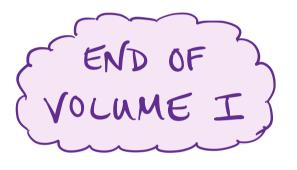
MATH 466/566 SPRING 2024

CHELSEA WALTON RICE U.

LASTTIME

- · INTERNAL END ALGEBRAS
- · OSTRIK'S THEOREM

LECTURE #22



TOPICS:

I. NOTIONS OF SAMENESS

(RECAP)

II. PROPERTIES OF ALGEBRAS IN &

(84.9)

II. BIMODULES AND BEYOND

(84.10.1)

II. CATEGORICAL MORITA EQUIVALENCE

(54.10.2)

FOR IK-ALGEBRAS

A = B

EQUALITY

OF ALGEBRAS

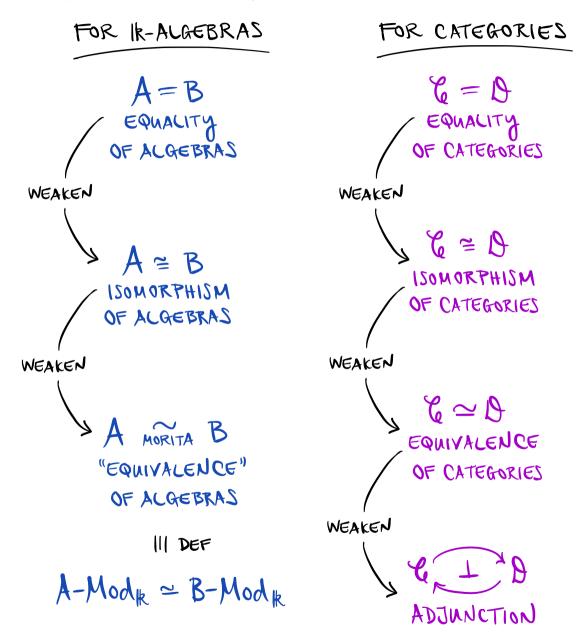
WEAKEN

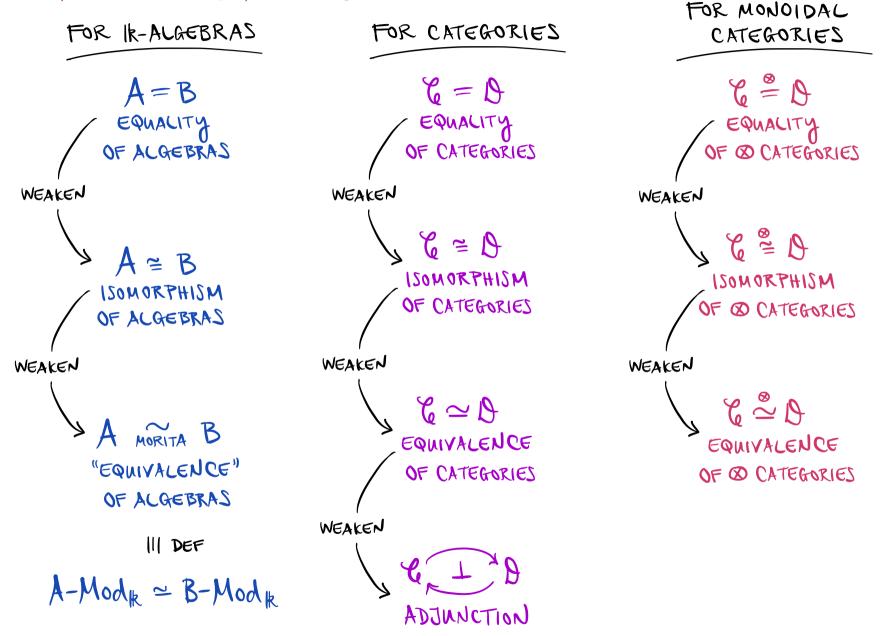
A = B

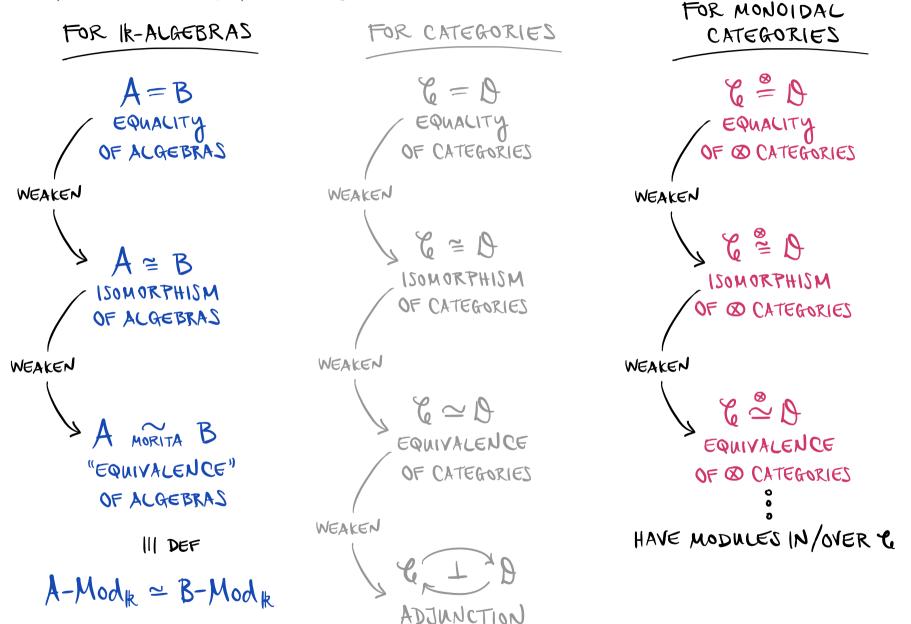
ISOMORPHISM

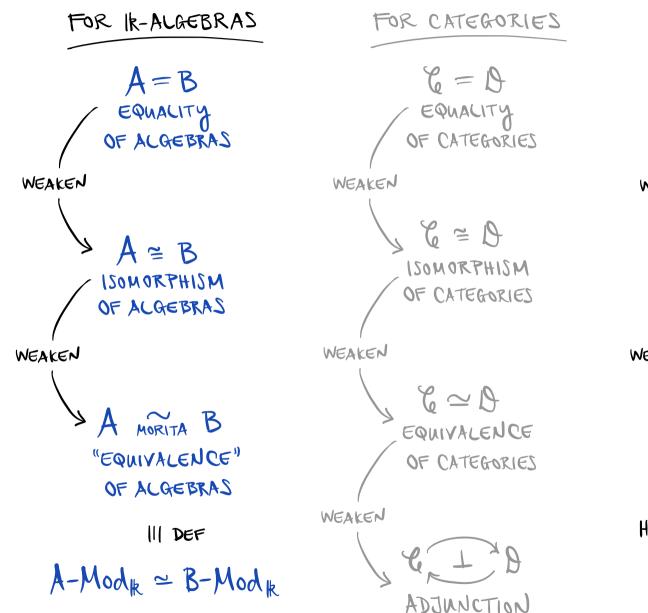
OF ALGEBRAS

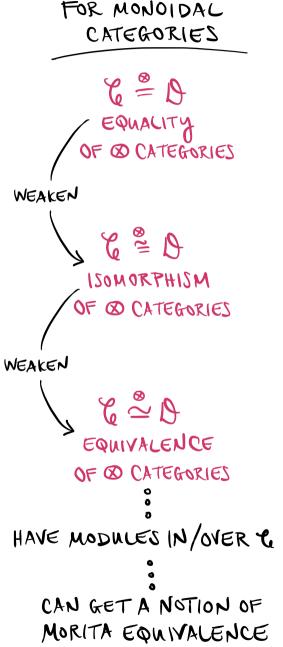
FOR IK-ALGEBRAS WEAKEN WEAKEN "EQUIVALENCE" OF ALGEBRAS III DEF A-Modk = B-Modk





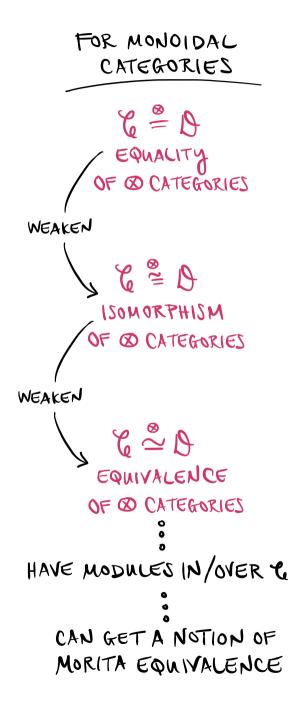






FOR IK-ALGEBRAS

A MORITA B
"EQUIVALENCE"
OF ALGEBRAS
III DEF
A-Modik = B-Modik



FOR IK-ALGEBRAS

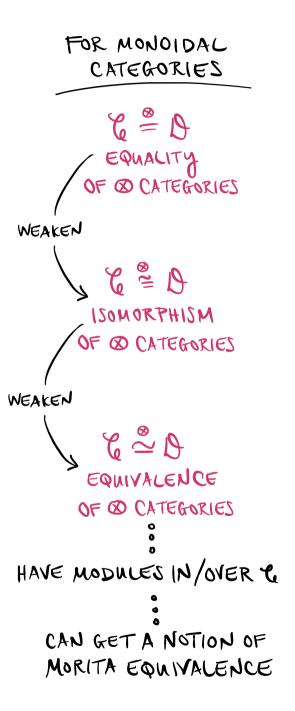
A MORITA B
"EQUIVALENCE"
OF ALGEBRAS
III DEF
A-Modr = B-Modr

MORITAISTHM 1

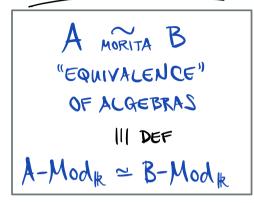
JBIMODULES/R: APB, BQA

BROWN A-BiMODIK

BROWN B-BIMODIK



FOR IK-ALGEBRAS



MORITAISTHM 1

JBIMODULES/R: APB, BQA

J. POBQ = Arg IN A-Bimodik

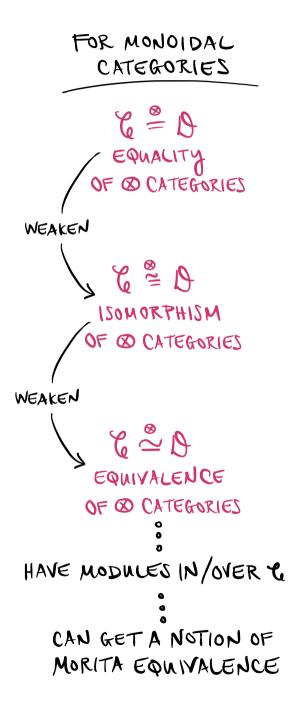
QOBAP = Brg IN B-Bimodik

EXER. 2.36

JFIN. GEN. PROJ. M ∈ A-Modik
WITH HOMA-Modik

.>. BOP = Enda-Mod (M)

AS IR-ALGEBRAS



FOR IK-ALGEBRAS

A MORITA B
"EQUIVALENCE"
OF ALGEBRAS
III DEF
A-Modk = B-Modk

MORITAISTHM 1

JBIMODULES/IR: APB, BQA

J. POBQ = Arg IN A-Bimodik

QOAP = Brg IN B-Bimodik

EXER. 2.36

JFIN. GEN. PROJ. MEA-MODIR
WITH HOMA-MODIR

J. BOP = ENDA-MODIC (M)

AS IR-ALGEBRAS

FOR MONOIDAL WANT CATEGORIES III DEF C-Mod = D-Mod WEAKEN ISOMORPHISM OF @ CATEGORIES WEAKEN EQUIVALENCE OF @ CATEGORIES HAVE MODILES IN/OVER & CAN GET A NOTION OF

MORITA EQUIVALENCE

FOR IK-ALGEBRAS

A MORITA B
"EQUIVALENCE"
OF ALGEBRAS
III DEF
A-Modk ~ B-Modk

MORITAIS THM

JBIMODINCES/R: APB, BQA

J. POBQ = Arg IN A-Bimodik

QOAP = Brog IN B-Bimodik

EXER. 2.36

J FIN. GEN. PROJ. M ∈ A-Modik
WITH HOMA-Modik

AS IR-ALGEBRAS

WANT

C ~ D

MORITA

III DEF

C-Mod ~ D-Mod

FOR MONOIDAL CATEGORIES

C-Mod

OBJECTS: LEFT &-MODULE CATEGS.

