

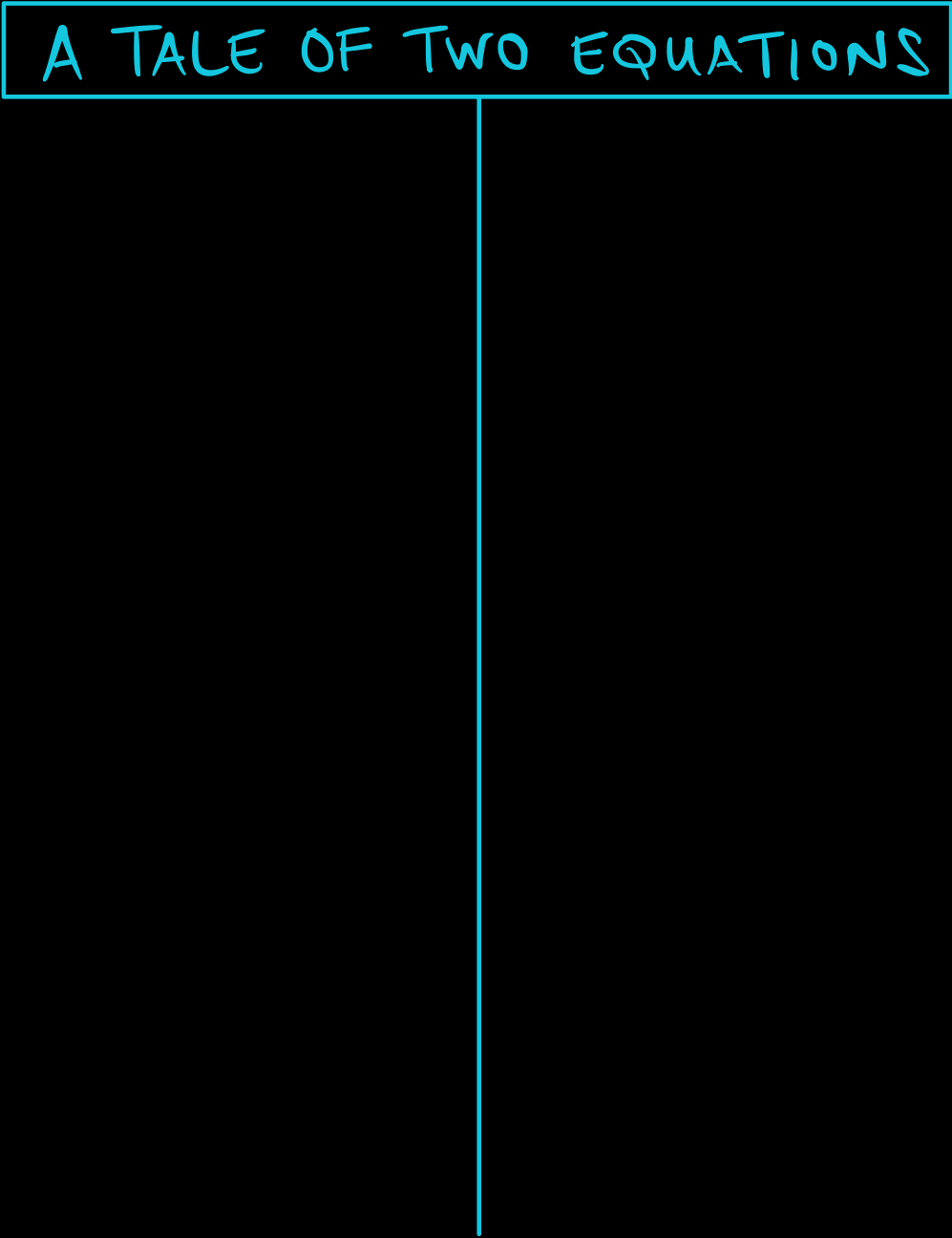
VENTURING TO THE BOUNDARY!

WITH BRAIDED MODULE CATEGORIES

CHELSEA WALTON
RICE UNIVERSITY

INTERNATIONAL CATEGORY THEORY CONFERENCE, JULY 2026

A TALE OF TWO EQUATIONS



YANG (1967)
BAXTER (1972)

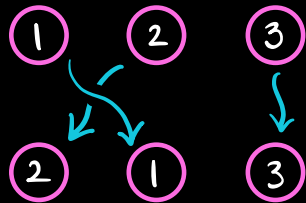
A TALE OF TWO EQUATIONS

↳ STUDIED HOW PARTICLES
SCATTER IN A PHYSICAL SYSTEM
WITHOUT BOUNDARY

YANG (1967)
BAXTER (1972)

A TALE OF TWO EQUATIONS

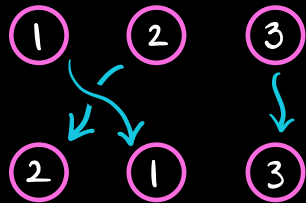
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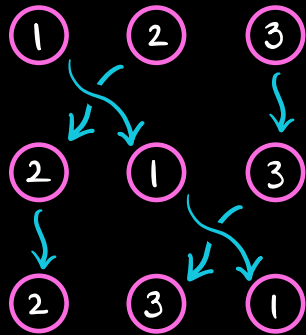
R_{ab} = PARTICLE a PASS PARTICLE b

R_{12}

YANG (1967)
BAXTER (1972)

A TALE OF TWO EQUATIONS

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SCATTER IN A PHYSICAL SYSTEM
WITHOUT BOUNDARY



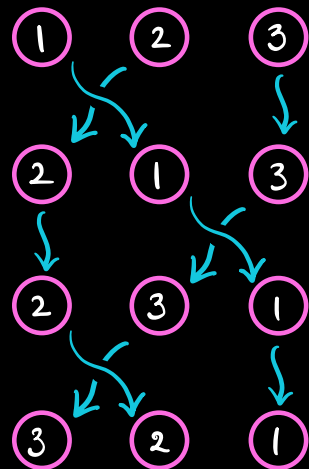
R_{ab} = PARTICLE a PASS PARTICLE b

$R_{12} R_{13}$

YANG (1967)
BAXTER (1972)

A TALE OF TWO EQUATIONS

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WITHOUT BOUNDARY



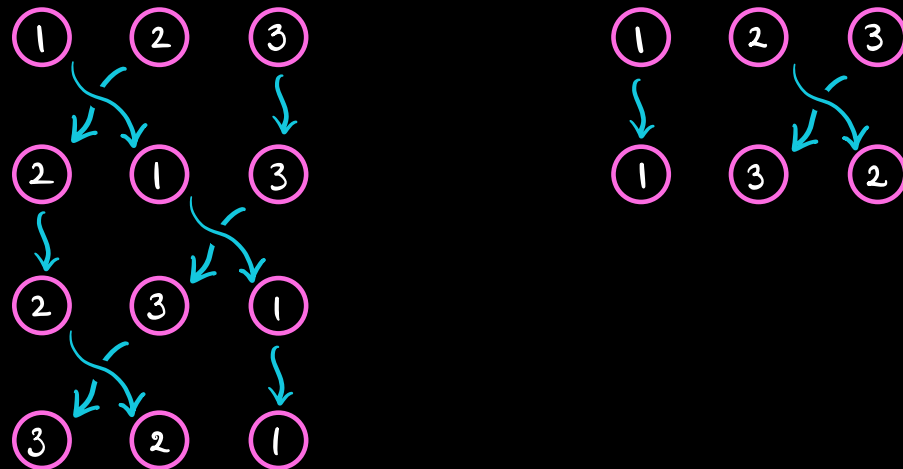
R_{ab} = PARTICLE a PASS PARTICLE b

$$R_{12} R_{13} R_{23}$$

YANG (1967)
BAXTER (1972)

A TALE OF TWO EQUATIONS

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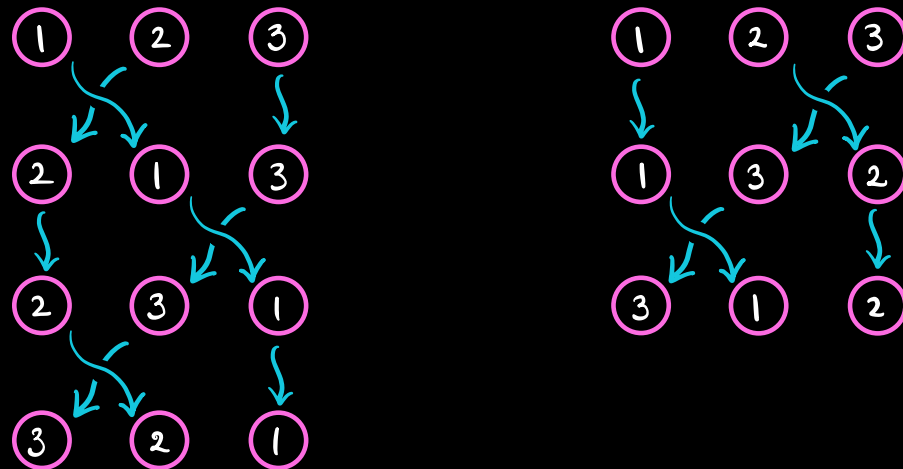
R_{ab} = PARTICLE a PASS PARTICLE b

$R_{12} R_{13} R_{23}$ R_{23}

YANG (1967)
BAXTER (1972)

A TALE OF TWO EQUATIONS

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R_{ab} = PARTICLE a PASS PARTICLE b

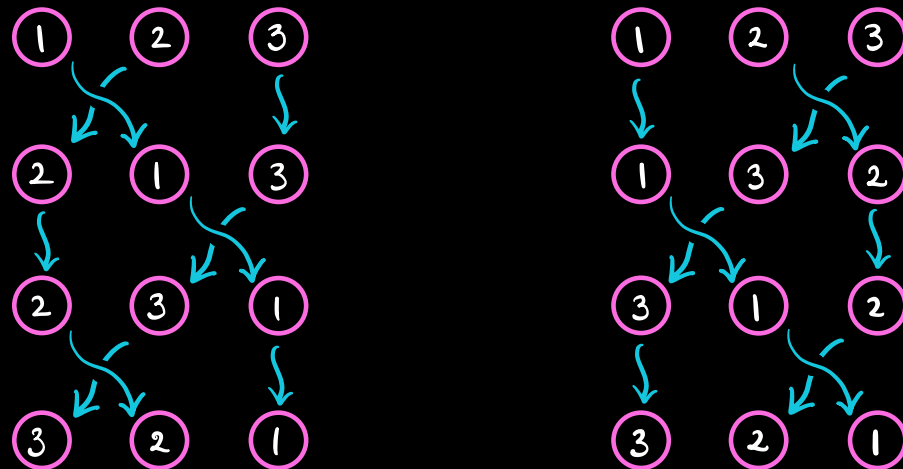
$R_{12} R_{13} R_{23}$

$R_{23} R_{13}$

YANG (1967)
BAXTER (1972)

A TALE OF TWO EQUATIONS

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SCATTER IN A PHYSICAL SYSTEM
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R_{ab} = PARTICLE a PASS PARTICLE b

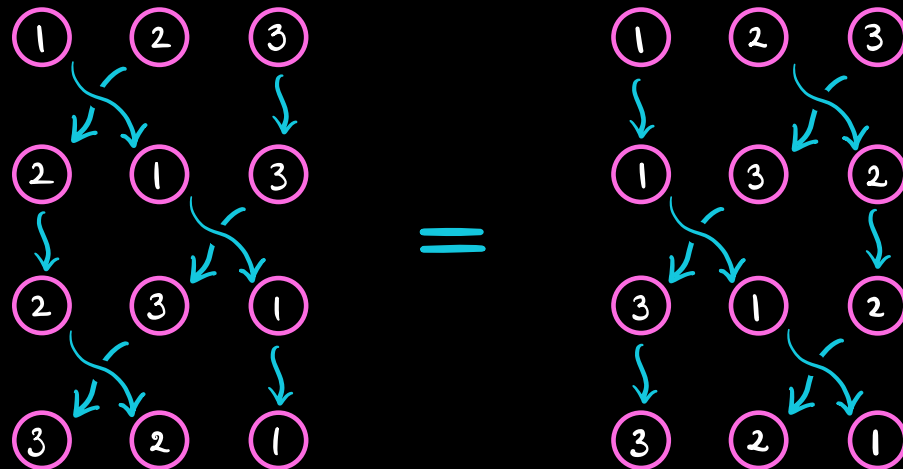
$R_{12} R_{13} R_{23}$

$R_{23} R_{13} R_{12}$

YANG (1967)
BAXTER (1972)

A TALE OF TWO EQUATIONS

↳ STUDIED HOW PARTICLES
SCATTER IN A PHYSICAL SYSTEM
WITHOUT BOUNDARY



R_{ab} = PARTICLE a PASS PARTICLE b

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

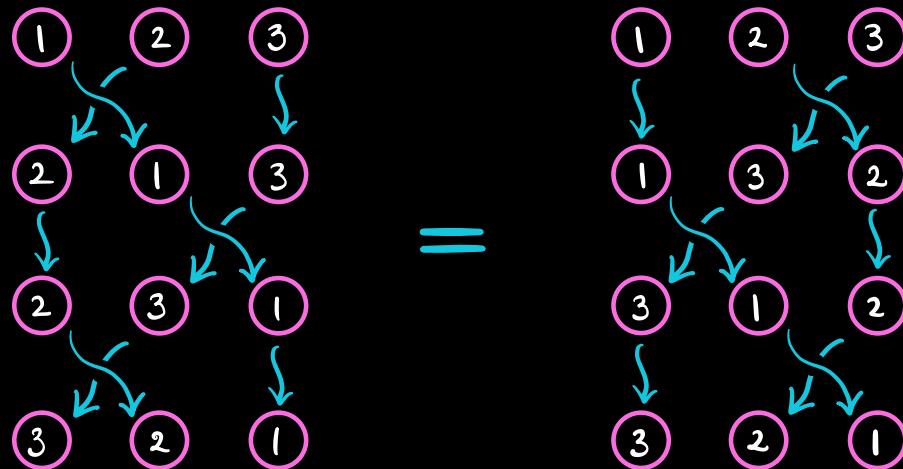
QUANTUM YANG-BAXTER EQ. (qYBE)

