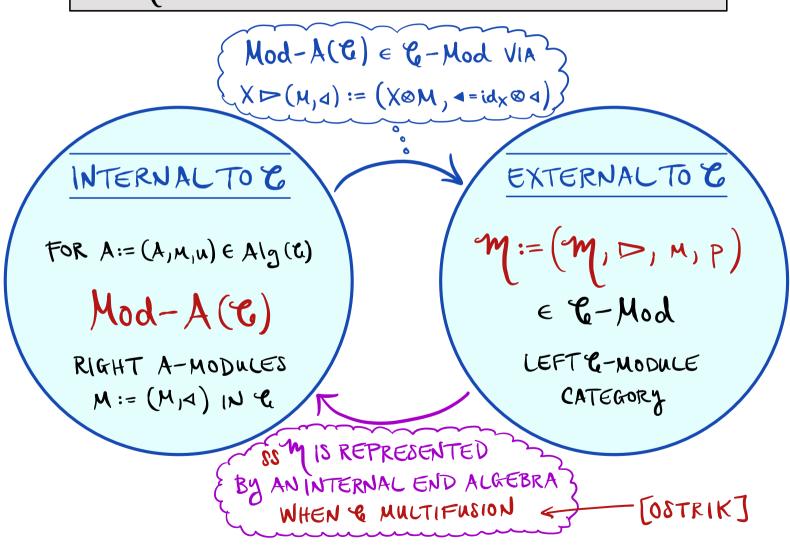
NOW:

How Mod-A(E)  $(X \otimes A, Z \otimes A) \cong Hom_{Mod-A(E)}$  (F(X D M), F(Z D M)) (X D M, Z D M) (X D M, Z D M)

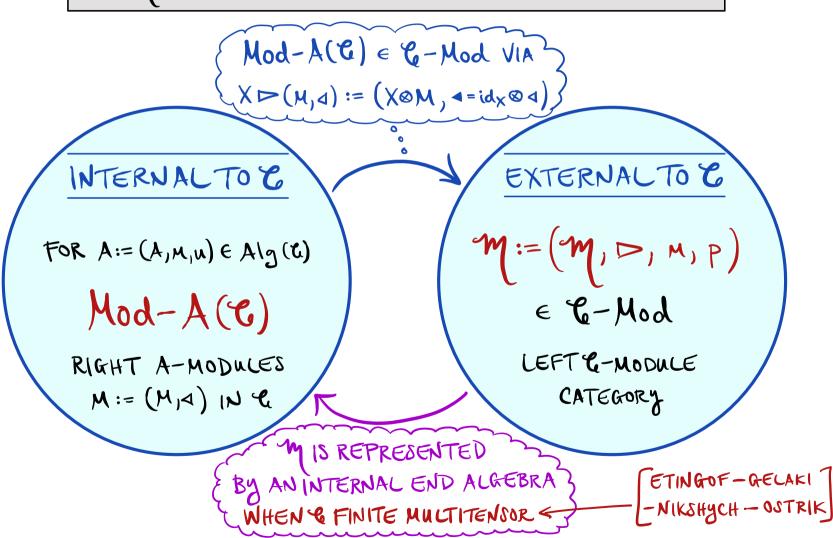
 $F(\operatorname{coker} \emptyset') \stackrel{=}{=} \operatorname{coker} (F(\emptyset')) \stackrel{=}{=} Z.$ 

: FIS ESSENTIALLY SURJECTIVE

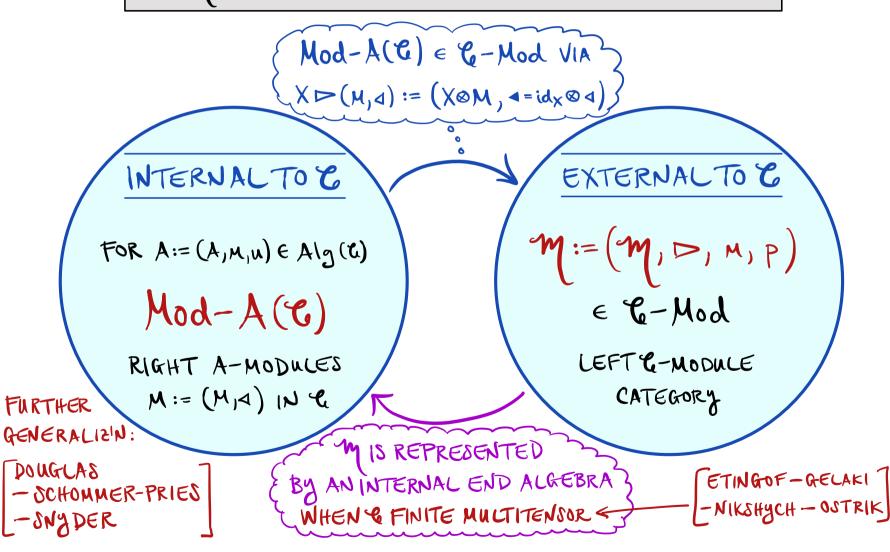
Mee-Mod is represented by Aealg(c) IF  $M \simeq Mod-A(c)$  as left e-module categories.



Mee-Mod is represented by Aealg(c) IF  $M \simeq Mod-A(c)$  as left e-module categories.



MeG-Mod is represented by Aealg(G) IF M = Mod-A(G) As left G-Module Categories.



MATH 466/566 SPRING 2024

CHELSEA WALTON RICE U.

NEXT TIME
NICE PROPERTIES
OF ALGEBRAS
IN MONOIDAL CATEGORIES

LECTURE #21

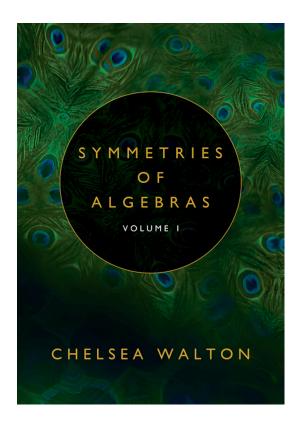
# TOPICS:

I. INTERNAL END ALGEBRAS (§4.8.1)

II. OSTRIK'S THEOREM (54.8.2)

# Enjoy this lecture? You'll enjoy the textbook!

# C. Walton's "Symmetries of Algebras, Volume 1" (2024)



**Available for purchase at:** 

619 Wreath (at a discount)

https://www.619wreath.com/

Also on Amazon & Google Play

<u>Lecture #21 keywords</u>: internal End algebra, internal Hom module, module category represented by an algebra, Ostrik's Theorem