MORPHISMS: &-MODULE FUNCTORS

FOR IK-ALGEBRAS

A MORITA B
"EQUIVALENCE"
OF ALGEBRAS
III DEF
A-Modk = B-Modk

MORITAISTHM 1

JBIMODULES/R: APB, BQA

J. POBQ = Arg IN A-Bimodik

QOAP = Brg IN B-Bimodik

EXER. 2.36

J FIN. GEN. PROJ. M ∈ A-Modik
WITH HOMA-Modik

→ BOP = End A-Mod (M)

AS IR-ALGEBRAS

WANT

C ~ D
MORITA

III DEF

C-Mod ~ D-Mod

FOR MONOIDAL CATEGORIES

C-Mod

OBJECTS:

LEFT &-MODULE CATEGS.

MORPHISMS:

&-MODULE FUNCTORS

THIS EQUIVALENCE
IS BEST HANDLED
IN A

"2-CATEGORICAL" SETTING

FOR IR-ALGEBRAS

A MORITA B
"EQUIVALENCE"
OF ALGEBRAS
III DEF
A-Modik = B-Modik

MORITAISTHM 1

JBIMODULES/K: APB, BQA

3. POBQ = Arg IN A-Bimodik

QOBAP = Brg IN B-Bimodik

EXER. 2.36

JFIN. GEN. PROJ. MEA-MODIR
WITH HOMA-MODIR

3. BOP = ENDA-MODIC (M)
AS IR-ALGEBRAS

WANT

C ~ D

MORITA

III DEF

C-Mod ~ D-Mod

FOR MONOIDAL CATEGORIES

C-Mod

OBJECTS:

LEFT C-MODULE CATEGS.

MORPHISMS:

&-MODULE FUNCTORS

CAN MIMIC VIA
BIMODULE CATEGORIES

Jelo, DQE

+ CONDITIONS

THIS EQUIVALENCE
IS BEST HANDLED
IN A

"2-CATEGORICAL" SETTING

FOR IR-ALGEBRAS

A MORITA B
"EQUIVALENCE"
OF ALGEBRAS
III DEF
A-Modk = B-Modk

MORITAIS THM

JBIMODINCES/R: APB, BQA

J. POBQ = Arg IN A-Bimodik

QOBAP = Brg IN B-Bimodik

EXER. 2.36

JFIN. GEN. PROJ. M ∈ A-MODIR
WITH HOMA-MODIR

→ BOP = ENDA-MODIC (M)

AS IR-ALGEBRAS

WANT

C ~ D

MORITA

III DEF

C-Mod ~ D-Mod

FOR MONOIDAL CATEGORIES

C-Mod

OBJECTS: LEFT &-MODULE CATEGS.

MORPHISMS: &-MODULE FUNCTORS

CAN MIMIC VIA

BIMODULE CATEGORIES

J & Pa, a Q &

+ CONDITIONS

CAN MIMIC VIA CERTAIN
ENDOFUNCTOR CATEGORIES
ENDOFUNCTOR CATEGORIES

THIS EQUIVALENCE
IS BEST HANDLED
IN A

"2-CATEGORICAL" SETTING

FOR IK-ALGEBRAS

A MORITA B
"EQUIVALENCE"
OF ALGEBRAS
III DEF
A-Modk = B-Modk

MORITAISTHM 1

JBIMODULES/R: APB, BQA

J. POBQ = Arg IN A-Bimodik

QOAP = Brg IN B-Bimodik

EXER. 2.36

JFIN. GEN. PROJ. MEA-MODIR
WITH HOMA-MODIR

J. BOP = ENDA-MODIC (M)

AS IR-ALGEBRAS

WANT

C-Mod = D-Mod

VIA ALGEBRAS IN ⊗ CATS.

CAN USE OSTRIK'S THEOREM

ON MODULE CATEGORIES OVER FUSION CATEGS.

CAN MIMIC VIA

BIMODULE CATEGORIES

J & Pa, a Q &

+ CONDITIONS

CAN MIMIC VIA CERTAIN
ENDOFUNCTOR CATEGORIES
ENDOFUNCTOR CATEGORIES

THIS EQUIVALENCE
IS BEST HANDLED
IN A

"2-CATEGORICAL" SETTING

FOR IK-ALGEBRAS

A MORITA B
"EQUIVALENCE"

OF ALGEBRAS

III DEF

A-Modk = B-Modk

WANT

 $C\sim$ A

III DEF

C-Mod = D-Mod

VIA ALGEBRAS IN ⊗ CATS.

CAN USE
OSTRIK'S THEOREM
(OR A GENERALIZATION)

ON MODULE CATEGORIES OVER FUSION CATEGS.

(OVER FINITE TENSOR CATEGS.)

MORITAISTHM 1

JBIMODULES/R: APB, BQA

3. POBQ = Arg IN A-Bimodik

QOBAP = Brg IN B-Bimodik

CAN MIMIC VIA

BIMODULE CATEGORIES

Jepa, a Qe

+ CONDITIONS

EXER. 2.36

JFIN. GEN. PROJ. MEA-MODIR
WITH HOMA-MODIR

J. BOP = ENDA-MODIC (M)

AS IR-ALGEBRAS

··· CAN MIMIC VIA CERTAIN
ENDOFUNCTOR CATEGORIES
ENDOFUNCTOR CATEGORIES

FOR IK-ALGEBRAS

A MORITA B
"EQUIVALENCE"
OF ALGEBRAS
III DEF
A-Modk = B-Modk

WANT

 $C \sim \beta$ MORITA

III DEF

C-Mod = D-Mod

VIA ALGEBRAS IN & CATS.

CAN USE
OSTRIK'S THEOREM
(OR A GENERALIZATION)

ON MODULE CATEGORIES OVER FUSION CATEGS.

(OVER FINITE TENSOR CATEGS.)

MORITAISTHM 1

JBIMODULES/R: APB, BQA

J. POBQ = Arg IN A-Bimodik

QOAP = Brg IN B-Bimodik

CAN MIMIC VIA

BIMODULE CATEGORIES

J & Pa , a Q &

+ CONDITIONS

m ~ Mod-A(G)
FOR SOME A & Alg(G)

EXER. 2.36

JFIN. GEN. PROJ. MEA-MODIR
WITH HOMA-MODIR

J. BOP = ENDA-MODIC (M)

AS IR-ALGEBRAS

CAN MIMIC VIA CERTAIN
ENDOFUNCTOR CATEGORIES
ENDOFUNCTOR CATEGORIES

FOR IK-ALGEBRAS

A MORITA B
"EQUIVALENCE"
OF ALGEBRAS
III DEF
A-Modk = B-Modk

WANT

 $C \sim \beta$

III DEF

C-Mod = D-Mod

VIA ALGEBRAS IN ⊗ CATS.

CAN USE
OSTRIK'S THEOREM
(OR A GENERALIZATION)

ON MODULE CATEGORIES OVER FUSION CATEGS.

(OVER FINITE TENSOR CATEGS.)

MORITAISTHM 1

JBIMODULES/R: APB, BQA

J. POBQ = Arg IN A-Bimodik

QOAP = Brg IN B-Bimodik

BIMODULE CATEGORIES

3 & PB, BQ&

+ CONDITIONS

m ~ Mod-A(G)
FOR SOME A & Alg(G)

EXER. 2.36

JFIN. GEN. PROJ. MEA-MODIR
WITH HOMA-ModIR

J. BOP = ENDA-Mod (M)

AS IR-ALGEBRAS

SHOOT FOR
TENSOR EQUIVALENCE

SOP = Rexe-Mod (M, M)

FOR IK-ALGEBRAS

A MORITA B "EQUIVALENCE" OF ALGEBRAS III DEF A-Modk = B-Modk WANT

C ~ B

III DEF

C-Mod ~ D-Mod

VIA ALGEBRAS IN & CATS.

CAN USE OSTRIK'S THEOREM (OR A GENERALIZATION)

AN MODULE CATEGORIES OVER FUSION CATEGS.

(OVER FINITE TENSOR CATEGS.)

MORITAISTHM 1

JBIMODULES/R: APB, BQA J. P⊗BQ = Areg IN A-Bimodik Q⊗AP = Brg IN B-Bimodik

CAN MIMIC VIA BIMODULE CATEGORIES JePa, aQy + CONDITIONS

m ~ Mod-A(2)

FOR SOME A E Alg (4)

GENERALIZED

EILENBERG-WATTS

EXER. 2.36

3 FIN. GEN. PROJ. MEA-MODIL WITH HOMA-MODIE (M, -) FAITHFUL .7. BOP = End A-Mod (M) AS K-ALGEBRAS

SHOOT FOR TENSOR EQUIVALENCE

THEOREM

WANT

III DEF

C-Mod = D-Mod

VIA ALGEBRAS IN & CATS.

CAN USE OSTRIK'S THEOREM (OR A GENERALIZATION)

ON MODULE CATEGORIES OVER FUSION CATEGS.

(OVER FINITE TENSOR CATEGS.)

FOR SOME A E Alg (C)

GENERALIZED SHOOT FOR
TENSOR EQUIVALENCE

BOOP = Rexe-mod (m, m) = (A-Bimod (c)) & OP

FOR THIS STUDY

OF SAMENESS,

CAND B

SHOULD LIE IN THE

SAME SETTING

E.G., BOTH FUSION/
FINITE TENSOR

WANT

C ~ D

MORITA

III DEF

C-Mod ~ D-Mod

VIA ALGEBRAS IN ⊗ CATS.

CAN USE
OSTRIK'S THEOREM
(OR A GENERALIZATION)

ON MODULE CATEGORIES OVER FUSION CATEGS.

(OVER FINITE TENSOR CATEGS.)

m ~ Mod-A(C)
FOR SOME A & Alg(C)

SHOOT FOR
TENSOR EQUIVALENCE

SOP = Rexe-Mod (m, m)

GENERALIZED
EILENBERG-WATTS
THEOREM

MACHINE (C)

MORE MACHINE MACHINE

FOR THIS STUDY OF SAMENESS, & dua & SHOULD LIE IN THE SAME SETTING E.G., BOTH FUSION/ FINITE TENSOR

NEED TO STUDY WHEN

A-Binod(E)

IS FUSION FINITE TENSOR

WANT

 $C \sim \beta$ III DEF

C-Mod = D-Mod

VIA ALGEBRAS IN & CATS.