YANG (1967)
BAXTER (1972)

A TALE OF TWO EQUATIONS

CHEREDNIK (1984)
SKLYANIN (1988)

↪ STUDIED HOW PARTICLES
SCATTER IN A PHYSICAL SYSTEM
WITHOUT BOUNDARY



R_{ab} = PARTICLE a PASS PARTICLE b

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (QYBE)

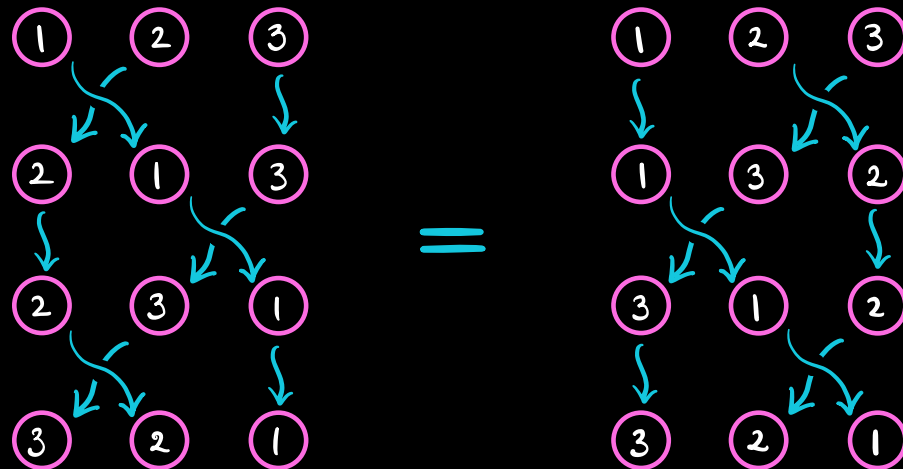
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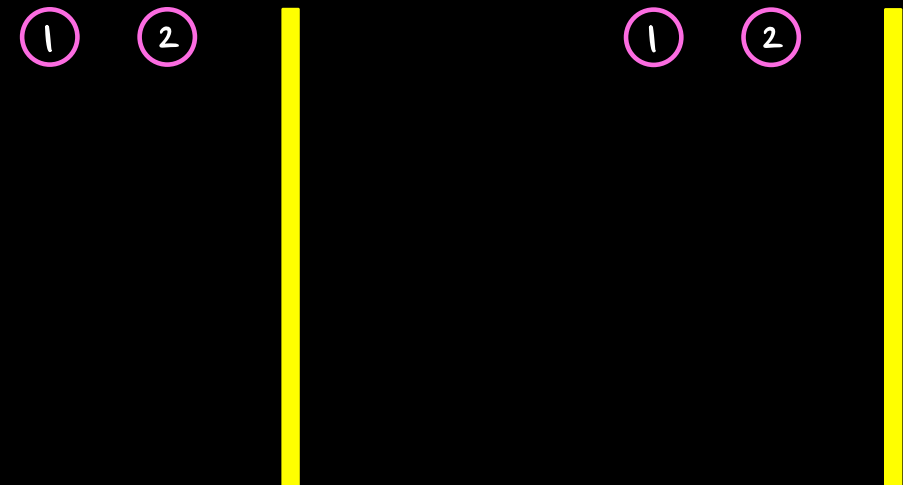


R_{ab} = PARTICLE a PASS PARTICLE b

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)

STUDIED HOW PARTICLES ↪
SCATTER IN A PHYSICAL SYSTEM
WITH BOUNDARY



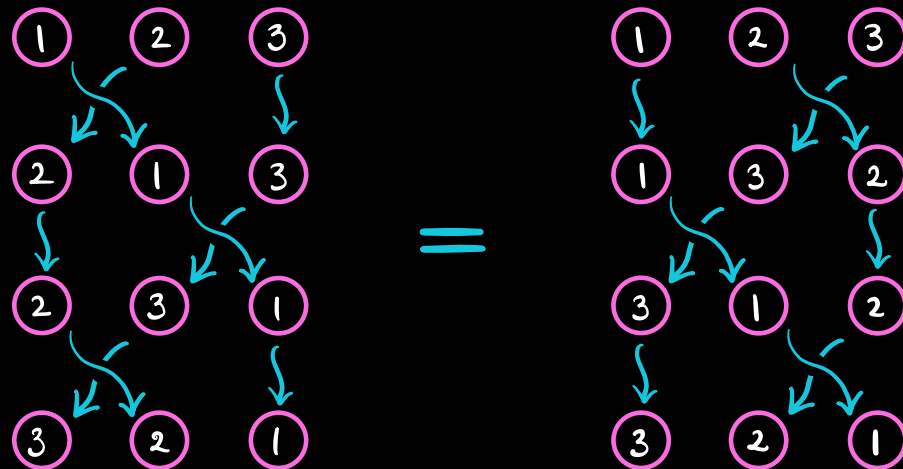
K_c = PARTICLE c AGAINST BOUNDARY

YANG (1967)
BAXTER (1972)

A TALE OF TWO EQUATIONS

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SKLYANIN (1988)

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WITHOUT BOUNDARY

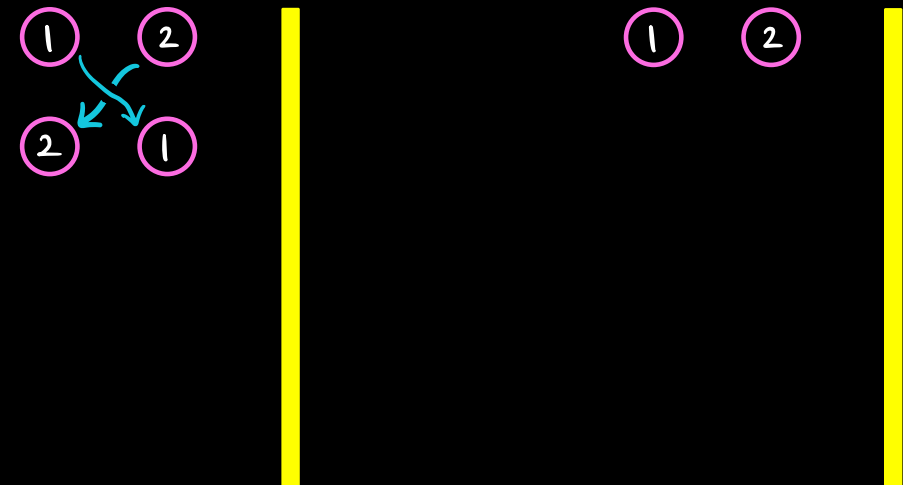


R_{ab} = PARTICLE a PASS PARTICLE b

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)

STUDIED HOW PARTICLES
SCATTER IN A PHYSICAL SYSTEM
WITH BOUNDARY



K_c = PARTICLE c AGAINST BOUNDARY

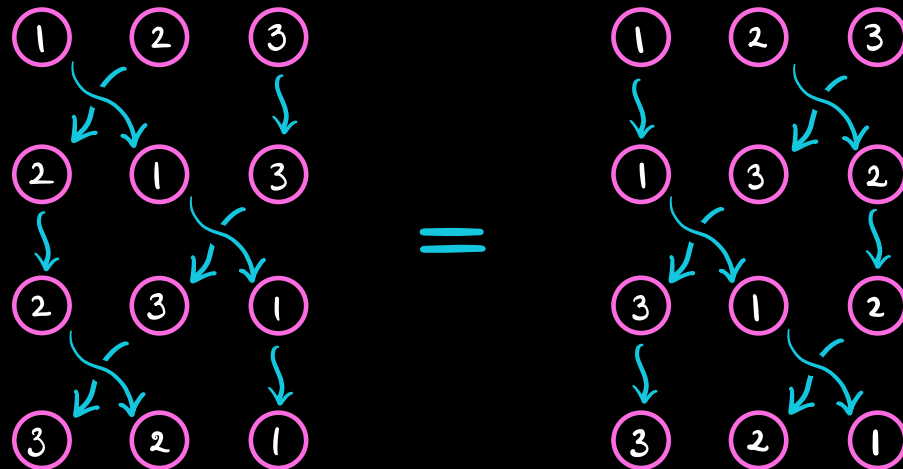
$$R_{12}$$

YANG (1967)
BAXTER (1972)

A TALE OF TWO EQUATIONS

CHEREDNIK (1984)
SKLYANIN (1988)

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WITHOUT BOUNDARY

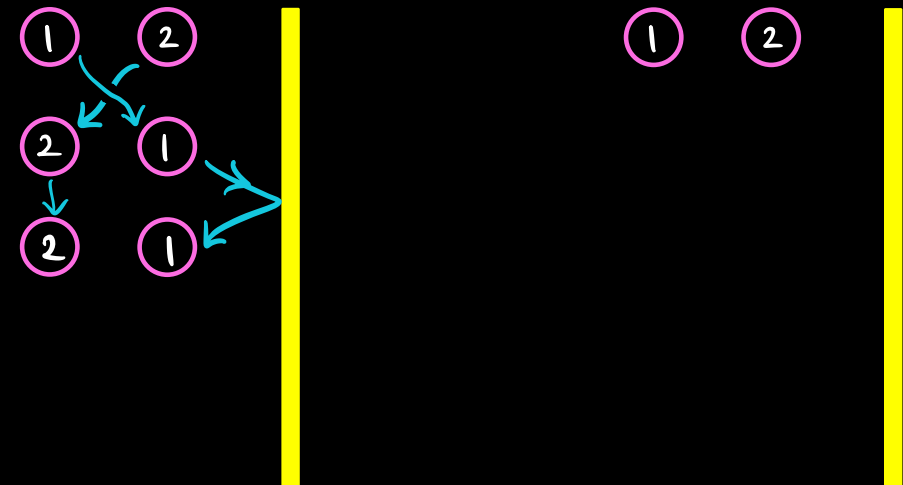


R_{ab} = PARTICLE a PASS PARTICLE b

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (QYBE)

STUDIED HOW PARTICLES
SCATTER IN A PHYSICAL SYSTEM
WITH BOUNDARY



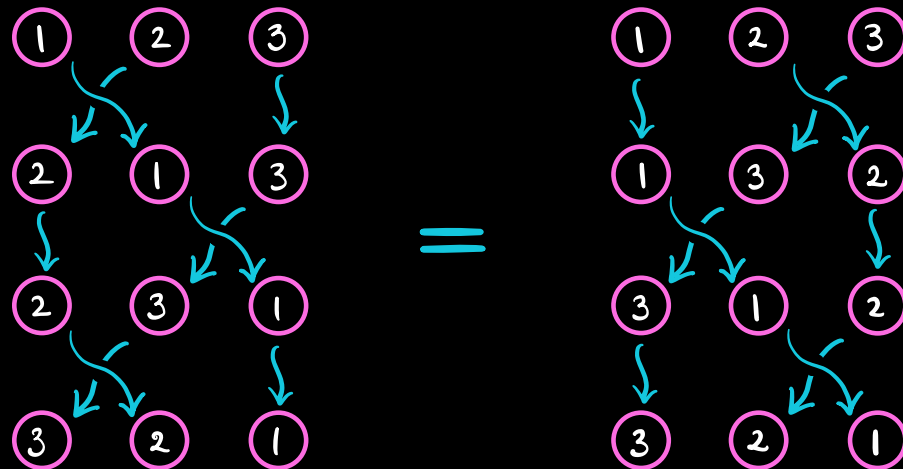
K_c = PARTICLE c AGAINST BOUNDARY

$$R_{12} K_1$$

YANG (1967)
BAXTER (1972)

A TALE OF TWO EQUATIONS

↪ STUDIED HOW PARTICLES
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WITHOUT BOUNDARY



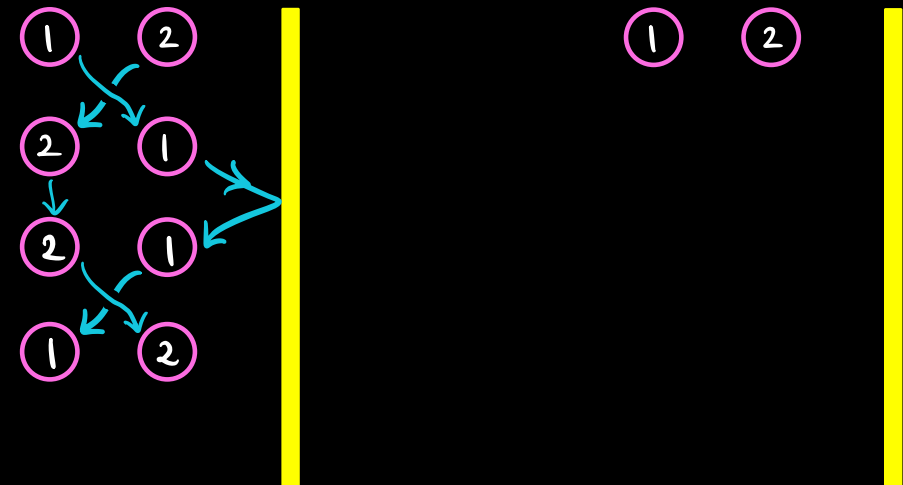
R_{ab} = PARTICLE a PASS PARTICLE b

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)

CHEREDNIK (1984)
SKLYANIN (1988)

