Preview for Lecture #4 of C. Walton's Spring 2024 course on "Symmetries of Algebras" at Rice U.
Keywords: Bimodule Tensor-Hom adjunction, coinduction, direct product/direct sum of algebras, direct product/direct sum of modules, Frobenius Reciprocity, induction, restriction, tensor product of modules
https://math.rice.edu/~notlaw/symalgbook.html, https://www.619wreath.com/

III. HOM AND DUAL OF ALGEBRAS & MODULES

ALGEBRA

A vs

$m: A \otimes A \to A$

$u: \mathbb{k} \otimes A \to A$

& COMP. AXIOMS

LEFT A-MODULE $_AV$

V vs

$\triangleright: A \otimes V \to V$

& COMP. AXIOMS

RIGHT A-MODULE V_A

V vs

$\triangleleft: V \otimes A \to V$

& COMP. AXIOMS

(B_1, B_2)-BIMODULE $_{B_1}V_{B_2}$

V vs

$\triangleright: B_1 \otimes V \to V$

$\triangleleft: V \otimes B_2 \to V$

& COMP. AXIOMS

$$\boxed{\mathrm{Hom}_{A\text{-mod}}(\underset{_AV_{B_1}}{V}, \underset{_AW_{B_2}}{W}) \equiv (B_1, B_2)\text{-BIMODULE}}$$

RECALL:

$$\boxed{\underset{_{B_1}V_A}{V} \otimes_A \underset{_AW_{B_2}}{W} \equiv (B_1, B_2)\text{-BIMODULE}}$$

TENSOR ADJUNCTION FOR VECTOR SPACES:

$$\boxed{\mathrm{Hom}_{\mathbb{k}}(U \otimes V, W) \cong \mathrm{Hom}_{\mathbb{k}}(V, \mathrm{Hom}_{\mathbb{k}}(U, W))}$$

EXER. 1.25 DERIVE TENSOR-HOM ADJUNCTION FOR BIMODULES

You do!

$$\mathrm{Hom}_{??}(U \otimes_{??} V, W) \cong \mathrm{Hom}_{??}(V, \mathrm{Hom}_{??}(U, W))$$

$$U = {}_{??}U_{??} \qquad V = {}_{??}V_{??} \qquad W = {}_{??}W_{??}$$