CAN INSE OSTRIK'S THEOREM (OR A GENERALIZATION)

AN MODULE CATEGORIES OVER FUSION CATEGS.

(OVER FINITE TENSOR CATEGS.)

m ~ Mod-A(2) FOR SOME A E Alg (4)

SHOOT FOR TENSOR EQUIVALENCE

Dop ~ Rexe-mod (m, m) ~ (A-Bimod (c)) op

GENERALIZED EILENBERG-WATTS THEOREM

FOR THIS STUDY OF SAMENESS, & dua & SHOULD LIE IN THE SAME SETTING E.G., BOTH FUSION/ FINITE TENSOR

NEED TO STUDY WHEN A-Binod(E) IS FUSION FINITE TENSOR WANT

 $C \sim \beta$ III DEF

C-Mod ~ D-Mod

PROPERTIES OF A & Alg(C) ARE NEEDED

SHOOT FOR TENSOR EQUIVALENCE Boop ~ Rexe-Mod (m, m) ~ (A-Bimod (E)) op

VIA ALGEBRAS IN & CATS.

CAN INSE OSTRIK'S THEOREM (OR A GENERALIZATION)

ON MODULE CATEGORIES OVER FUSION CATEGS.

(OVER FINITE TENSOR CATEGS.)

m ~ Mod-A(2) FOR SOME A & Alg (C)

GENERALIZED EILENBERG-WATTS THEOREM

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

CONNECTEDNESS

INDECOMPOSABILITY

SIMPLICITY

EXACTNESS

SEMISIMPLICITY

SEPARABILITY

 $A := (\mathcal{L}, \otimes, \mathcal{L})$ TENSOR CATEGORY $A := (A, M_A: A \otimes A \rightarrow A, U_A: \mathcal{L} \rightarrow A) \in Alg(\mathcal{L})$

CONNECTEDNESS

EX.

PROP.

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

CONNECTEDNESS

A CONNECTED

$$\Leftrightarrow$$
 dim_{|k} A = dim_{|k} Hom_{|k}(|k, A) = 1

PROP.

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \otimes A \rightarrow A) \in Alg(Y)$

CONNECTEDNESS

(STRONG CONDITION)

DEF.

A IS CONNECTED IF

dink Home (1, A) = 1.

EX. & = Vec -> A IS A IR-ALGEBRA.

A CONNECTED

 \Leftrightarrow dim_{|k} A = dim_{|k} Hom_{|k}(|k, A) = 1

⇔ A=k.

PROP.

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \otimes A \rightarrow A) \in Alg(Y)$

CONNECTEDNESS

(STRONG CONDITION)

A IS CONNECTED IF

dink Home (1, A) = 1.

A CONNECTED

$$\Leftrightarrow$$
 dim_{|k} A = dim_{|k} Hom_{|k}(|k, A) = 1

PROP. (a) Homy (1, A) IS A IR-ALGEBRA WITH:
$$M(f \otimes f'): 1 \xrightarrow{J_{u}^{-1}} 1 \otimes 1 \xrightarrow{f \otimes f'} A \otimes A \xrightarrow{MA} A \Leftrightarrow M(f) := U_{A}: 1 \longrightarrow A$$

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

CONNECTEDNESS

(STRONG CONDITION)

A IS CONNECTED IF

dink Home (1, A) = 1.

A CONNECTED

$$\Leftrightarrow$$
 dim_{|k} A = dim_{|k} Hom_{|k}(|k, A) = 1

PROP. (a) HOMY (1, A) IS A IK-ALGEBRA WITH:

$$M(f \otimes f'): 1 \xrightarrow{J_{u}^{-1}} 1 \otimes 1 \xrightarrow{f \otimes f'} A \otimes A \xrightarrow{MA} A \notin M(f) := M_{A}: 1 \longrightarrow A$$

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

CONNECTEDNESS

(STRONG CONDITION)

A IS CONNECTED IF

dink Home (1, A) = 1.

A CONNECTED

$$\Leftrightarrow$$
 dim_{|k} A = dim_{|k} Hom_{|k}(|k, A) = 1

PROP. (a) HOMY (1,A) IS A IK-ALGEBRA WITH:

$$M(f \otimes f'): 1 \xrightarrow{J_{\mathcal{U}}^{-1}} 1 \otimes 1 \xrightarrow{f \otimes f'} A \otimes A \xrightarrow{MA} A \Leftrightarrow M(f) := M_A: 1 \longrightarrow A$$

- (b) Home (U, A) = End, -mod(e) (A Ares) As Ik-ALGS
- (c) A CONNECTED (THESE ALGS ARE 1-DIMENSIONAL.

4:= (4,0,1) TENSOR CATEGORY A := (A, MA: A & A -> A, WA: 1 -> A) & Alg(Y)

CONNECTEDNESS

ding Home (11, 1) = 1

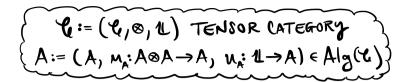
INDECOMPOSABILITY

SIMPLICITY

EXACTNESS

SEMISIMPLICITY

SEPARABILITY



INDECOMPOSABILITY

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

INDECOMPOSABILITY

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \otimes A \rightarrow A) \in Alg(Y)$

CONNECTEDNESS => INDECOMPOSABILITY

dimpleme(11, A) = 1

DEF. A IS
INDECOMPOSABLE
IF IT IS NOT

= A, DAz AS ALGS

FOR A^{*0}, A^{*0} Alg(&).

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

CONNECTEDNESS

dink Home (11, A) = 1



INDECOMPOSABILITY

A IS DECOMPOSABLE

$$\Rightarrow 1 \xrightarrow{u_{A_1}} \xrightarrow{A_1} \xrightarrow{\alpha_1} \xrightarrow{A_1} \xrightarrow{A_2} \xrightarrow{\alpha_2} A_1 \square A_2$$

ARE LINEARLY INDEP. ELTS OF Home (11, A)

⇒ A IS NOT CONNECTED

INDECOMPOSABLE

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

CONNECTEDNESS

dink Home (1L, A) = 1



A IS DECOMPOSABLE

INDECOMPOSABLE

IF IT IS NOT = A1 DA2 AS ALGS FOR A1, A2 & A1g(2).

EX. &=Vec: Matn(lk) IS INDECOMP.

$$\Rightarrow 1 \xrightarrow{u_{A_1}} \xrightarrow{A_1} \xrightarrow{\alpha_1} \xrightarrow{A_1} \xrightarrow{A_2} \xrightarrow{A_2} \xrightarrow{\alpha_2} A_1 \square A_2$$

ARE UNEARLY INDEP. ELTS OF Home (11, A)

⇒ A IS NOT CONNECTED

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \otimes A \rightarrow A) \in Alg(Y)$

CONNECTEDNESS

dink Home (1L, A) = 1



INDECOMPOSABILITY

A IS DECOMPOSABLE

DEF. A 13
INDECOMPOSABLE

FIT IS NOT ≅A₁ □A₂ AS ALGS FOR A₁^{*0}, A₂^{*0} Alg(€).

EX. &=Vec: Matn(lk) IS INDECOMP.

$$\Rightarrow \underbrace{1}_{A_{2}} \underbrace{A_{1}}_{A_{2}} \underbrace{A_{1}}_{A_{2}} \underbrace{A_{1}}_{A_{2}} \underbrace{A_{1}}_{A_{2}} \underbrace{A_{2}}_{A_{2}}$$

OF HOME (11, A)

⇒ A IS NOT CONNECTED

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

CONNECTEDNESS

dink Home (1L, A) = 1



INDECOMPOSABILITY

A IS DECOMPOSABLE

DEF. A 1S
INDECOMPOSABLE

IF IT IS NOT

= A1 DA2 AS ALGS

FOR A10 A20 Alg(e).

EX. &=Vec:

Matn(IR) IS INDECOMP.

& IN GENERAL:

1 AND XXX E Alg(C)

ARE INDECOMP.

$$\Rightarrow \underbrace{1}_{A_{2}} \underbrace{A_{1}}_{A_{2}} \underbrace{A_{1}}_{A_{2}} \underbrace{A_{1}}_{A_{2}} \underbrace{A_{2}}_{A_{2}} \underbrace{A_{1}}_{A_{2}} \underbrace{A_{2}}_{A_{2}}$$

OF Home (11, A)

⇒ A IS NOT CONNECTED

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

CONNECTEDNESS

ding Hong (1L, A) = 1



INDECOMPOSABILITY

DEF. A 1S
INDECOMPOSABLE

IF IT IS NOT = A, DAZ AS ALGO FOR A, A, A, E Alg(e).

EX. &=Vec:

Mata(IR) IS INDECOMP.

& IN GENERAL:

1 AND XXX E Alg(C)

ARE INDECOMP.

PROP.

A-Mod(&) \in Mod-&

IS INDECOMP.

A \in Alg(&)

IS INDECOMP.

Mod-A(&) \in C-Mod

IS INDECOMP.

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

CONNECTEDNESS

dink Home (11,2) = 1



INDECOMPOSABILITY

HAVE:

(A, DA2)-Mod(C)

 $\simeq A_1 - Mod(\mathcal{C}) \times A_2 - Mod(\mathcal{C})$

DEF. A 1S

INDECOMPOSABLE

IF IT IS NOT

= A1 DA2 AS ALGS

FOR A; Aze Alg(&).

EX. &=Vec:

Mata(IR) IS INDECOMP.

& IN GENERAL:

1 AND XXX E Alg(e)

ARE INDECOMP.

PROP.

= A-Mod(&) ∈ Mod-& IS INDECOMP.

A = Alg(6)

13 INDECOMP.

Mod-A(&) ∈ G-Mod

IS INDECOMP.

V:= (V, ⊗, 1L) TENSOR CATEGORY A := (A, MA: A & A -> A, WA: 1 -> A) & Alg(4)

INDECOMPOSABILITY

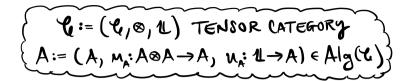
 \neq $A_1 \square A_2$ AS ALRS IN \mathcal{C} FOR $A_1^{\dagger,0}A_2^{\dagger,0} \land A_3^{\dagger,0} \land A_3^{$

SIMPLICITY

EXACTNESS

SEMISIMPLICITY

SEPARABILITY



SIMPLICITY

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

DEF. A IS SIMPLE IF

THE ONLY IDEALS OF A

ARE THE ZERO IDEAL

AND A ITSELF

SIMPLICITY

EX.

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

DEF. A IS SIMPLE IF

THE ONLY IDEALS OF A

ARE THE ZERO IDEAL

AND A ITSELF

EX.

1 ∈ Alg(&) IS SIMPLE

(1 IS A SIMPLE OBJ.)

SIMPLICITY

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \otimes A \rightarrow A) \in Alg(Y)$

DEF. A IS SIMPLE IF
THE ONLY IDEALS OF A
ARE THE ZERO IDEAL
AND A ITSELF

INDECOMPOSABILITY = SIMPLICITY

EX.

1 = Alg(&) IS SIMPLE

(1 IS A SIMPLE OBJ.)

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

A IS DECOMPOSABLE => A = A, DA2 AS ALGS.

⇒ (A1) reg □ 0 IS A PROPER IDEAL OF A ⇒ A IS NOT SIMPLE

DEF. A IS SIMPLE IF
THE ONLY IDEALS OF A
ARE THE ZERO IDEAL
AND A ITSELF

INDECOMPOSABILITY & SIMPLICITY

EX. 1 = Alg(E) IS SIMPLE (1 IS A SIMPLE OBJ.)