STUDIED HOW PARTICLES ↪
SCATTER IN A PHYSICAL SYSTEM
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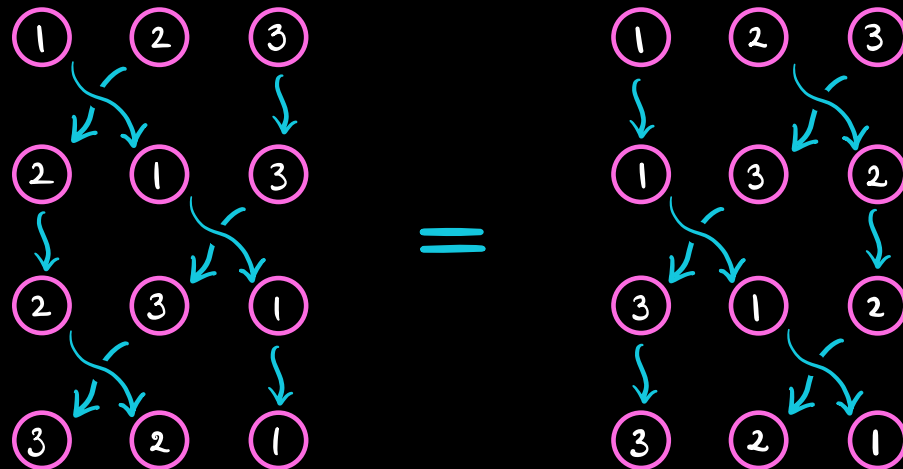
K_c = PARTICLE c AGAINST BOUNDARY

$$R_{12} K_1 R_{21}$$

YANG (1967)
BAXTER (1972)

A TALE OF TWO EQUATIONS

↪ STUDIED HOW PARTICLES
SCATTER IN A PHYSICAL SYSTEM
WITHOUT BOUNDARY



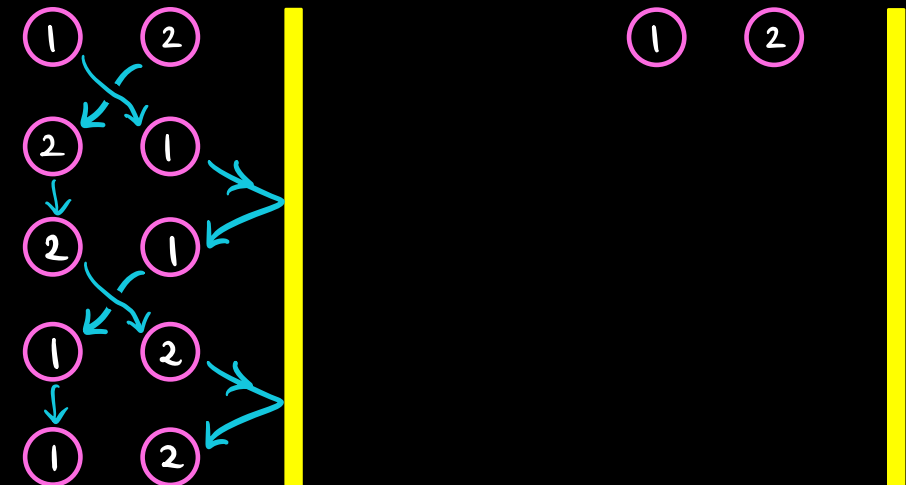
R_{ab} = PARTICLE a PASS PARTICLE b

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (QYBE)

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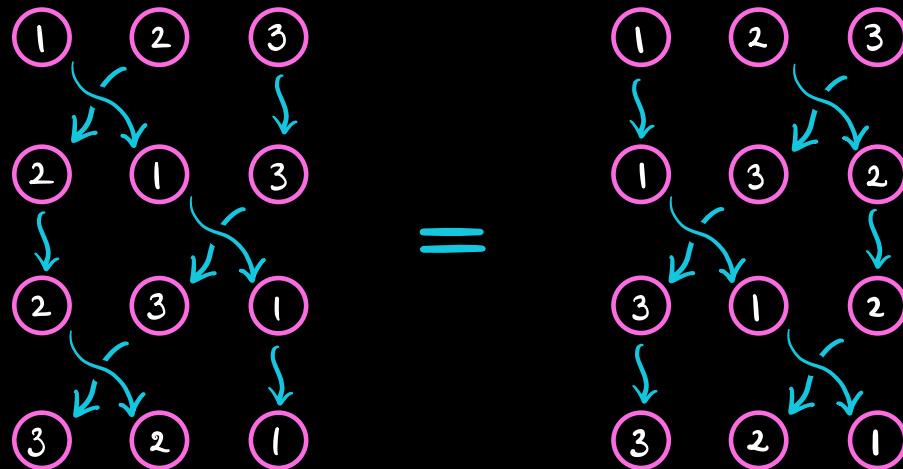
K_c = PARTICLE c AGAINST BOUNDARY

$$R_{12} K_1 R_{21} K_2$$

YANG (1967)
BAXTER (1972)

A TALE OF TWO EQUATIONS

STUDIED HOW PARTICLES
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WITHOUT BOUNDARY



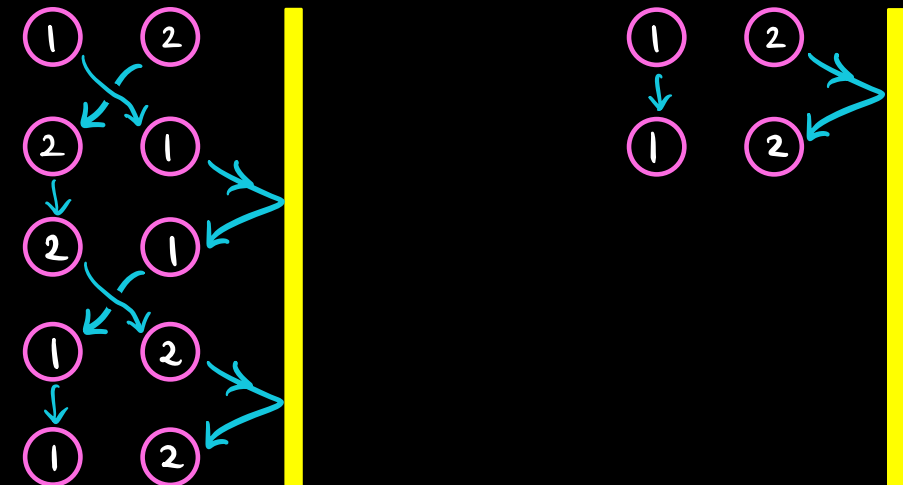
R_{ab} = PARTICLE a PASS PARTICLE b

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)

CHEREDNIK (1984)
SKLYANIN (1988)

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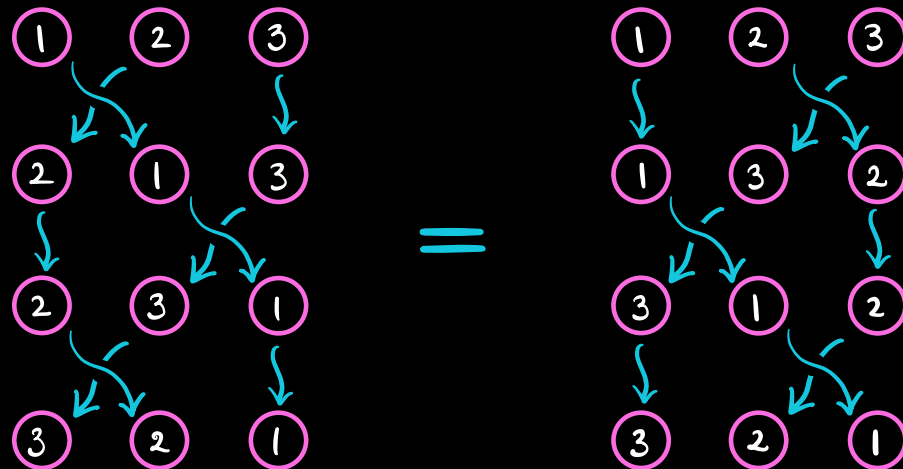
K_c = PARTICLE c AGAINST BOUNDARY

$$R_{12} K_1 R_{21} K_2 = K_2$$

YANG (1967)
BAXTER (1972)

A TALE OF TWO EQUATIONS

STUDIED HOW PARTICLES
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WITHOUT BOUNDARY



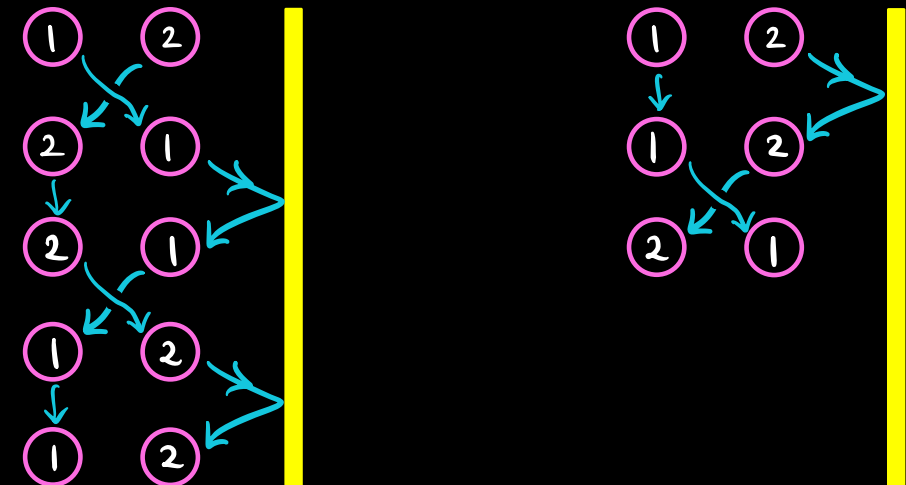
R_{ab} = PARTICLE a PASS PARTICLE b

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)

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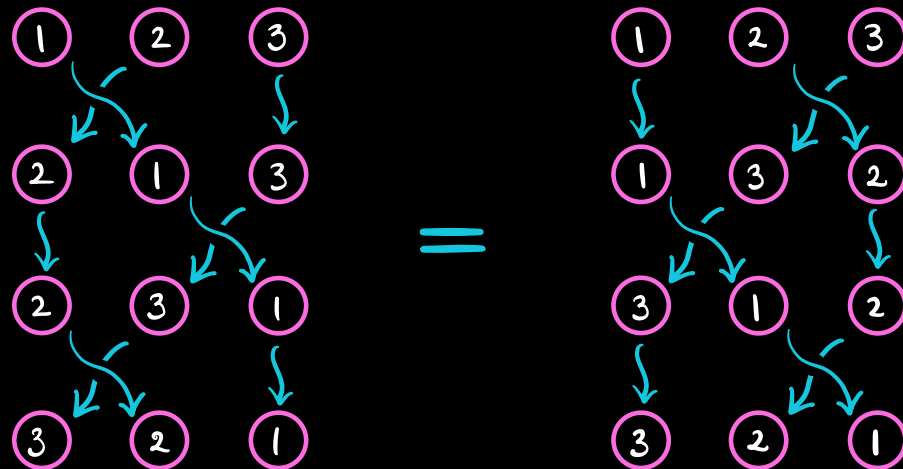
K_c = PARTICLE c AGAINST BOUNDARY

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12}$$

YANG (1967)
BAXTER (1972)

A TALE OF TWO EQUATIONS

STUDIED HOW PARTICLES
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WITHOUT BOUNDARY



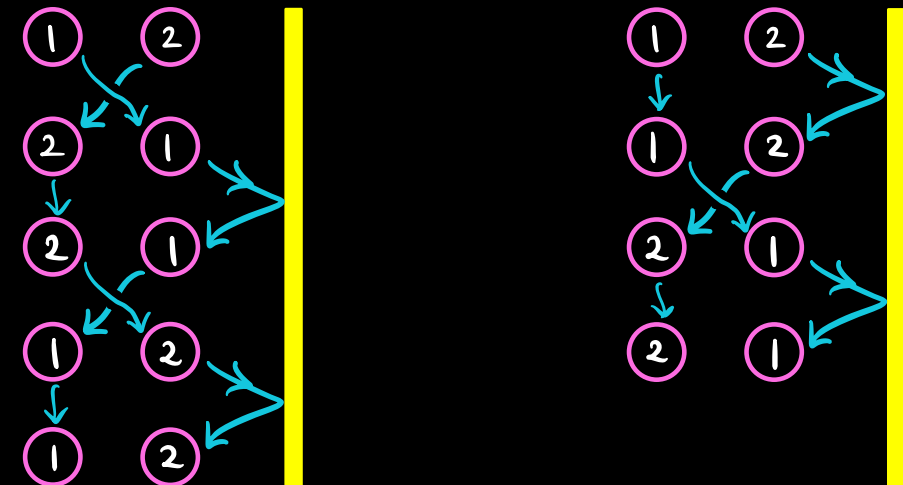
R_{ab} = PARTICLE a PASS PARTICLE b

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)

CHEREDNIK (1984)
SKLYANIN (1988)

