

I. FUNCTORS

ALGEBRAIC FUNCTORS...

/ \mathbb{R} FIELD

A V FUNCTOR $F: \mathcal{C} \rightarrow \mathcal{D}$
(RESP., CONTRAVARIANT)

CONSISTS OF:

- (a) $F(x) \in \mathcal{D} \quad \forall x \in \mathcal{C}$.
- (b) $F(g) \in \text{Hom}_{\mathcal{D}}(F(x), F(y))$
(RESP.,
 $F(g) \in \text{Hom}_{\mathcal{D}}(F(y), F(x))$)
 $\forall g: x \rightarrow y \in \mathcal{C}$.

RESPECTING:

- $F(\text{id}_x) = \text{id}_{F(x)} \quad \forall x \in \mathcal{C}$
- $F(hg) = F(h)F(g)$
 $\forall g: x \rightarrow y, h: y \rightarrow z \in \mathcal{C}$
(RESP.,
 $F(gf) = F(f)F(g)$)
 $\forall f: w \rightarrow x, g: x \rightarrow y \in \mathcal{C}$)

$$\text{Hom}_{\mathbb{R}}(V, -) : \text{Vec} \rightarrow \text{Vec} \quad \text{FOR FIXED } V \in \text{Vec}$$

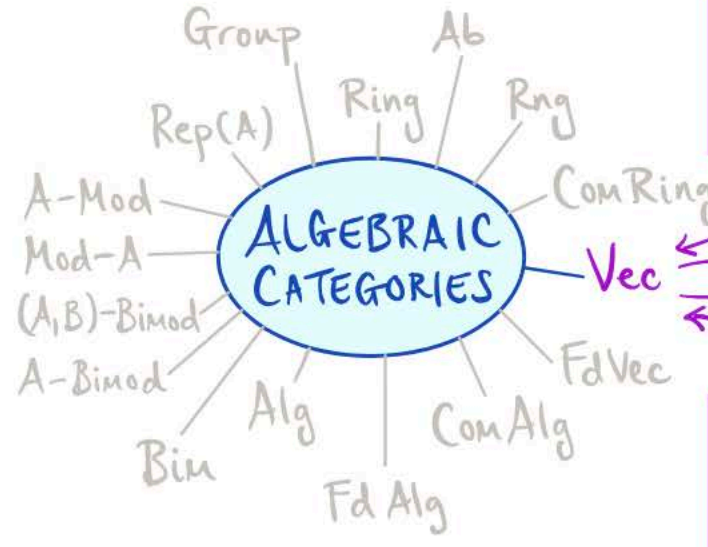
$$W \mapsto \text{Hom}_{\mathbb{R}}(V, W)$$

$$[W \xrightarrow{g} W'] \mapsto [\text{Hom}_{\mathbb{R}}(V, W) \rightarrow \text{Hom}_{\mathbb{R}}(V, W')]$$

$$f \mapsto gf$$

COVARIANT

CONTRA-VARIANT



$$\text{Hom}_{\mathbb{R}}(V, -)$$

FOR $V \in \text{Vec}$

$$\text{Hom}_{\mathbb{R}}(-, W)$$

FOR $W \in \text{Vec}$

$$\text{Hom}_{\mathbb{R}}(-, W) : \text{Vec} \rightarrow \text{Vec} \quad \text{FOR FIXED } W \in \text{Vec}$$

$$V \mapsto \text{Hom}_{\mathbb{R}}(V, W)$$

$$[V \xrightarrow{g} V'] \mapsto [\text{Hom}_{\mathbb{R}}(V', W) \rightarrow \text{Hom}_{\mathbb{R}}(V, W)]$$

$$f \mapsto fg$$