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

A IS DECOMPOSABLE => A = A, DA2 AS ALGS.

⇒ (A1) reg □ 0 IS A PROPER IDEAL OF A ⇒ A IS NOT SIMPLE

DEF. A IS SIMPLE IF
THE ONLY IDEALS OF A
ARE THE ZERO IDEAL
AND A ITSELF

INDECOMPOSABILITY & SIMPLICITY

(SAW THIS FOR &= Vec)

EX. 1 = Alg(C) IS SIMPLE (1 IS A SIMPLE OBJ.)

 $A := (L, \otimes, L)$ TENSOR CATEGORY $A := (A, M_A: A \otimes A \rightarrow A, M_A: L \rightarrow A) \in Alg(Y)$

A IS DECOMPOSABLE => A = A D A AS ALGS.

⇒ (A1) reg □ 0 IS A PROPER IDEAL OF A ⇒ A IS NOT SIMPLE

DEF. A IS SIMPLE IF
THE ONLY IDEALS OF A
ARE THE ZERO IDEAL
AND A ITSELF

INDECOMPOSABILITY & SIMPLICITY

(SAW THIS FOR &= Vec)

EX. 1 = Alg(E) IS SIMPLE (1 IS A SIMPLE OBJ.) PROP. SIMPLICITY IS MORITA INVARIANT:

TAKE A, B & Alg(&) WITH A & B.

MORITA

THEN A IS SIMPLE AS AN ALG. IN &

B IS SIMPLE AS AN ALG. IN &.

 $\begin{cases} & \text{$\ell := (\ell, \otimes, 1L)$ TENSOR CATEGORY} \\ & \text{$A := (A, M_A: } A \otimes A \rightarrow A, \text{ $U_A: } 1L \rightarrow A) \in Alg(L) \end{cases}$

A IS DECOMPOSABLE => A = A D A AS ALGS.

⇒ (A1) reg □ 0 IS A PROPER IDEAL OF A ⇒ A IS NOT SIMPLE

DEF. A IS SIMPLE IF
THE ONLY IDEALS OF A
ARE THE ZERO IDEAL
AND A ITSELF

INDECOMPOSABILITY

SIMPLICITY **

(SAW THIS FOR &= Vec)

EX.

1 ∈ Alg(C) IS SIMPLE
(1 IS A SIMPLE OBJ.)

PROP. SIMPLICITY IS MORITA INVARIANT:

TAKE A, B & Alg (&) WITH A ~ B.

THEN A IS SIMPLE AS AN ALG. IN &

B IS SIMPLE AS AN ALG. IN C.

" JAPB AND BQA J. P®BQ = A AND Q®AP = B.

FOR A PROPER IDEAL I OF A, GET

Q®AI®AP IS = TO A PROPER IDEAL OF B.

A IS DECOMPOSABLE \Rightarrow A = A, DA2 AS ALGS.

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

⇒ (A1) RO I O IS A PROPER IDEAL OF A ⇒ A IS NOT SIMPLE

DEF. A IS SIMPLE IF
THE ONLY IDEALS OF A
ARE THE ZERO IDEAL
AND A ITSELF

INDECOMPOSABILITY <

SIMPLICITY *

(SAW THIS FOR &= Vec)

EX.

1 ∈ Alg(E) IS SIMPLE

(1 IS A SIMPLE OBJ.)

 \bigvee

XXX* EAIg(C) IS SIMPLE FOR ANY XEC. PROP. SIMPLICITY IS MORITA INVARIANT:

TAKE A, B & Alg (C) WITH A ~ B.

THEN A IS SIMPLE AS AN ALG. IN &

B IS SIMPLE AS AN ALG. IN C.

" JAPB AND BQA J. P®BQ = A AND Q®AP = B.

FOR A PROPER IDEAL I OF A, GET

Q®AI®AP IS = TO A PROPER IDEAL OF B.

V:= (V, ⊗, 1L) TENSOR CATEGORY A := (A, MA: A & A -> A, WA: 1 -> A) & Alg(4)



CONNECTEDNESS

ding Home (1, A) = 1



 $A = A_1 = A_2$ AS ALGS IN &
FOR $A_1^{\sharp,0}$, $A_2^{\sharp,0}$ $A_2^{\sharp,0}$ Alg(%).

EXACTNESS

SEMISIMPLICITY

SEPARABILITY

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

SEMISIMPLICITY

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \otimes A \rightarrow A) \in Alg(Y)$

EX. REM. SEMISIMPLICITY DEF. A IS SEMISIMPLE IF A-Mod(&) IS A SEMISIMPLE CATEGORY.

 $A := (L, \otimes, L)$ TENSOR CATEGORY $A := (A, M_A: A \otimes A \rightarrow A, M_A: L \rightarrow A) \in Alg(L)$

EX. REM. 1 ∈ Alg (℃) IS SEMISIMPLE & IS SEMISIMPLE [1-Mod(2)=2] SEMISIMPLICITY DEF. A IS SEMISIMPLE

IF A-Mod(C) IS A

SEMISIMPLE CATEGORY.

 $A := (L, \otimes, L)$ TENSOR CATEGORY $A := (A, M_A: A \otimes A \rightarrow A, M_A: L \rightarrow A) \in Alg(L)$

REM. SEMISIMPLICITY IS MORITA INVARIANT:

TAKE A, B & Alg (&) WITH A & B.

MORITA B.

THEN A IS SEMISIMPLE AS AN ALG. IN &

B IS SEMISIMPLE AS AN ALG. IN &.

EX.

L = Alg(x)

IS SEMISIMPLE

C1-Mod(x)=x]

SEMISIMPLICITY

DEF. A IS SEMISIMPLE

IF A-Mod(C) IS A

SEMISIMPLE CATEGORY.

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

REM. SEMISIMPLICITY IS MORITA INVARIANT:

TAKE A, B & Alg (G) WITH A B B.

MORITA B.

THEN A IS SEMISIMPLE AS AN ALG. IN G.

B IS SEMISIMPLE AS AN ALG. IN G.

SEMISIMPLICITY

DEF. A IS SEMISIMPLE

IF A-Mod(&) IS A

SEMISIMPLE CATEGORY.

EX. 1 ∈ Alg(&) IS SEMISIMPLE & IS SEMISIMPLE [1-Mod(2)=2] $\rightarrow X \otimes X^* \in Alg(\mathcal{C})$ IS SEMISIMPLE FOR ANY XEC. & IS SEMISIMPLE

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

REM. SEMISIMPLICITY IS MORITA INVARIANT:

TAKE A, B & Alg (G) WITH A B.

MORITA B.

THEN A IS SEMISIMPLE AS AN ALG. IN G.

B IS SEMISIMPLE AS AN ALG. IN G.

GOOD NOTION OF

ARTIN-WEDDERBURN

THEOREM

... UNRESOLVED G = FdVecA SEMISIMPLE A = TI = Mathi(Ik)

SEMISIMPLICITY

DEF. A IS SEMISIMPLE

IF A-Mod(&) IS A

SEMISIMPLE CATEGORY.

EX. 1 ∈ Alg(&) IS SEMISIMPLE & IS SEMISIMPLE [1-Mod(2)=2] $\rightarrow X \otimes X^* \in Alg(\mathcal{C})$ IS SEMISIMPLE FOR ANY XEC. & IS SEMISIMPLE

V:= (V, ⊗, 1L) TENSOR CATEGORY A := (A, MA: A & A -> A, WA: 1 -> A) & Alg(4)

CONNECTEDNESS

dink Home (11, A) = 1

INDECOMPOSABILITY

FOR A, \$0 A2 \$ 6 Alg (8).

SIMPLICITY

EXACTNESS

SEMISIMPLICITY

A-Mod(&) IS A SEMISIMPLE CATEGORY SEPARABILITY

 $A := (L, \otimes, L)$ TENSOR CATEGORY $A := (A, M_A: A \otimes A \rightarrow A, U_A: L \rightarrow A) \in Alg(C)$

PROP

EX.

DEF A IS SEPARABLE IF $\exists \varphi : A \longrightarrow A \otimes A$ IN A-Bimod(C) .3. $M \otimes = id_A$

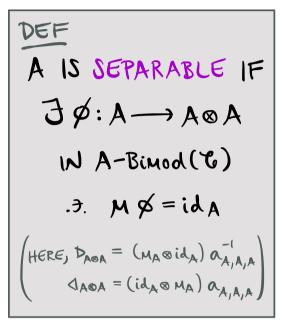
SEPARABILITY

EX.

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

PROP

EX.



SEPARABILITY

EX.

 $A := (L, \otimes, L)$ TENSOR CATEGORY $A := (A, M_A: A \otimes A \rightarrow A, M_A: L \rightarrow A) \in Alg(L)$

PROP

EX.

SEPARABILITY

EX.

(1, le, ide) & Alg(x)

(3 SEPARABLE

WITH ØL := La!

W:= (4, ∞, 1L) TENSOR CATEGORY

A:= (A, MA: A⊗A → A, MA: 1L → A) ∈ Alg(4)

PROP

EX. & PIVOTAL (STRICT FOR EASE)

TAKE $X \in \mathcal{C} \ni .$ $dim_{j} X := \begin{cases} x^{v} \\ y^{v} \\ y^{v} \\ y^{v} \end{cases}$ IS AN 180 $(\Longrightarrow \neq 0)$

DEF A IS SEPARABLE IF $J \varnothing : A \longrightarrow A \otimes A$ IN A-Bimod(C) .3. $M \varnothing = id_A$ $(A \otimes A = (id_A \otimes M_A) \alpha_{A,A,A}^{-1}$ $(A \otimes A = (id_A \otimes M_A) \alpha_{A,A,A}^{-1})$

SEPARABILITY

((4, ∞, 1) TENSOR CATEGORY

A:= (A, MA: A⊗A → A, MA: 11 → A) ∈ Alg(1)

PROP

DEF A IS SEPARABLE IF $\exists \, \varphi : A \longrightarrow A \otimes A$ IN A-Bimod(C) .3. $M \otimes = id_A$ $\begin{pmatrix}
HERE, b_{A\otimes A} = (M_A \otimes id_A) \alpha_{A,A,A}^{-1} \\
d_{A\otimes A} = (id_A \otimes M_A) \alpha_{A,A,A}
\end{pmatrix}$

SEPARABILITY

EX.