STUDIED HOW PARTICLES
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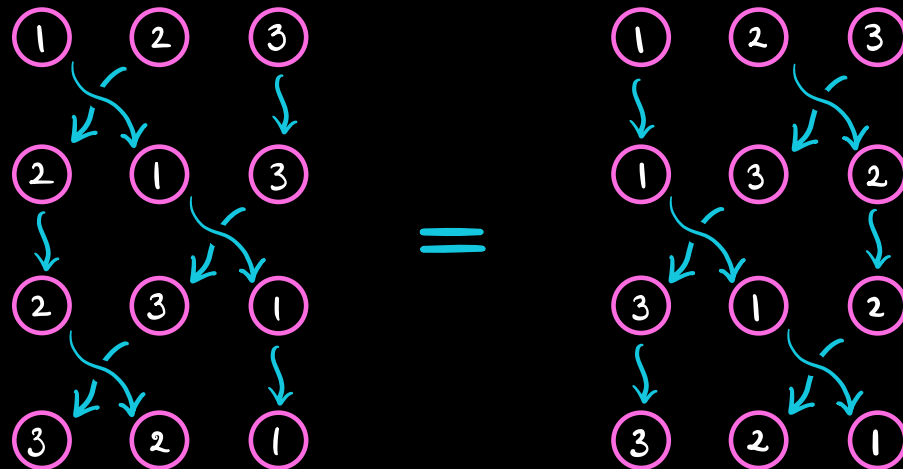
K_c = PARTICLE c AGAINST BOUNDARY

$$R_{12} K_1 R_{21} K_2 \quad K_2 R_{12} K_1$$

YANG (1967)
BAXTER (1972)

A TALE OF TWO EQUATIONS

STUDIED HOW PARTICLES
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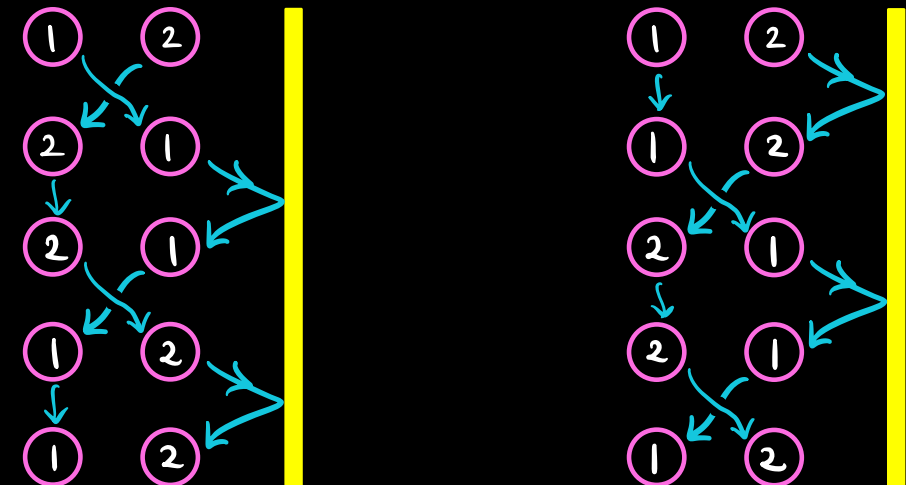
R_{ab} = PARTICLE a PASS PARTICLE b

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

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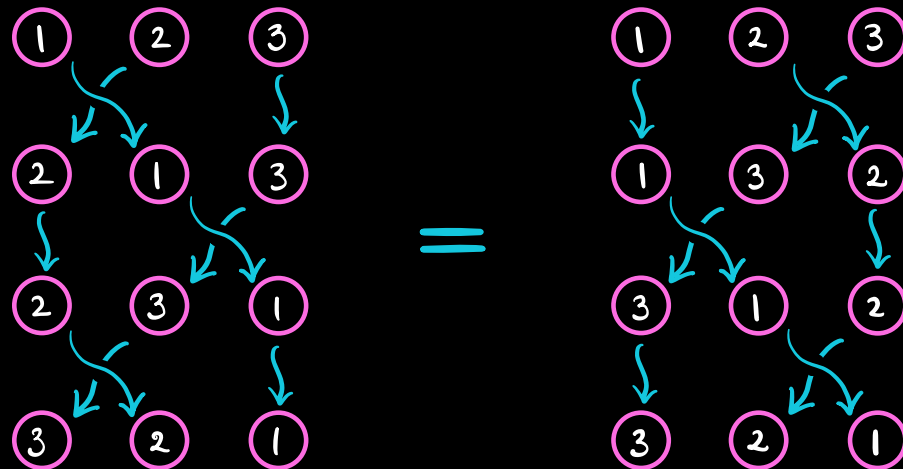
K_c = PARTICLE c AGAINST BOUNDARY

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

YANG (1967)
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A TALE OF TWO EQUATIONS

STUDIED HOW PARTICLES
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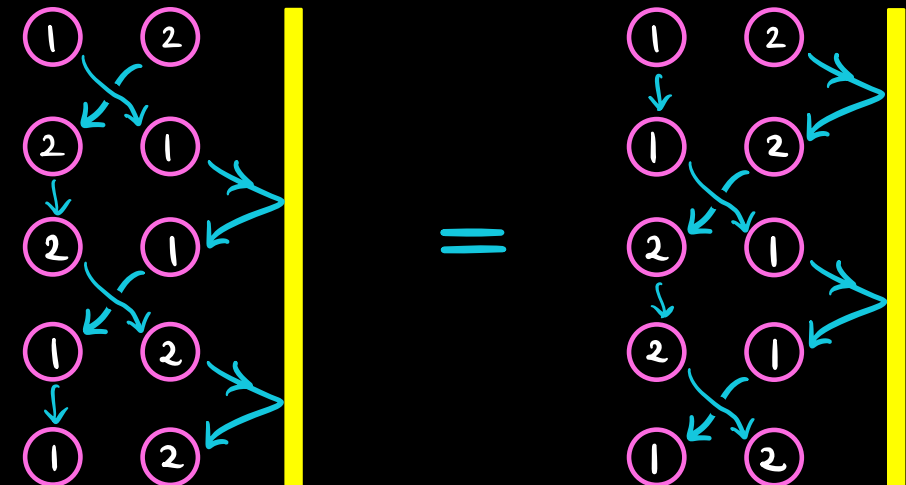
R_{ab} = PARTICLE a PASS PARTICLE b

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)

CHEREDNIK (1984)
SKLYANIN (1988)

STUDIED HOW PARTICLES
SCATTER IN A PHYSICAL SYSTEM
WITH BOUNDARY



K_c = PARTICLE c AGAINST BOUNDARY

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)

YANG (1967)
BAXTER (1972)

STUDIED HOW PARTICLES
SCATTER IN A PHYSICAL SYSTEM
WITHOUT BOUNDARY

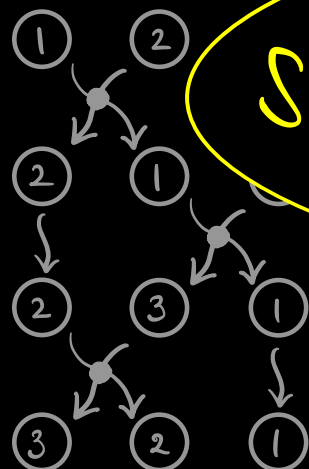
A TALE OF TWO EQUATIONS

CHEREDNIK (1984)
SKLYANIN (1988)

STUDIED HOW PARTICLES
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WITH BOUNDARY

GOAL

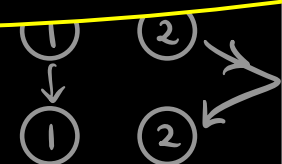
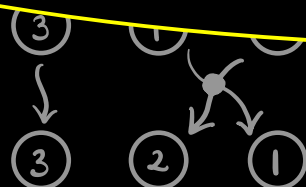
STUDY SOLUTIONS TO THESE EQUATIONS



R_{ab} = PARTICLE a PASS PARTICLE b

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

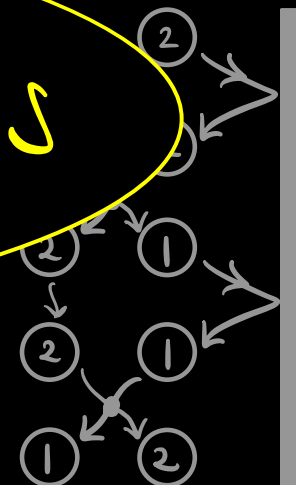
QUANTUM YANG-BAXTER EQ. (qYBE)



K_c = PARTICLE c AGAINST BOUNDARY

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)



YANG (1967)
BAXTER (1972)

STUDIED HOW PARTICLES
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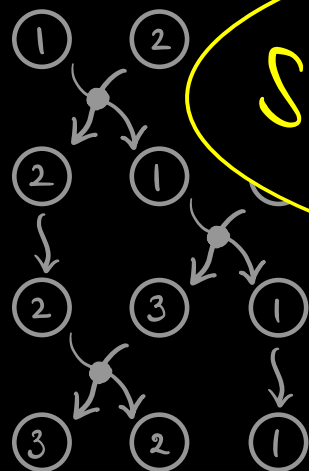
A TALE OF TWO EQUATIONS

CHEREDNIK (1984)
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STUDIED HOW PARTICLES
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GOAL

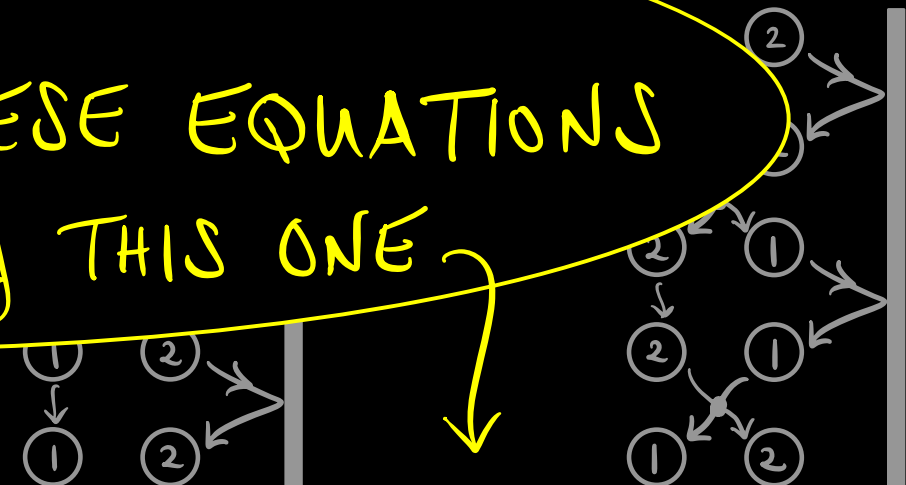
STUDY SOLUTIONS TO THESE EQUATIONS
ESPECIALLY THIS ONE



R_{ab} = PARTICLE a PASS PARTICLE b

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)

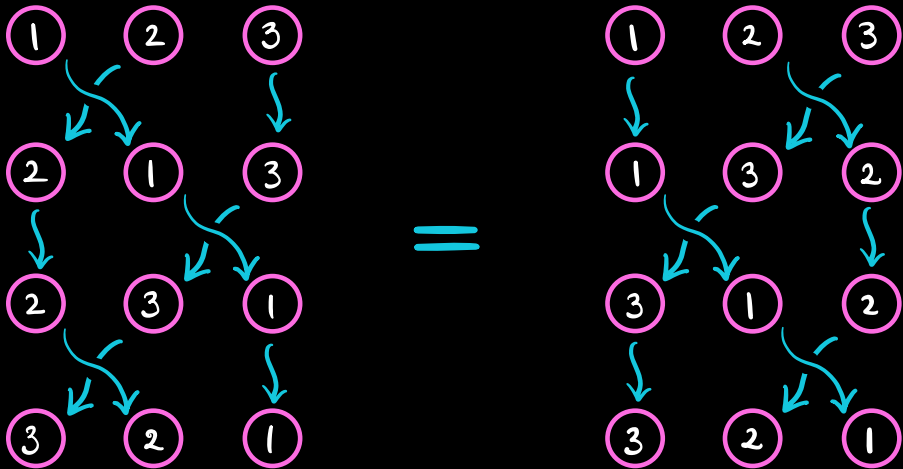


K_c = PARTICLE c AGAINST BOUNDARY

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)

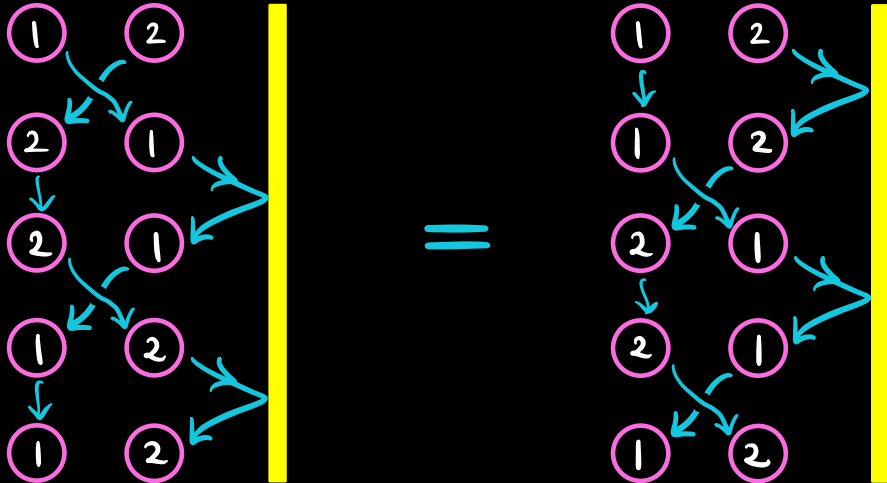
A TALE OF TWO EQUATIONS



$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)

