II. REPRESENTATIONS OF ALGEBRAS & GROUPS

$S \equiv$ ALGEBRAIC STRUCTURE E.G. GROUP, RING, ALGEBRA

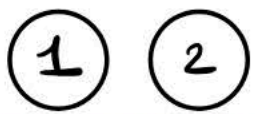
A REPRESENTATION OF S IS A STRUCTURE MAP $S \xrightarrow{\rho} \mathrm{End}(U)$

FOR $U \equiv$ ANOTHER STRUCTURE E.G. SET, ABELIAN GROUP, VSPACE (↑ WE CHOOSE)

FAITHFULNESS ENSURES THAT S DOES THIS ON THE NOSE, NOT UNNECESARILY BIG

CAN THINK OF ρ AS S CAPTURING SYMMETRIES OF U

$\mathrm{End}(U) \equiv \mathrm{Sym}(U)$

Eg. 

SYMMETRIES CAPTURED BY

THE RIGHT CHOICE $C_2, C_4, C_6, \cdots$

$=$

$\{e, g, g^2, g^3\}$

SWAP SWAP

DO NOTHING DO NOTHING

ρ IS FAITHFUL IF ρ IS INJECTIVE

$\Updownarrow$

THE ONLY ELEMENT OF S THAT DOES NOTHING TO U IS THE IDENTITY ELT OF S