(1, lu, idu)

Alg(x)

IS SEPARABLE

WITH

Ø1:= Ju'

(:= (\(\lambda \), \(\lambda \) TENSOR CATEGORY

A:= (\(A \), \(M_A : A \otimes A \rangle A \), \(M_A : A \otimes A \) \(A \otimes A \)

A:= (\(A \), \(M_A : A \otimes A \rangle A \rangle A \), \(M_A : A \otimes A \rangle A

PROP

DEF A IS SEPARABLE IF $J \not A : A \longrightarrow A \otimes A$ IN A-Bimod(C) .3. $M \not A = id_A$ $(MA \otimes A) = (MA \otimes id_A) \alpha_{A,A,A}^{-1}$ $(MA \otimes A) = (id_A \otimes MA) \alpha_{A,A,A}$

SEPARABILITY

EX. $(1,l_1,id_1) \in Alg(x)$ IS SEPARABLE
WITH $\emptyset_1 := J_1^{-1}$

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: 1 \rightarrow A) \in Alg(Y)$

PROP SUPPOSE

& MULTIFUSION.
THEN TFAE:

- · A SEPARABLE
- · A-Mod(&) IS SEMISIMPLE
- Mod-A(V) IS
 SEMISIMPLE
- A-Bimod(C) IS SEMISIMPLE

CDAVYDOV-MÜGER -NIKSHYCH-OSTRIKJ EX. & PIVOTAL (STRICT FOR EASE)
TAKE XEC 3.

$$dim_{\dot{j}}\chi := \bigvee_{\dot{j}^{-1}}^{\chi^{\nu}} 13 \text{ AN } 180$$

$$(\Leftrightarrow \neq 0)$$

TAKE A:= $X \otimes X^{V}$ WITH $M_A := X \otimes X^{V}$ UA := $X \otimes X^{V}$

THEN A IS SEPARABLE WITH

$$\oint_{A} := \left(\operatorname{dim}_{j} X \right)^{-1} \left| X \left(X^{v} \right) X^{vv} \right| X^{v}$$

DEF

A IS SEPARABLE IF

 $\exists \phi: A \longrightarrow A \otimes A$

IN A-Bimod(C)

.7. M & = idA

HERE, $b_{A\otimes A} = (M_A \otimes id_A) \alpha_{A,A,A}^{-1}$ $d_{A\otimes A} = (id_A \otimes M_A) \alpha_{A,A,A}$

SEPARABILITY

EX.

(1, lu, idu) & Alg(x)

(S SEPARABLE

WITH Ø11:= 11

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \otimes A \rightarrow A) \in Alg(Y)$



dink Hone (11, x) = 1

> INDECOMPOSABILITY

AS ALRS IN CFOR $A_1^{\dagger 0}$, $A_2^{\dagger 0}$ Alg(C).

SIMPLICITY

ONLY IDEALS ARE O AND A

EXACTNESS

SEMISIMPLICITY

A-Mod(G) IS A SEMISIMPLE CATEGORY

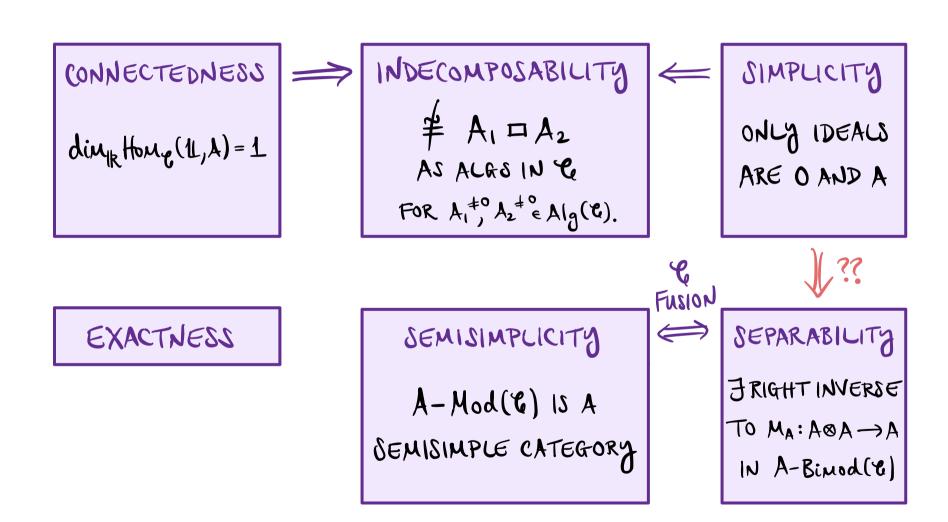
Fusion

SEPARABILITY

 $\exists RIGHT INVERSE$ TO $M_A: A \otimes A \longrightarrow A$

IN A-Bimod(&)

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \otimes A \rightarrow A) \in Alg(Y)$



$$A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$$

EXACTNESS

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: 1 \rightarrow A) \in Alg(Y)$

DEF. A IS EXACT IF A-Mod(C) IS AN EXACT RIGHT G-MOD. CATEG.

THAT IS YPE & PROJECTIVE,

YME A-MODIC),

MUAP = M&P & A-MODIC)

IS PROJECTIVE.

REM.

EX.

PROP.

EXACTNESS

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

DEF. A IS EXACT IF A-Mod(C) IS AN EXACT RIGHT G-MOD. CATEG.

THAT IS YPE & PROJECTIVE,

YME A-MODICO),

MAAP = M&P & A-MODICO)

IS PROJECTIVE.

REM.

EX. 11 = Alg(e) IS EXACT

EXACTNESS

DEF. A IS EXACT IF A-Mod(&) IS AN EXACT RIGHT &-MOD. CATEG.

THAT IS YPEC PROJECTIVE,

YME A-MODIC),

MAAP = M&P & A-MODIC)

IS PROJECTIVE.

EXACTNESS

 $U := (U, \otimes, U)$ TENSOR CATEGORY $A := (A, M_A: A \otimes A \rightarrow A, U_A: U \rightarrow A) \in Alg(U)$

REM. EXACTNESS IS MORITA INVARIANT

 $U := (U, \otimes, U)$ TENSOR CATEGORY $A := (A, M_A: A \otimes A \rightarrow A, U_A: U \rightarrow A) \in Alg(U)$

DEF. A IS EXACT IF A-Mod(C) IS AN EXACT RIGHT G-MOD. CATEG.

THAT IS YPER PROJECTIVE,

YME A-MODIC),

MUAP = M&P & A-Mod(e)

IS PROJECTIVE.

REM. EXACTNESS IS MORITA INVARIANT

PROP.

EXACTNESS



SEMISIMPLICITY

 $A := (\mathcal{C}_{1} \otimes_{1} \mathcal{L})$ TENSOR CATEGORY $A := (A, M_{A} : A \otimes A \rightarrow A, U_{A} : \mathcal{L} \rightarrow A) \in Alg(\mathcal{C})$

DEF. A IS EXACT IF A-Mod(C) IS AN EXACT RIGHT &-MOD. CATEG.

THAT IS YPE & PROJECTIVE,

YME A-Mod(C),

MAAP = M&P & A-Mod(C)

IS PROJECTIVE.

REM. EXACTNESS IS MORITA INVARIANT

EX. 11 = Alg(E) IS EXACT

> X & X * = Alg(E) IS EXACT

PROP.

EXACTNESS

= SEMISIMPLICITY

A SEMISIMPLE

A-Mod(C) SEMISIMPLE

OBJECTS IN A-Mod(8)
ARE PROJECTIVE

 $\begin{cases} \mathcal{C} := (\mathcal{C}, \otimes, \mathcal{L}) \text{ TENSOR CATEGORY} \\ A := (A, M_A: A \otimes A \rightarrow A, U_A: \mathcal{L} \rightarrow A) \in Alg(\mathcal{C}) \end{cases}$

DEF. A IS EXACT IF A-Mod(C) IS AN EXACT RIGHT &-MOD. CATEG.

THAT IS YPE & PROJECTIVE,

YME A-MODIC),

MUAP = M&P & A-MODIC)

IS PROJECTIVE.

REM. EXACTNESS IS MORITA INVARIANT

EXACTNESS

SEMISIMPLE

A-Mod(e) SEMISIMPLE

OBJECTS IN A-Mod(e)

ARE PROJECTIVE

PROP.

SAY & FINITE TENSOR.

THEN:

A IS EXACT

ANY RIGHT

C-MODULE FUNCTOR

A-MODULE FUNCTOR

IS EXACT

 $A := (A, M_A: A \otimes A \rightarrow A, M_A: A \rightarrow A) \in Alg(Y)$

DEF A IS EXACT IF A-Mod(C) IS AN EXACT RIGHT G-MOD. CATEG.

THAT IS YPE & PROJECTIVE,

YME A-MODIC),

MAAP = M&P & A-MODIC)

IS PROJECTIVE.

REM. EXACTNESS IS MORITA INVARIANT

EXACTNESS



SEMISIMPLICITY

WOULD LIKE THIS TO BE LEFT-RIGHT SYMMETRIC. INTRINSIC CHARACTERIZATION OF EXACTNESS A SEMISIMPLE

A-Mod(C) SEMISIMPLE

U

OBJECTS IN A-Mod(C)

ARE PROJECTIVE

PROP.

SAY & FINITE TENSOR.

THEN:

A IS EXACT

ANY RIGHT

C-MODILE FUNCTOR

A-MODILE FUNCTOR

IS EXACT

 $\left\{\begin{array}{l} \mathcal{U} := (\mathcal{U}, \otimes, \mathcal{U}) \text{ TENSOR CATEGORY} \\ A := (A, M_A: A \otimes A \rightarrow A, U_A: \mathcal{U} \rightarrow A) \in Alg(\mathcal{U}) \end{array}\right\}$

DEF A IS EXACT IF A-Mod(&) IS AN EXACT RIGHT &-MOD. CATEG.

THAT IS YPEG PROJECTIVE,

YMEA-MODIC),

MUAP = M&P & A-Mod(e)

IS PROJECTIVE.

REM. EXACTNESS IS MORITA INVARIANT

EXACTNESS



SEMISIMPLICITY

WOULD LIKE THIS TO BE
LEFT-RIGHT SYMMETRIC.
INTRINSIC CHARACTERIZATION
OF EXACTNESS
.... UNRESOLVED

A SEMISIMPLE

A-Mod(B) SEMISIMPLE

U

OBJECTS IN A-Mod(B)

ARE PROJECTIVE

PROP.

SAY & FINITE TENSOR.

THEN:

A IS EXACT

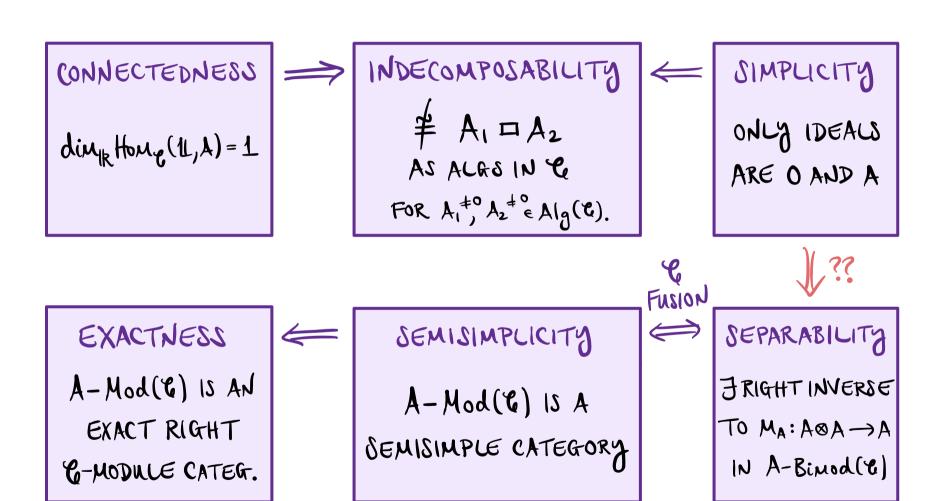
ANY RIGHT

C-MODULE FUNCTOR

A-MODILE FUNCTOR

IS EXACT

 $A := (\mathcal{L}, \otimes, \mathcal{L})$ TENSOR CATEGORY $A := (A, M_A: A \otimes A \rightarrow A, U_A: \mathcal{L} \rightarrow A) \in Alg(\mathcal{L})$



TAKE (C, &, L) TENSOR & (A, M, N) & Alg(&).