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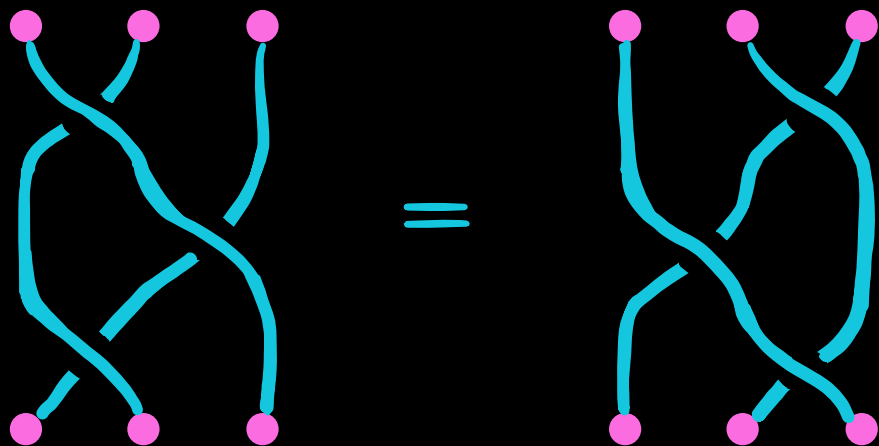
$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)

• VENTURING... BOUNDARY w/ BRAIDED MOD. CATS •

• INTL CT CONF., JULY 2026 •

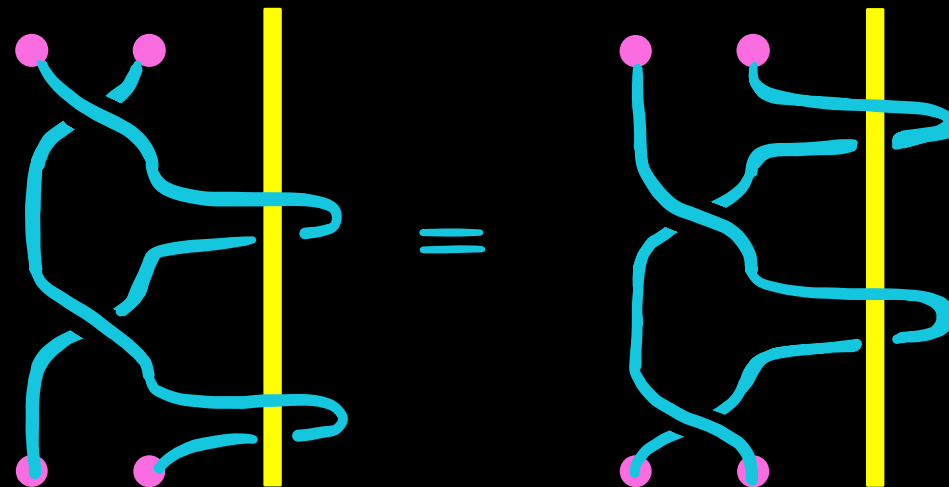
STRING MODELS



$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)

• CHELSEA WALTON (RICE UNIV.) •



$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

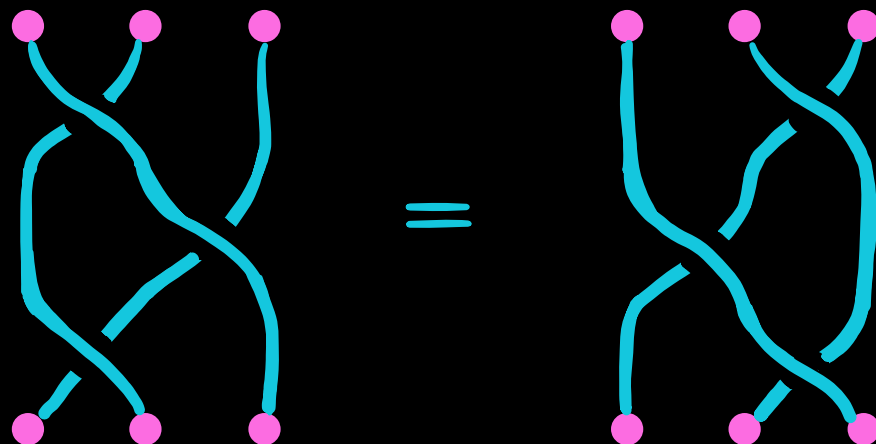
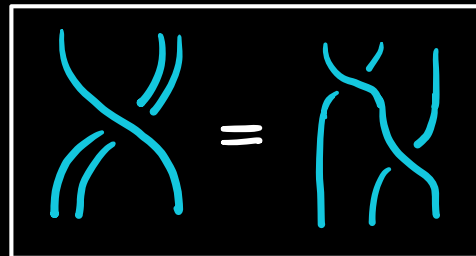
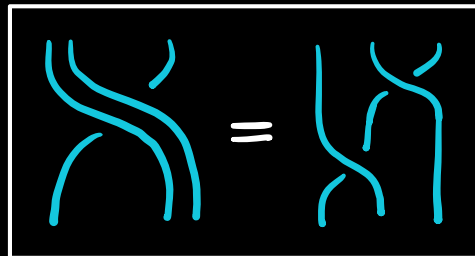
QUANTUM REFLECTION EQ (qRE)

• VENTURING... BOUNDARY w/ BRAIDED MOD. CATS •

• INTL CT CONF., JULY 2026 •

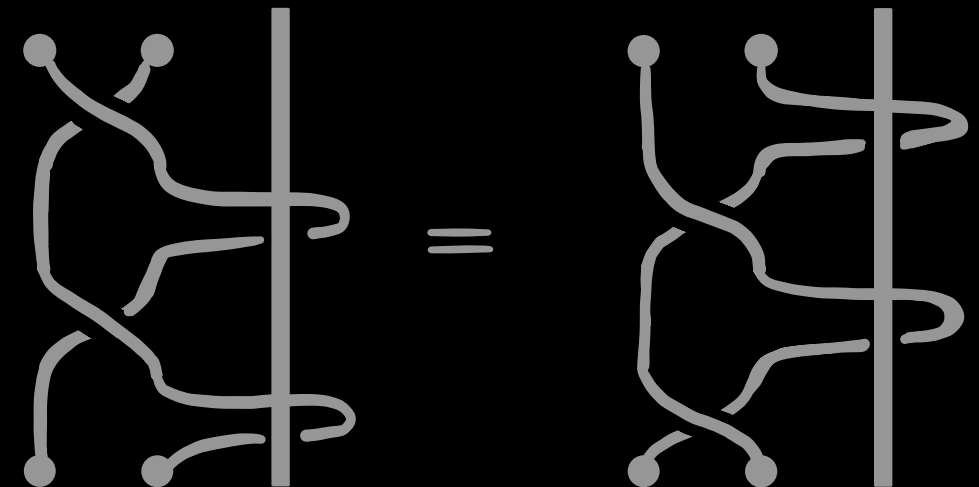
"BRAID AXIOMS"

STRING MODELS



$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)

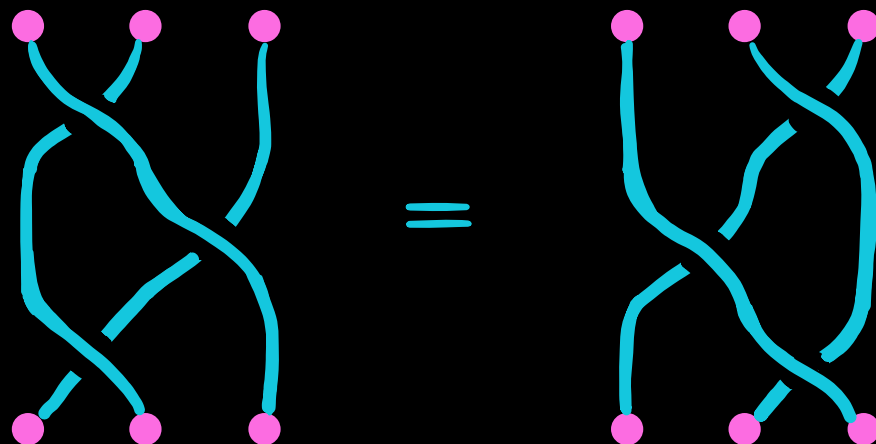
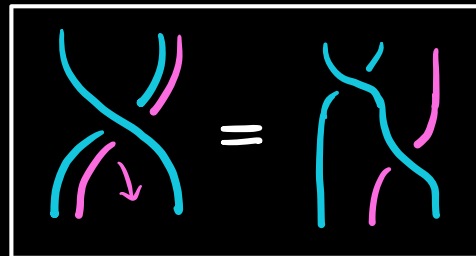
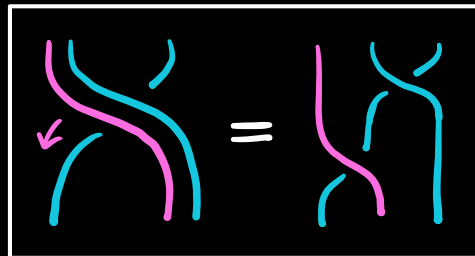


$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)

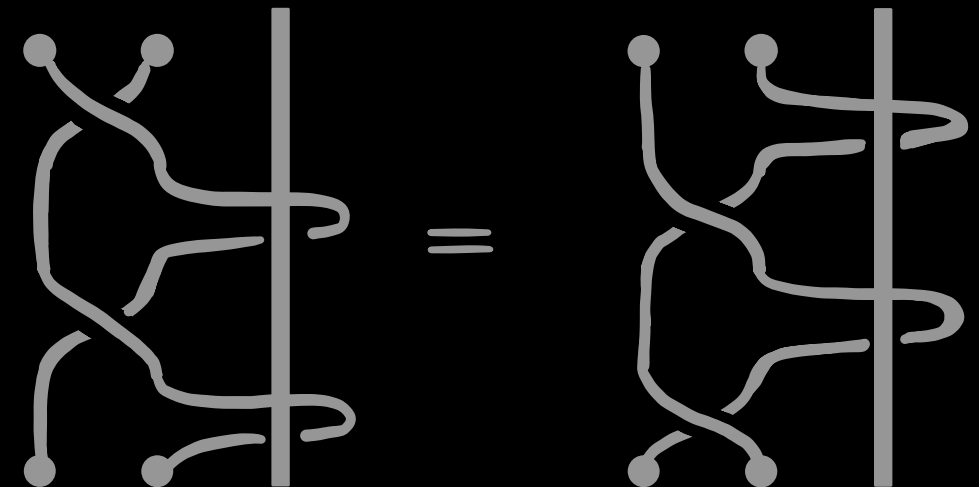
"BRAID AXIOMS"

STRING MODELS



$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)

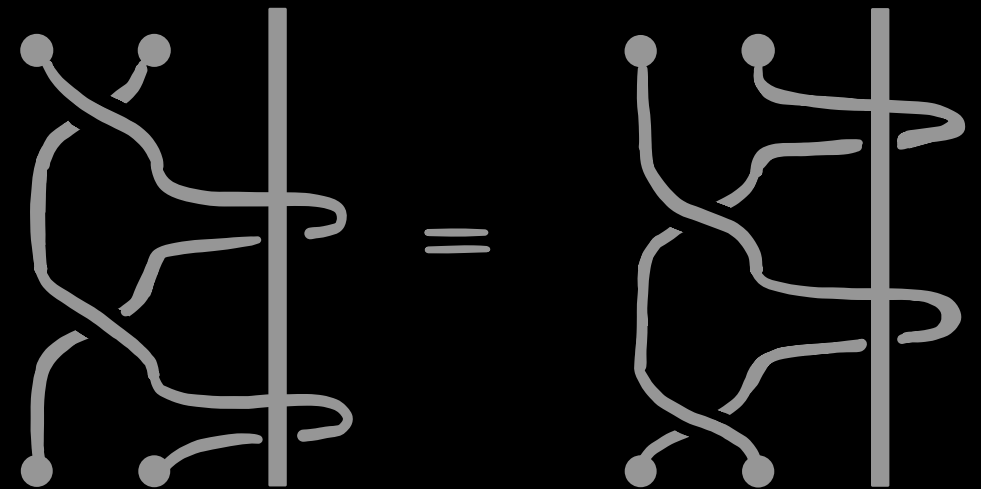
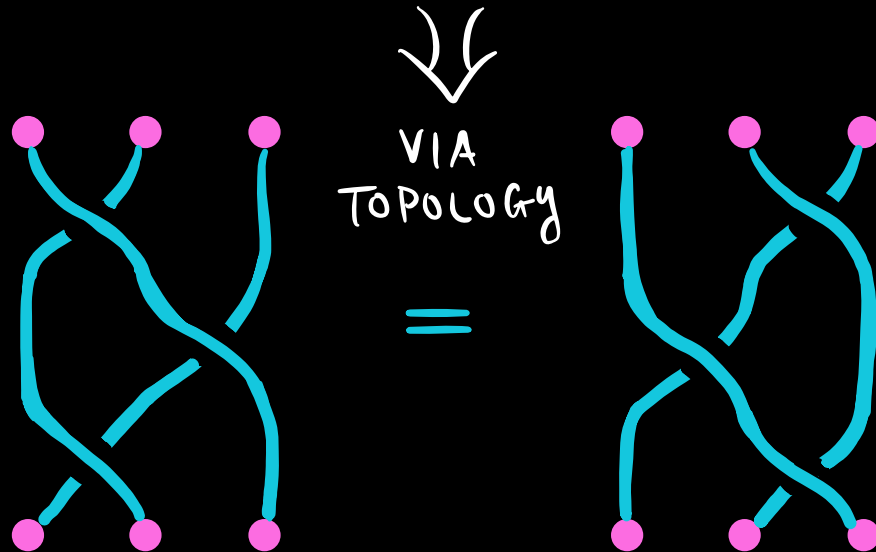
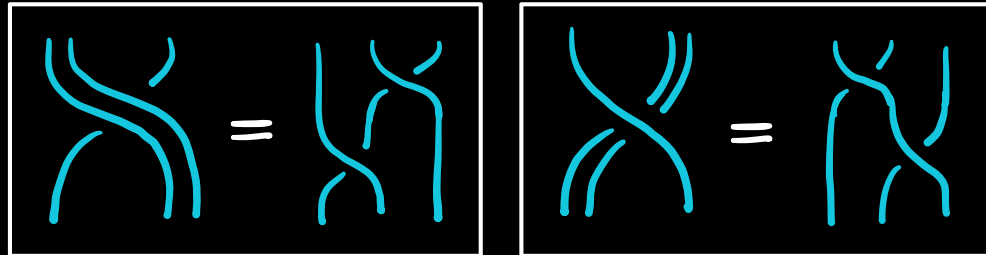