LET'S STUDY WHEN

(A-Bimod(&), &, Areg)

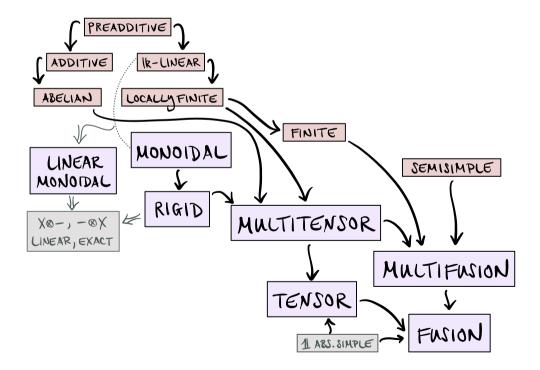
IS FUSION/FINITE TENSOR.

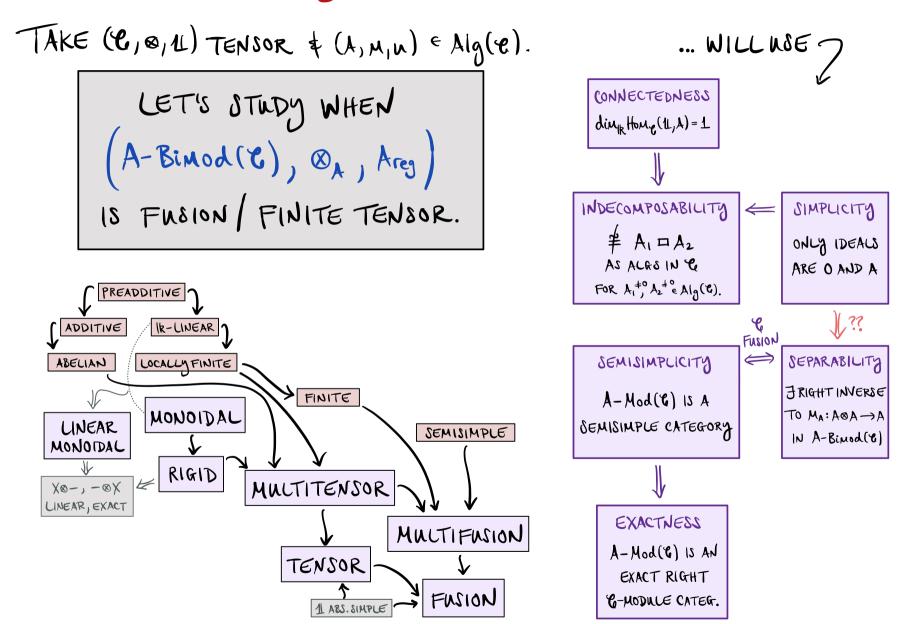
TAKE (C, 8, 1L) TENSOR & (A, M, N) & Alg(e).

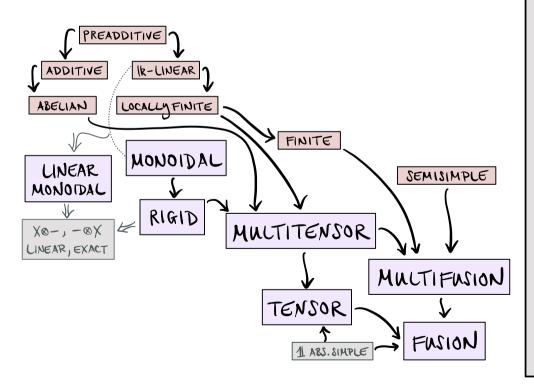
LET'S STUDY WHEN

(A-Bimod(C), &, Areg)

IS FUSION/ FINITE TENSOR.







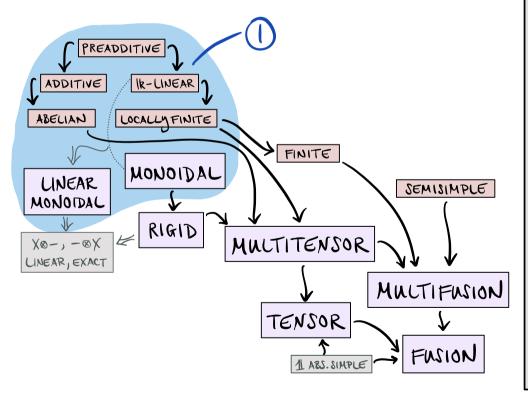
THEOREM:

TAKE (C, Ø, LL) TENSOR

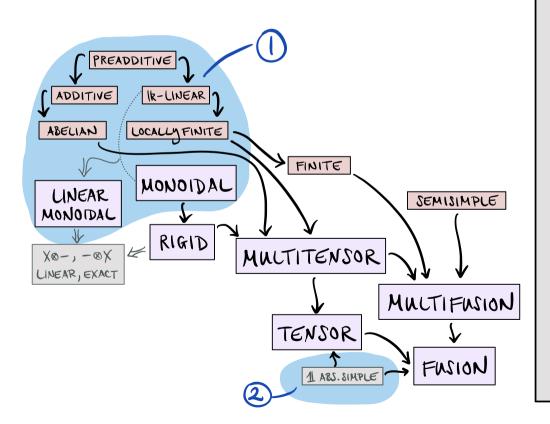
\$ (A, M, u) \in Alg(\varphi).

THEN (A-Bimod(\varphi), \omega_A, Areg)

SATISFIES:



THEOREM: TAKE (C, O, L) TENSOR $\neq (\lambda, \mu, \mu) \in Alg(e).$ THEN (A-Binod(G), OA, Areg) SATISFIES: (1) ALWAYS ;



THEOREM:

TAKE (C, 0, 11) TENSOR \$ (A, M, N) & Alg(e).

THEN (A-Binod(G), OA, Areg)

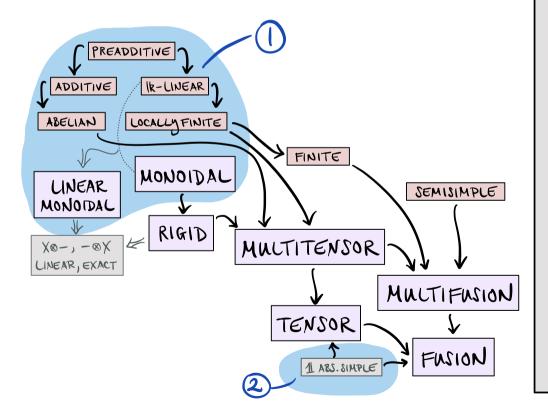
- 1 ALWAYS ;
- 2 WHEN A IS CONNECTED;

CONNECTEDNESS dimphone(11,2)=1

End A-Bimod(&) (Areg)

(S A SUBSPACE OF

End A-Mod (Areg) = Home (1,A)

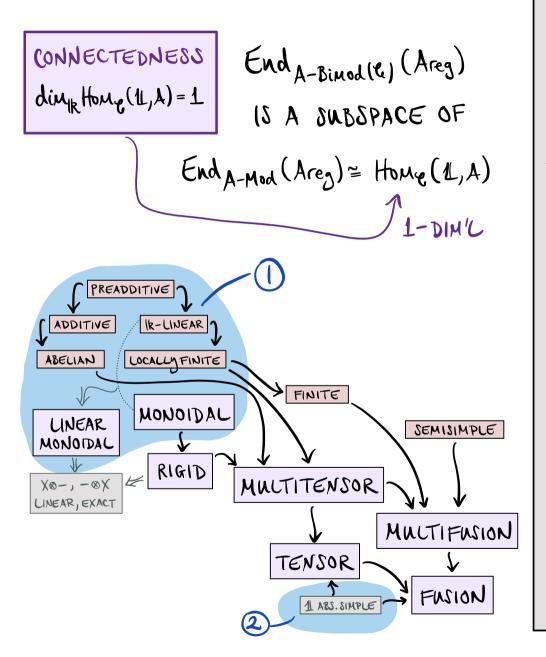


THEOREM:

TAKE (C, 8, 1L) TENSOR \$ (A, M, N) & Alg(e).

THEN (A-Bimod(&), OA, Areg)

- (1) ALWAYS ;
- 2 WHEN A IS CONNECTED;

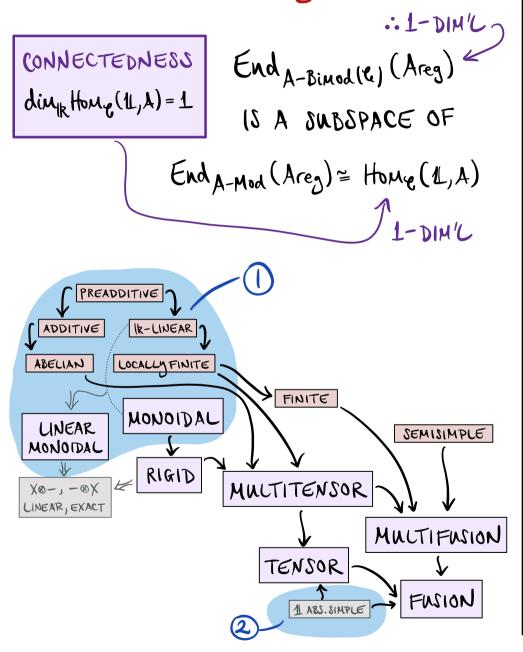


THEOREM:

TAKE (C, 0, 1L) TENSOR \$ (A, M, N) & Alg(e).

THEN (A-Bimod(&), OA, Areg)

- O ALWAYS ;
- 2 WHEN A IS CONNECTED;

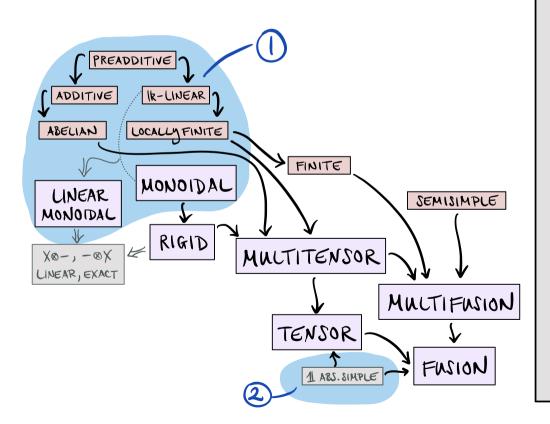


THEOREM:

TAKE (C, Ø, LL) TENSOR \$ (A, M, N) € Alg(e).

THEN (A-Bimod(&), OA, Areg)

- (1) ALWAYS ;
- 2 WHEN A IS CONNECTED;

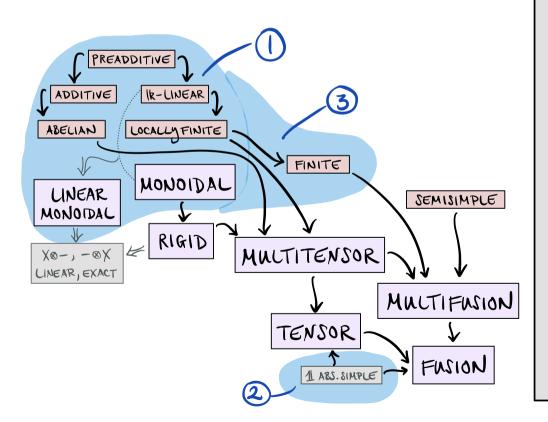


THEOREM:

TAKE (C, 0, 11) TENSOR \$ (A, M, N) & Alg(e).

THEN (A-Binod(G), OA, Areg)

- 1 ALWAYS ;
- 2 WHEN A IS CONNECTED;

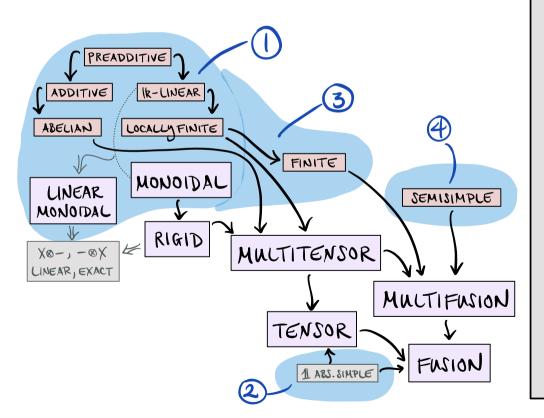


THEOREM:

TAKE (C, Q, LL) TENSOR \$ (A, M, N) & Alg(e).

THEN (A-Binod(6), OA, Areg)

- (1) ALWAYS ;
- 2 WHEN A IS CONNECTED;
- 3 WHEN & IS FINITE;



THEOREM:

TAKE (C, Ø, LL) TENSOR \$ (A, M, N) € Alg(E).

THEN (A-Binod(&), OA, Areg)

- O ALWAYS ;
- 2 WHEN A IS CONNECTED;
- 3 WHEN & IS FINITE;
- # A IS SEPARABLE;

SEPARABILITY

FRIGHT INVERSE

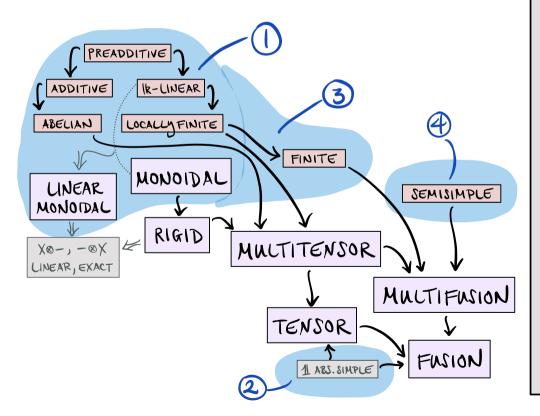
TO MA: A&A -> A

IN A-Bimod(&)

PROP: SUPPOSE & MULTIFUSION.
THEN TFAE:

- A SEPARABLE MOD-A(C) IS SEMISIMPLE
- A-Mod(C) IS A-Bimod(C) IS SEMISIMPLE

CDAVYDOV-MÜGER-NIKSHYCH-OSTRIK]

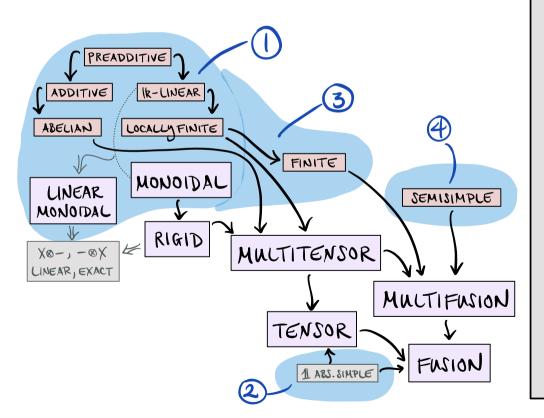


THEOREM:

TAKE (C, 0, 1L) TENSOR \$ (A, M, N) & Alg(&).

THEN (A-Binod(&), OA, Areg)

- 1 ALWAYS ;
- 2 WHEN A IS CONNECTED;
- 3 WHEN & IS FINITE;
- # A IS SEPARABLE;

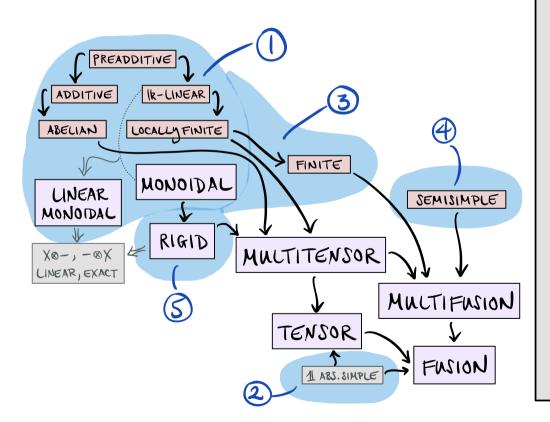


THEOREM:

TAKE (C, Ø, LL) TENSOR \$ (A, M, N) € Alg(E).

THEN (A-Binod(&), OA, Areg)

- O ALWAYS ;
- 2 WHEN A IS CONNECTED;
- 3 WHEN & IS FINITE;
- # A IS SEPARABLE;



THEOREM:

TAKE (C, Ø, LL) TENSOR \$ (A, M, N) € Alg(E).

THEN (A-Bimod(&), OA, Areg)

- 1 ALWAYS ;
- 2 WHEN A IS CONNECTED;
- 3 WHEN & IS FINITE;
- WHEN & IS SEMISIMPLE
 \$ A IS SEPARABLE;
- S WHEN & IS FINITE A IS EXACT.

PROP. EXACTNESS SAY & FINITE TENSOR. THEN: A-Mod(&) IS AN A IS EXACT EXACT RIGHT G-MODILLE CATEG. ANY RIGHT C-MODULE FUNCTOR A-Mod(&) -> m IS EXACT PREADDITIVE ADDITIVE IK-LINEAR 1 ABELIAN LOCALLYFINITE FINITE MONOIDAL LINEAR SEMISIMPLE MONOIDAL RIGID MULTITENSOR X⊗-, -⊗X ∠ LINEAR, EXACT MULTIFUSION S TENSOR 1 ABS. SIMPLE FUSION

THEOREM:

TAKE (€, Ø, LL) TENSOR \$ (A, M, N) € Alg(E).

THEN (A-Bimod(&), OA, Areg)

- (1) ALWAYS ;
- 2 WHEN A IS CONNECTED;
- 3 WHEN & IS FINITE;
- WHEN & IS SEMISIMPLE
 \$ A IS SEPARABLE;
- S WHEN & IS FINITE A IS EXACT.

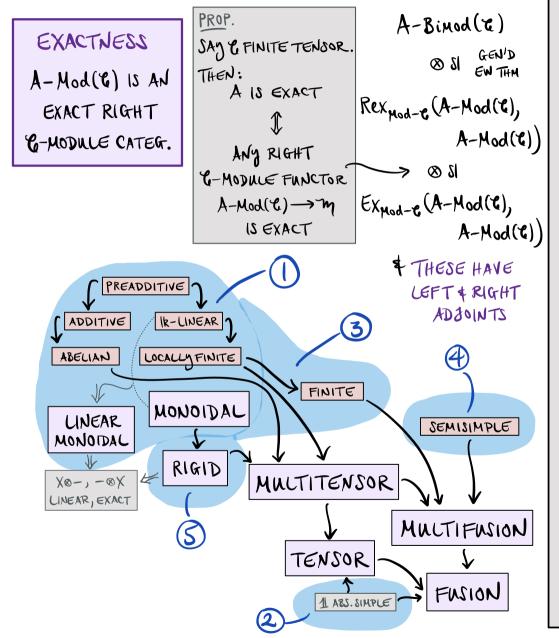
PROP. A-Binod(&) EXACTNESS SAY & FINITE TENSOR ⊗ SI GEN'D EW THM THEN: A-Mod(&) IS AN A IS EXACT Rexmod-e (A-Mod(E), EXACT RIGHT A-Mod(E)) C-MODILLE CATEG. ANY RIGHT **→** ⊗ ऽI C-MODULE FUNCTOR Exmod-e (A-Mod(e), A-Mod(&) -> m IS EXACT A-Mod(C)) PREADDITIVE ADDITIVE IK-LINEAR 7 LOCALLYFINITE ABELIAN FINITE MONOIDAL LINEAR SEMISIMPLE MONODAL RIGID MULTITENSOR $X \otimes -, - \otimes X$ LINEAR, EXACT MULTIFUSION S TENSOR 1 ABS. SIMPLE FUSION

THEOREM:

TAKE (C, Ø, LL) TENSOR \$ (A, M, N) & Alg(&).

THEN (A-Binod(&), OA, Areg)

- (1) ALWAYS ;
- 2 WHEN A IS CONNECTED;
- 3 WHEN & IS FINITE;
- WHEN & IS SEMISIMPLE
 \$ A IS SEPARABLE;
- S WHEN & IS FINITE A IS EXACT.

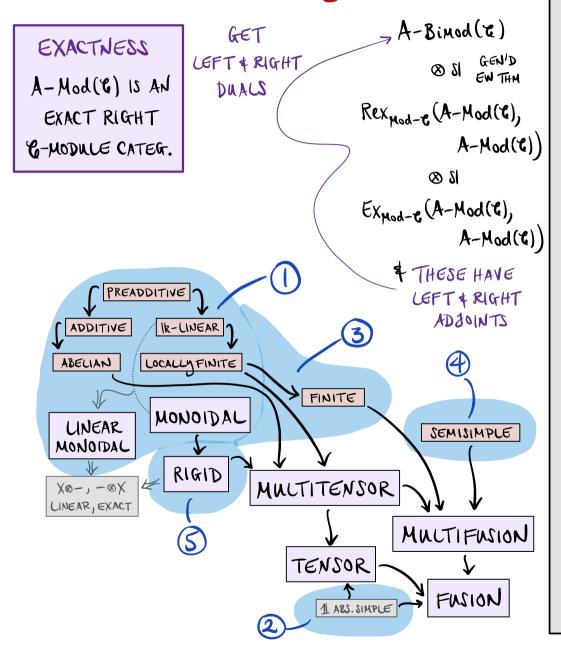


THEOREM:

TAKE (C, Ø, LL) TENSOR \$ (A, M, N) & Alg(&).

THEN (A-Binod(6), OA, Areg)

- (1) ALWAYS ;
- 2 WHEN A IS CONNECTED;
- 3 WHEN & IS FINITE;
- WHEN & IS SEMISIMPLE
 \$ A IS SEPARABLE;
- S WHEN & IS FINITE A IS EXACT.

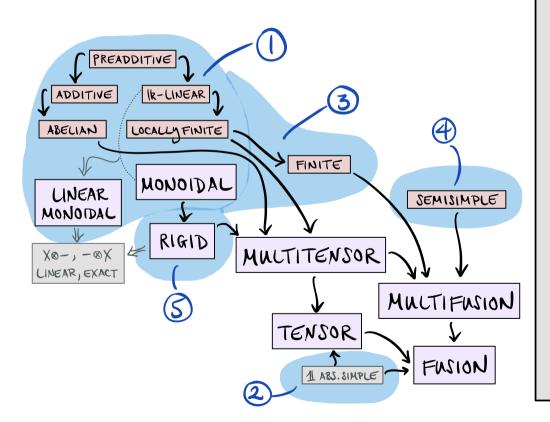


THEOREM:

TAKE (C, Ø, LL) TENSOR \$ (A, M, N) & Alg(e).

THEN (A-Binod(6), OA, Areg)

- 1 ALWAYS ;
- 2 WHEN A IS CONNECTED;
- 3 WHEN & IS FINITE;
- WHEN & IS SEMISIMPLE
 \$ A IS SEPARABLE;
- S WHEN & IS FINITE A IS EXACT.