$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)

"BRAID AXIOMS"

STRING MODELS

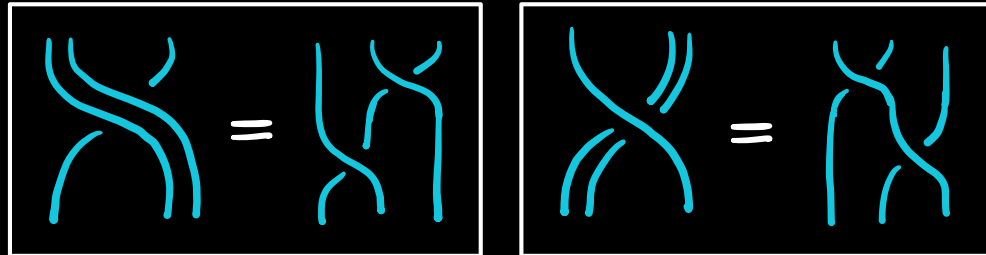


$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

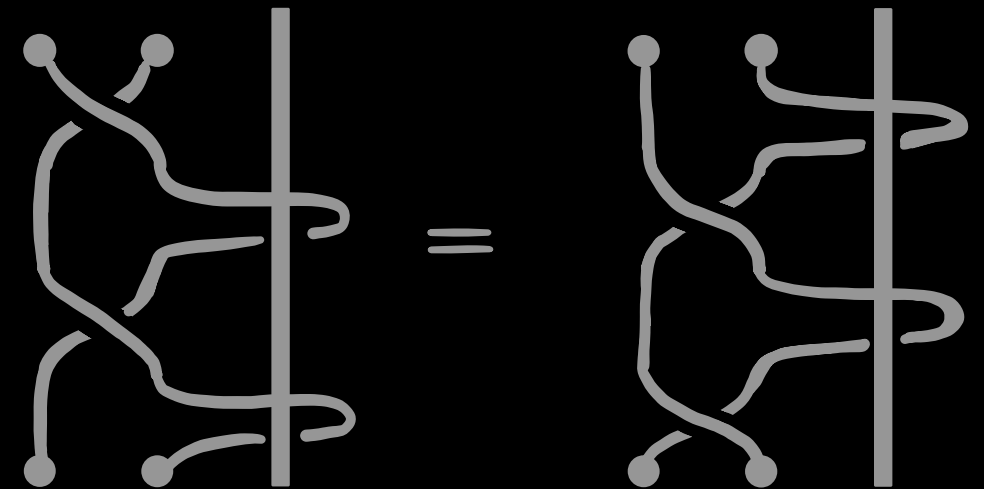
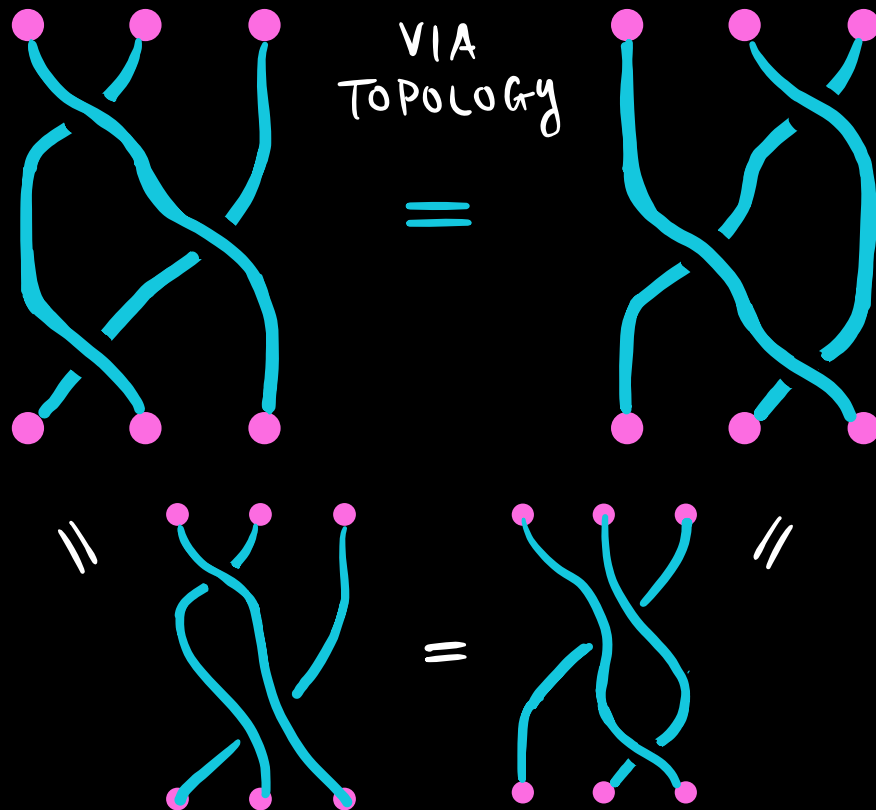
QUANTUM REFLECTION EQ (qRE)

"BRAID AXIOMS"

STRING MODELS



VIA
TOPOLOGY



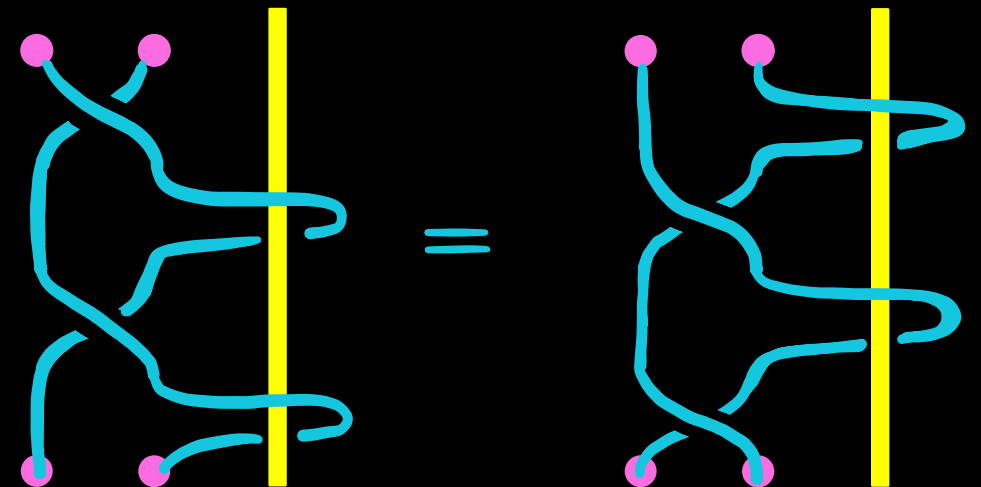
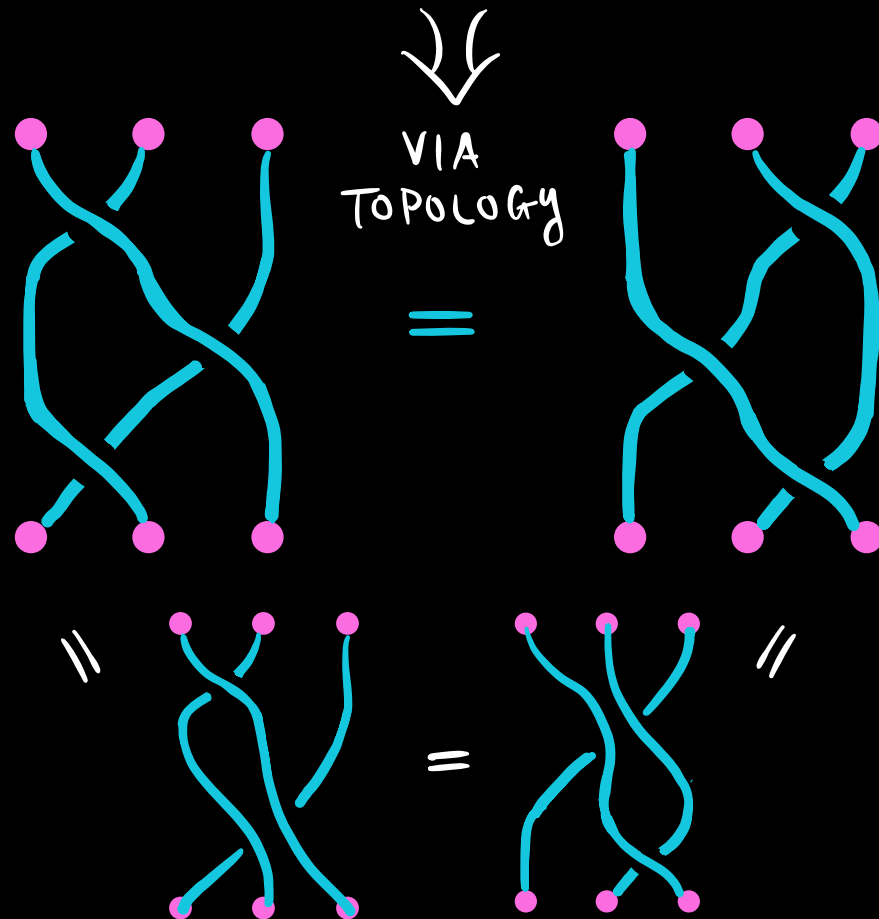
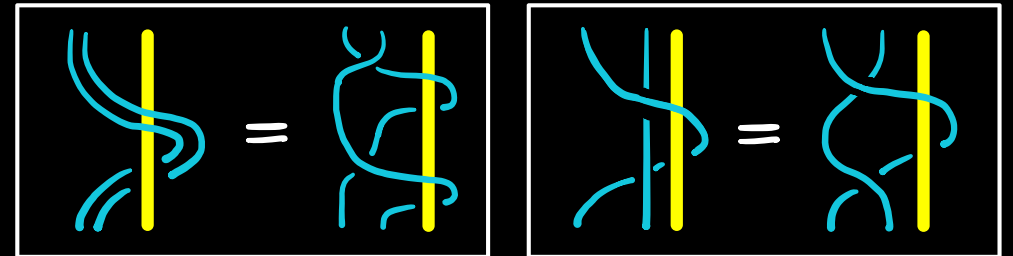
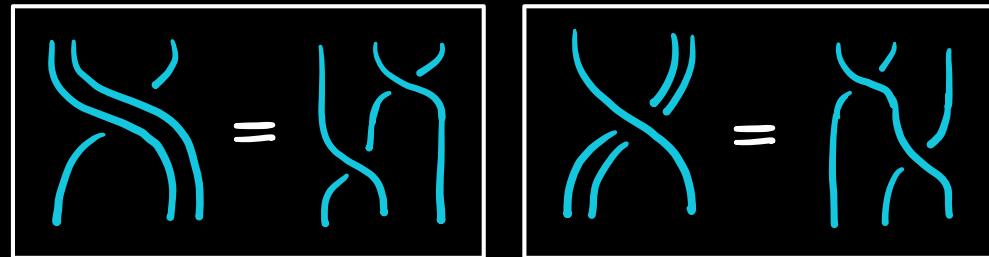
$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)

"BRAID AXIOMS"