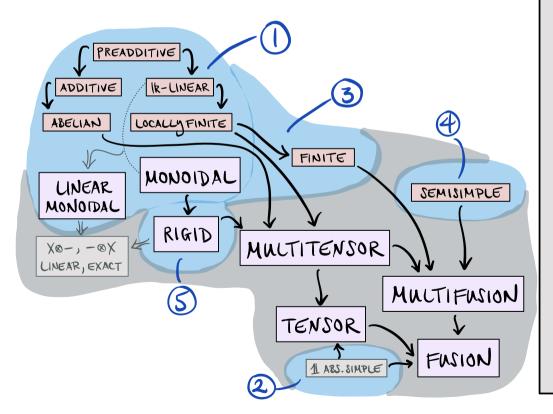
THEOREM:

TAKE (C, Ø, LL) TENSOR \$ (A, M, N) € Alg(E).

THEN (A-Bimod(&), OA, Areg)

- 1 ALWAYS ;
- 2 WHEN A IS CONNECTED;
- 3 WHEN & IS FINITE;
- WHEN & IS SEMISIMPLE
 \$ A IS SEPARABLE;
- S WHEN & IS FINITE A IS EXACT.

NOW WE GET THE REST ...



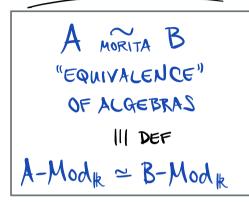
THEOREM:

TAKE (C, 0, 1L) TENSOR \$ (A, M, N) & Alg(&).

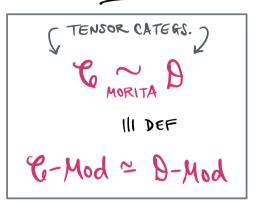
THEN (A-Bimod(&), OA, Areg)

- (1) ALWAYS ;
- 2 WHEN A IS CONNECTED;
- 3 WHEN & IS FINITE;
- WHEN & IS SEMISIMPLE
 \$ A IS SEPARABLE;
- S WHEN & IS FINITE A IS EXACT.

FOR IK-ALGEBRAS



WANT



MORITAISTHM 1

JBIMODULES/R: APB, BQA

J. POBQ = Arg IN A-Bimodik

QOAP = Brg IN B-Bimodik



JFIN. GEN. PROJ. M ∈ A-Modik
WITH HOMA-Modik

→. B°P = Enda-Mod (M)

AS IR-ALGEBRAS

FOR IR-ALGEBRAS

A MORITA B
"EQUIVALENCE"
OF ALGEBRAS
III DEF
A-Modik = B-Modik

WANT

TENSOR CATEGS. 2

C ~ D

MORITA

III DEF

C-Mod ~ D-Mod

MORITAISTHM 1

JBIMODULES/R: APB, BQA

JBIMODULES/R: APB, BQA

ABIMODULES/R: APB, BQA

ABIMODULES/R: APB, BQA

BROWN B-BIMODIR

CAN MIMIC VIA

BIMODULE CATEGORIES

J&Pa, aQ&

+ CONDITIONS

1

JFIN. GEN. PROJ. M ∈ A-MODIR
WITH HOMA-MODIR

J. BOP = ENDA-MODIC (M)

AS IR-ALGEBRAS

SHOOT FOR
TENSOR EQUIVALENCE

SEOP = Rexy-Mod (M, M)

FOR IR-ALGEBRAS

A MORITA B
"EQUIVALENCE"
OF ALGEBRAS
III DEF
A-Modk = B-Modk

C TENSOR C

CTENSOR CATERS. 2

COMORITA

III DEF

C-Mod = D-Mod

WANT

VIA ALGEBRAS IN & CATS.

CAN USE
OSTRIK'S THEOREM
(OR A GENERALIZATION)

ON MODULE CATEGORIES OVER FUSION CATEGS.)

FOR SOME A & Alg(E)

MORITAISTHM 1

JBIMODULES/R: APB, BQA

J. POBQ = Arg IN A-Bimodik

QOAP = Brg IN B-Bimodik

1

J FIN. GEN. PROJ. M ∈ A-MODIR
WITH HOMA-MODIR

J. BOP = ENDA-MODIC (M)

AS IR-ALGEBRAS

CAN MIMIC VIA

BIMODULE CATEGORIES

J&Pa, BQ&

+ CONDITIONS

SHOOT FOR
TENSOR EQUIVALENCE

BEOP = Rexy-Mod (M, M)

FOR IK-ALGEBRAS

A MORITA B
"EQUIVALENCE"
OF ALGEBRAS
III DEF
A-Modk = B-Modk

MORITAIS THM 1

JBIMODULES/R: APB, BQA

J. POBQ = Arg IN A-Bimodik

QOAP = Brg IN B-Bimodik

1

J FIN. GEN. PROJ. M ∈ A-Modik WITH Hom_{A-Modik}(M, -) FAITHFUL .>. B°P = End A-mod (M) AS IR-ALGEBRAS WANT

TENSOR CATEGS. 2

C ~ D

MORITA

III DEF

C-Mod ~ D-Mod

· CAN MIMIC VIA
BIMODULE CATEGORIES
A PO

Je Pa, a Qy E Zhoitidhoo +

SHOOT FOR
TENSOR EQUIVALENCE

800P = Rexy-Mod (M, M)

VIA ALGEBRAS IN ⊗ CATS.

CAN USE
OSTRIK'S THEOREM
(OR A GENERALIZATION)

ON MODULE CATEGORIES OVER FUSION CATEGS. (OVER FINITE TENSOR CATEGS.)

FOR SOME A & Alg(E)

GENERALIZED

EILENBERG-WATTS

THEOREM

· (A-Bimod(C)) op

TENSOR CATEGS. 6, B ARE

CATEGORICALLY

MORITA EQUIVALENT

IF

DOOP = Rexe-mod (M, M)

AS TENSOR CATEGS,

FOR SOME EXACT Me &-Mod.

CTENSOR CATEGS. 2

COMORITA

III DEF

C-Mod ~ D-Mod

WANT

CAN MIMIC VIA

BIMODULE CATEGORIES

Jepa, a Qe

+ CONDITIONS

SHOOT FOR
TENSOR EQUIVALENCE

SOP = Rexe-Mod (M, M)

VIA ALGEBRAS IN ⊗ CATS.

CAN USE
OSTRIK'S THEOREM
(OR A GENERALIZATION)

ON MODULE CATEGORIES OVER FUSION CATEGS. (OVER FINITE TENSOR CATEGS.)

m ~ Mod-A(G)
FOR SOME A & Alg(G)

GENERALIZED

EILENBERG-WATTS

THEOREM

MA-Binod(%))

OP

TENSOR CATEGS. 6, 8 ARE CATEGORICALLY MORITA EQUIVALENT Boop ~ Rexe-Mod (mm) AS TENSOR CATEGS, FOR SOME EXACT Me &-Mod. ° IN THE SETTING OF OSTRIK'S THEOREM (OR A GENERALIZATION) M ~ Mod-A(C) FOR SOME A∈ Alg(C) ~ (A-Bimod(C))^{⊗op}

TENSOR CATEGS. 6, 8 ARE CATEGORICALLY MORITA EQUIVALENT Book = Rexe-Mod (m, m) AS TENSOR CATEGS, FOR SOME EXACT Me &-Mod. " IN THE SETTING OF OSTRIK'S THEOREM (OR A GENERALIZATION) FOR SOME A & Alg (4) KNOW WHEN FUSION / FINITE TENSOR

TENSOR CATEGS. 6, B ARE

CATEGORICALLY

MORITA EQUIVALENT

IF

SOP REXE-MODITY

AS TENSOR CATEGS,

FOR SOME EXACT Me &-Mod.

EXAMPLE G FINITE GROWP

G = Vecg & D = G-Mod ARE

CATEGORICALLY MORITA EQUIVALENT

BECAUSE

G-Mod & P = Rex Vecg-Mod (Vec, Vec).

TENS

"IN THE SETTING OF

OSTRIK'S THEOREM

(OR A GENERALIZATION)

MOD-A(R)

FOR SOME A & AIJ(R)

L (A-BIMOD(R))

KNOW WHEN FUSION / FINITE TENSOR

TENSOR CATEGS. 6, 8 ARE CATEGORICALLY MORITA EQUIVALENT Boop = Rexe-Mod (m, m) AS TENSOR CATEGS, FOR SOME EXACT me C-Mod. " IN THE SETTING OF OSTRIK'S THEOREM (OR A GENERALIZATION) m ~ Mod-A(G) FOR SOME A & Alg(G) = (A-Bimod(C))

KNOW WHEN FUSION / FINITE TENSOR

EXAMPLE G FINITE GROWP

G = Vecg & D = G-Mod ARE

CATEGORICALLY MORITA EQUIVALENT

BECAUSE

G-Mod & P = Rex Vecg-Mod (Vecg, Vec).

TENS

= KEY REFERENCE =

\$7.12 OF BOOK "TENSOR CATEGORIES"

BY

ETINGOF-GELAKI-NIKSHYCH-OSTRIK

TENSOR CATEGS. 6, B ARE CATEGORICALLY MORITA EQUIVALENT Boop = Rexe-Mod (m, m) AS TENSOR CATEGS, FOR SOME EXACT me C-Mod.

" IN THE SETTING OF OSTRIK'S THEOREM (OR A GENERALIZATION)

m ~ Mod-A(G) FOR SOME A & Alg(G)

KNOW WHEN FUSION / FINITE TENSOR

EXAMPLE G FINITE GROUP C = Veca & D = G-Mod ARE CATEGORICALLY MORITA EQUIVALENT BECAUSE G-Mod & P = Rex Vec - Mod (Vec, Vec).

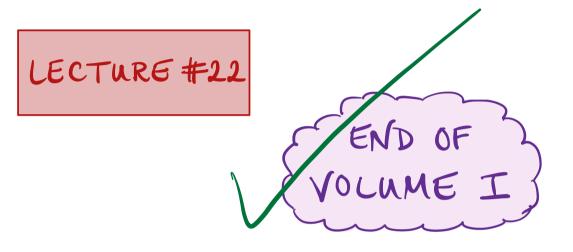
= KEY REFERENCE = §7.12 OF BOOK "TENSOR CATEGORIES" ETINGOF-GELAKI-NIKSHYCH-OSTRIK

VIA BIMODULE CATEGORIES = (A-Bimod(C)) & PB, BQ& + CONDITIONS

SEE E-N-0'S PAPER "FUSION CATS. AND HOMOTOPY THY"

MATH 466/566 SPRING 2024

CHELSEA WALTON RICE U.



TOPICS:

I. PROPERTIES OF ALGEBRAS IN & (RECAP)

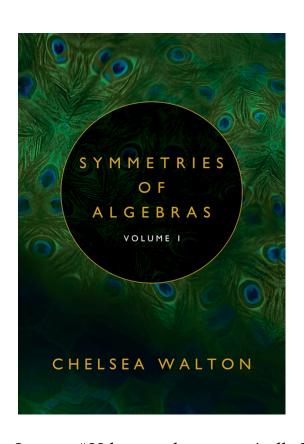
(849)

II. BIMODULES AND BEYOND (84.10.1)

IV. CATEGORICAL MORITA EQUIVALENCE (54.10.2)

Enjoy this lecture? You'll enjoy the textbook!

C. Walton's "Symmetries of Algebras, Volume 1" (2024)



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<u>Lecture #22 keywords</u>: categorically Morita equivalent, category of bimodules, connected algebra, exact algebra, indecomposable algebra, semisimple algebra, separable algebra, simple algebra