STRING MODELS

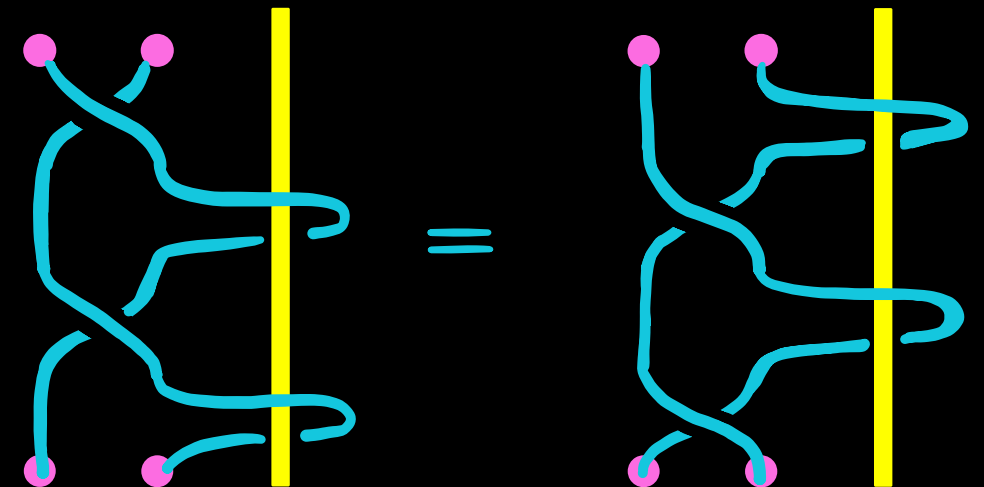
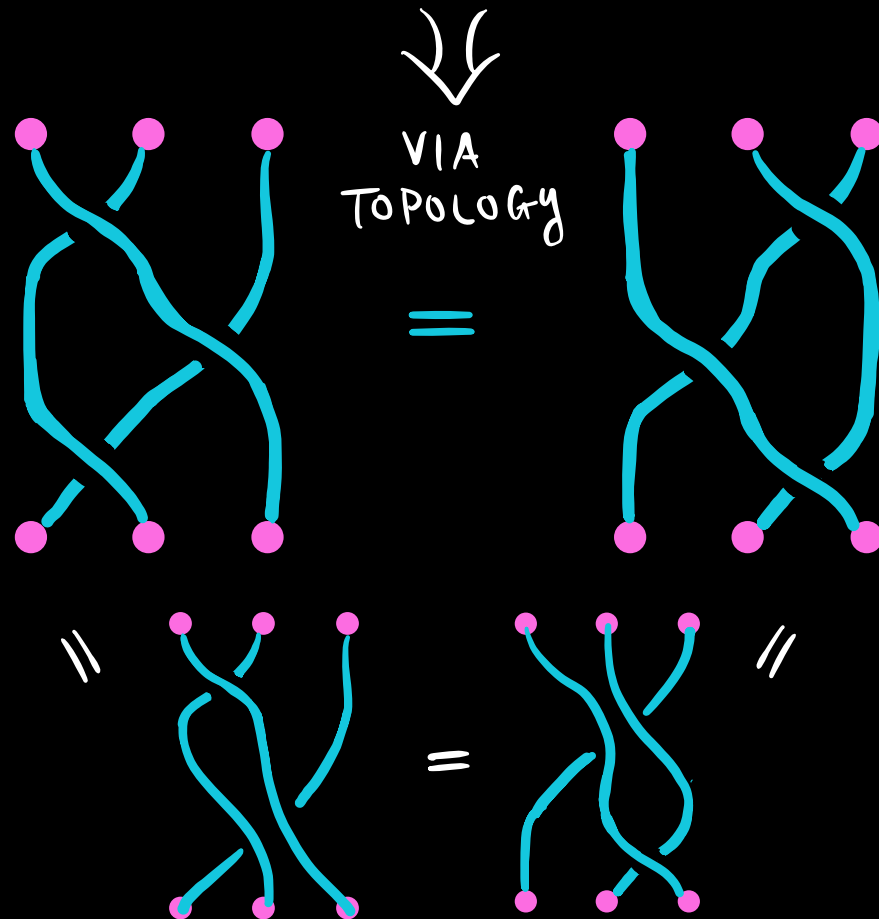
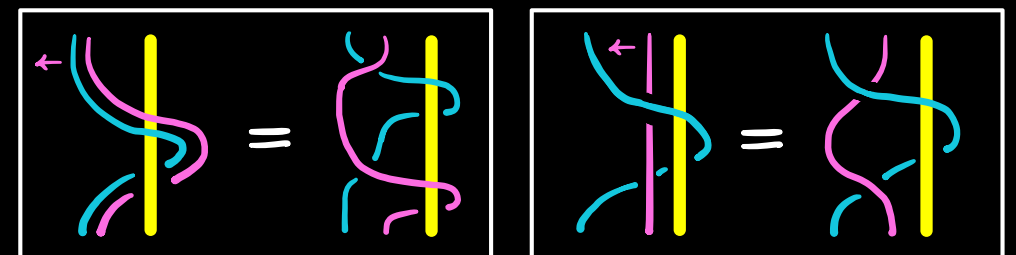
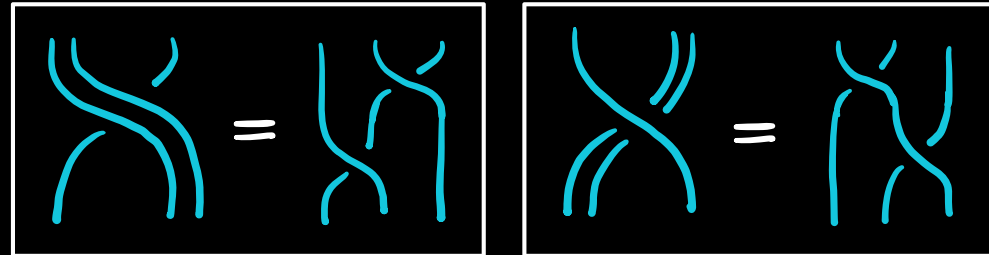
"BRAID AXIOMS"



"BRAID AXIOMS"

STRING MODELS

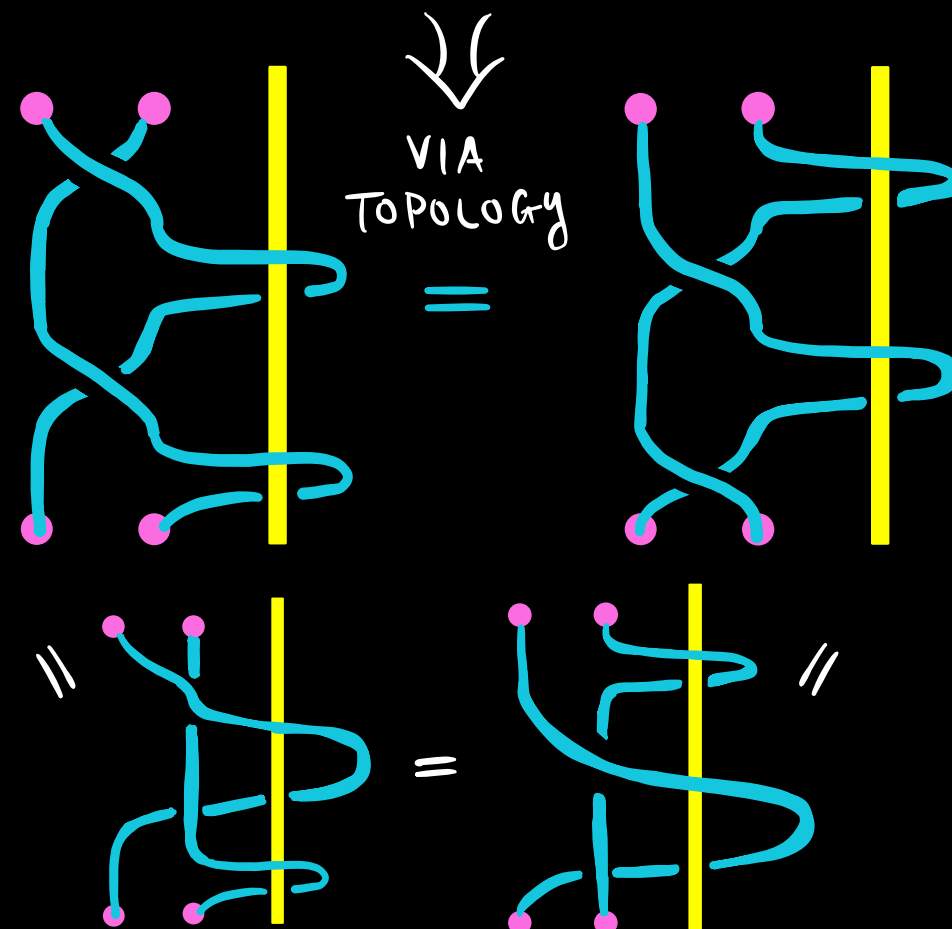
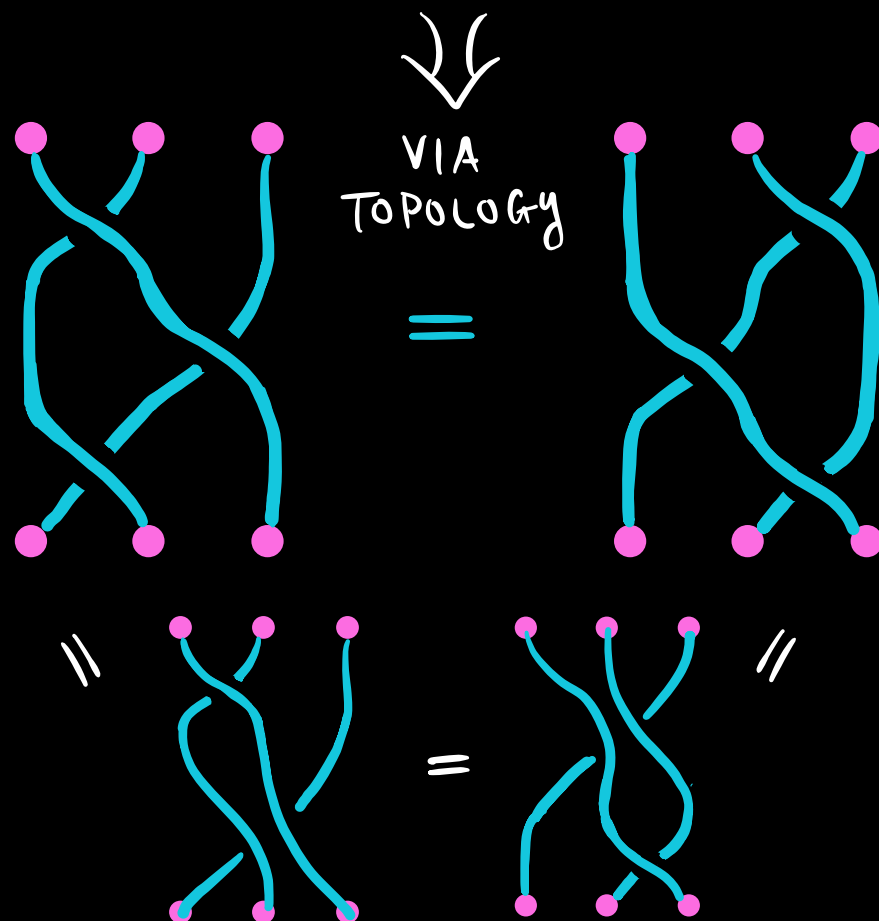
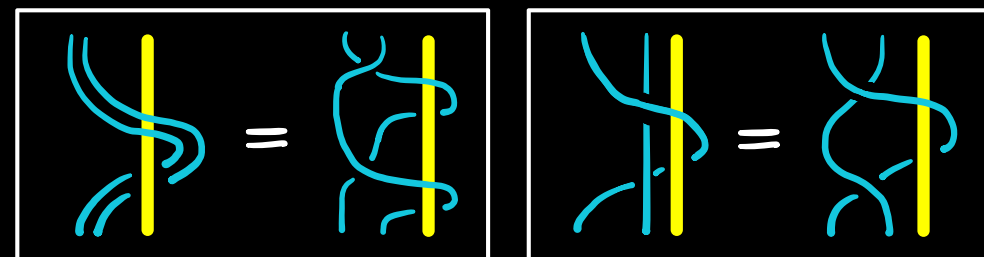
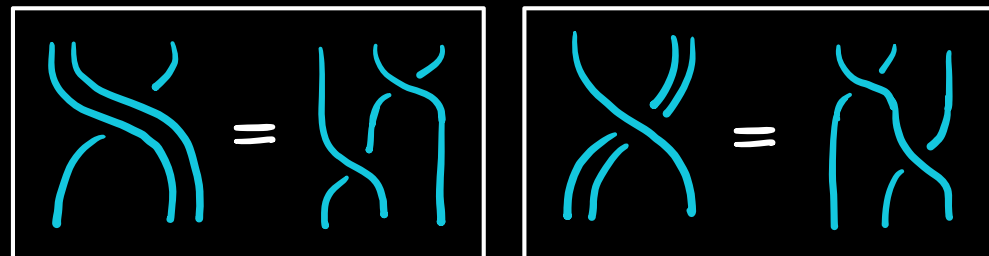
"BRAID AXIOMS"



"BRAID AXIOMS"

STRING MODELS

"BRAID AXIOMS"



"BRAID AXIOMS"

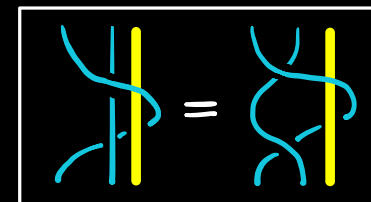
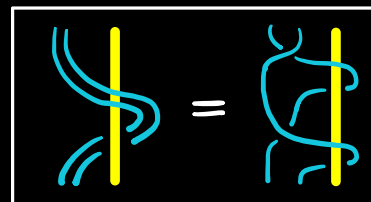
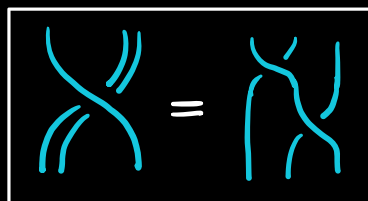
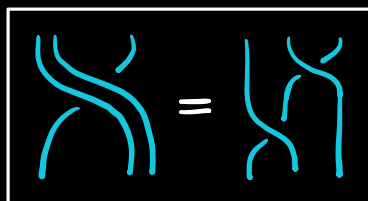
STRING MODELS

"BRAID AXIOMS"

LET'S CAPTURE THESE EQUATIONS
≡ CATEGORICALLY ≡

BRAIDED-TYPE CATEGORIES

"BRAID
AXIOMS"



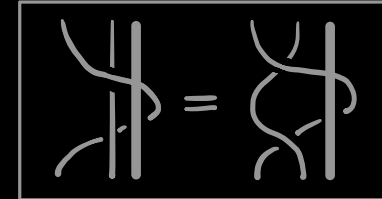
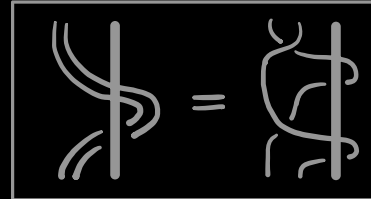
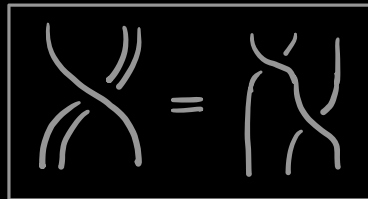
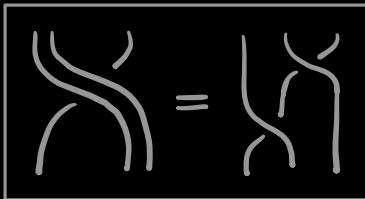
"BRAID
AXIOMS"

Joyal-Street
(1986, 1993)

BRAIDED-TYPE CATEGORIES

BRAIDED MONOIDAL CATEGORY
($\mathcal{C}, \otimes, \mathbb{1}, c$)

"BRAID
AXIOMS"



"BRAID
AXIOMS"

Joyal-Street
(1986, 1993)

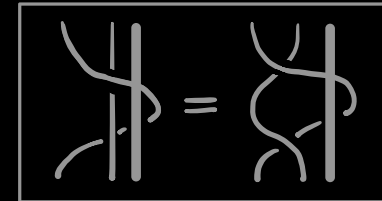
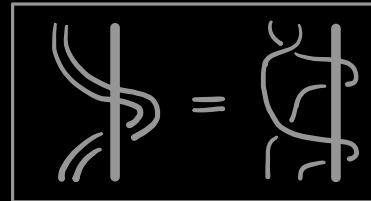
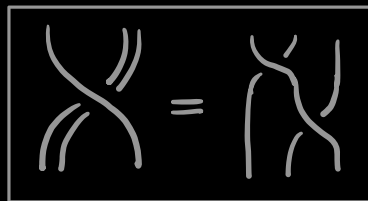
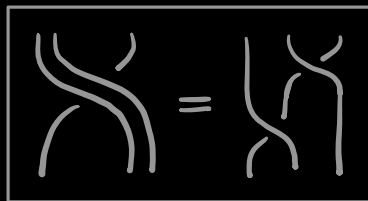
BRAIDED-TYPE CATEGORIES

BRAIDED MONOIDAL CATEGORY

$(\mathcal{C}, \otimes, \mathbb{1}, c)$

CATEGORY

"BRAID
AXIOMS"



"BRAID
AXIOMS"

Joyal-Street
(1986, 1993)

BRAIDED-TYPE CATEGORIES

BRAIDED MONOIDAL CATEGORY

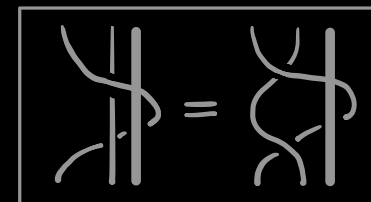
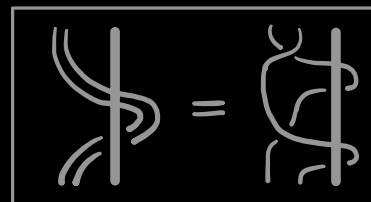
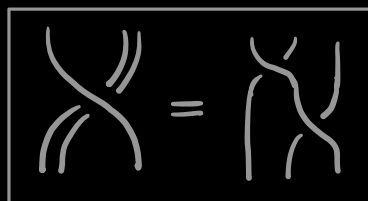
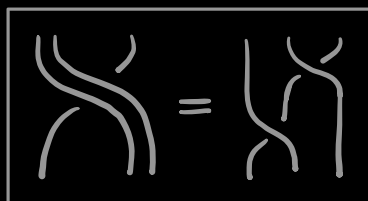
$(\mathcal{C}, \otimes, \mathbb{1}, c)$

CATEGORY

BIFUNCTOR

$\mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$

"BRAID
AXIOMS"



"BRAID
AXIOMS"

Joyal-Street
(1986, 1993)

BRAIDED-TYPE CATEGORIES

BRAIDED MONOIDAL CATEGORY

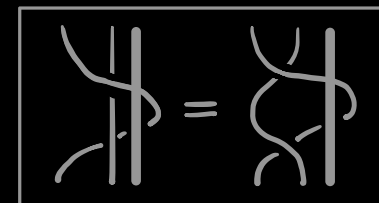
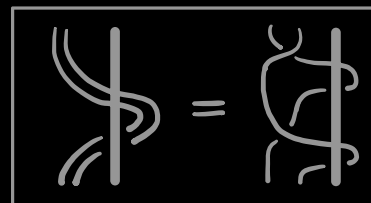
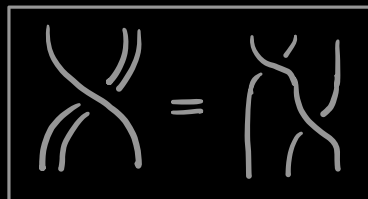
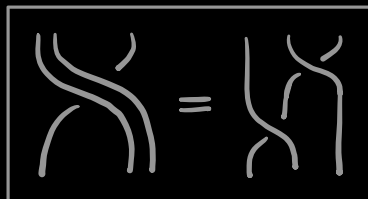
$(\mathcal{C}, \otimes, \mathbb{1}, c)$

CATEGORY

BIFUNCTOR
 $\mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$

DIST. OBJECT
IN $\mathcal{C}$

"BRAID
AXIOMS"



"BRAID
AXIOMS"

Joyal-Street
(1986, 1993)

BRAIDED-TYPE CATEGORIES

BRAIDED MONOIDAL CATEGORY

$(\mathcal{C}, \otimes, \mathbb{1}, c)$

CATEGORY

BIFUNCTOR

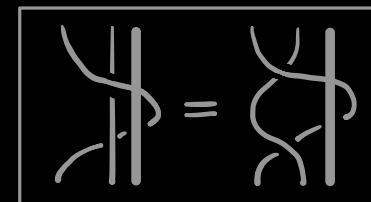
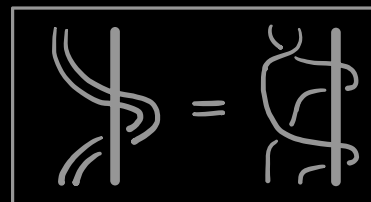
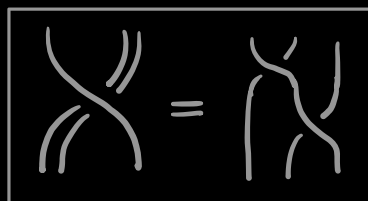
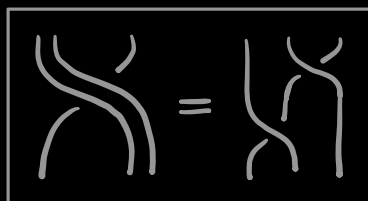
DIST. OBJECT

$\mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$

IN $\mathcal{C}$

BEHAVES LIKE A MONOID

"BRAID
AXIOMS"



"BRAID
AXIOMS"

Joyal-Street
(1986, 1993)

BRAIDED-TYPE CATEGORIES

BRAIDED MONOIDAL CATEGORY

$(\mathcal{C}, \otimes, \mathbb{1}, c)$

CATEGORY

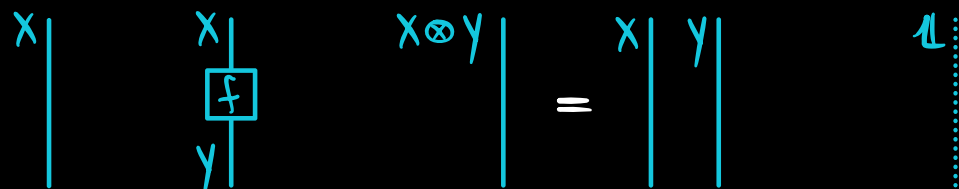
$f: x \rightarrow y$

BIFUNCTOR

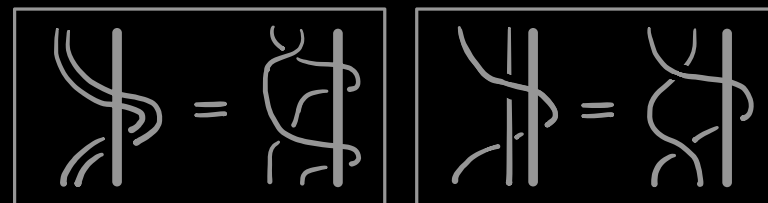
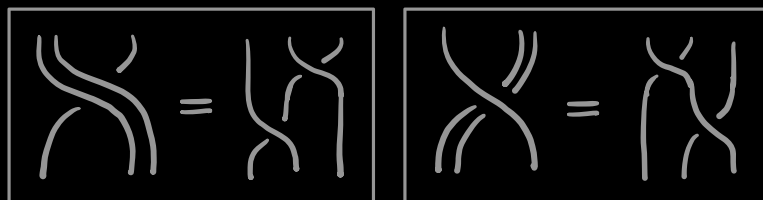
$\mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$

DIST. OBJECT

IN $\mathcal{C}$



"BRAID
AXIOMS"



"BRAID
AXIOMS"

Joyal-Street
(1986, 1993)

BRAIDED-TYPE CATEGORIES

BRAIDED MONOIDAL CATEGORY

$(\mathcal{C}, \otimes, \mathbb{1}, c)$

CATEGORY

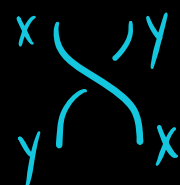
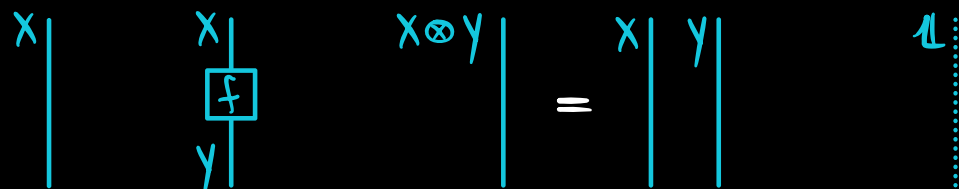
$f: x \rightarrow y$

BIFUNCTOR

$\mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$

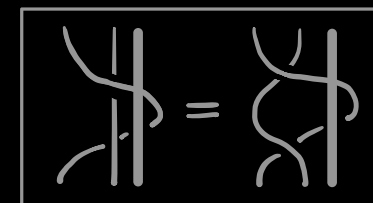
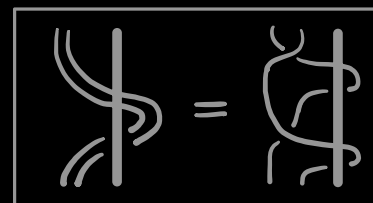
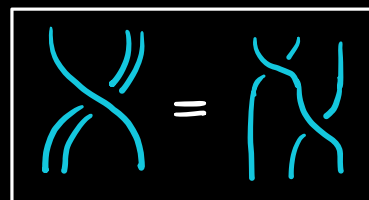
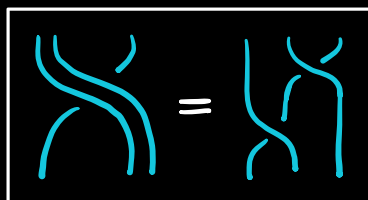
DIST. OBJECT

IN $\mathcal{C}$



$c := \{ c_{x,y} : x \otimes y \xrightarrow{\sim} y \otimes x \}_{x,y \in \mathcal{C}}$
NAT. ISOM.
SUCH THAT

"BRAID AXIOMS"



"BRAID AXIOMS"

Joyal-Street
(1986, 1993)

BRAIDED-TYPE CATEGORIES

BRAIDED MONOIDAL CATEGORY $(\mathcal{C}, \otimes, \mathbb{1}, c)$

CATEGORY
 $f: x \rightarrow y$

BIFUNCTOR
 $\mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$

DIST. OBJECT
IN $\mathcal{C}$

$$\begin{array}{c} x \\ | \\ y \end{array} = \begin{array}{c} x \\ | \\ \boxed{f} \\ | \\ y \end{array} \quad \begin{array}{c} x \otimes y \\ | \\ \mathbb{1} \end{array} = \begin{array}{c} x \\ | \\ y \end{array}$$

$$c := \{ c_{x,y} : x \otimes y \xrightarrow{\sim} y \otimes x \}_{x,y \in \mathcal{C}}$$

NAT. ISOM.

SUCH THAT

"BRAID AXIOMS"

$$\begin{array}{c} \text{Braid 1} \\ \text{Diagram 1} \end{array} = \begin{array}{c} \text{Braid 2} \\ \text{Diagram 2} \end{array}$$

$$\begin{array}{c} \text{Braid 3} \\ \text{Diagram 3} \end{array} = \begin{array}{c} \text{Braid 4} \\ \text{Diagram 4} \end{array}$$

$$\begin{array}{ccc} x \otimes y \otimes z & \xrightarrow{c_{x \otimes y, z}} & z \otimes x \otimes y \\ & \searrow \text{id}_x \otimes c_{y,z} & \nearrow c_{x,z} \otimes \text{id}_y \\ & x \otimes z \otimes y & \end{array}$$

$$\begin{array}{ccc} x \otimes y \otimes z & \xrightarrow{c_{x, y \otimes z}} & y \otimes z \otimes x \\ & \searrow c_{x,y} \otimes \text{id}_z & \nearrow \text{id}_y \otimes c_{x,z} \\ & y \otimes x \otimes z & \end{array}$$

$$\begin{array}{c} \text{Braid 5} \\ \text{Diagram 5} \end{array} = \begin{array}{c} \text{Braid 6} \\ \text{Diagram 6} \end{array}$$

$$\begin{array}{c} \text{Braid 7} \\ \text{Diagram 7} \end{array} = \begin{array}{c} \text{Braid 8} \\ \text{Diagram 8} \end{array}$$

"BRAID AXIOMS"

Joyal-STREET
(1986, 1993)

BRAIDED-TYPE CATEGORIES

BRAIDED MONOIDAL CATEGORY $(\mathcal{C}, \otimes, \mathbb{1}, c)$

CATEGORY

$$f: x \rightarrow y$$

BIFUNCTOR

$$\mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$$

DIST. OBJECT

IN $\mathcal{C}$

$$\begin{array}{c} x \\ | \\ y \end{array} = \begin{array}{c} x \\ | \\ \boxed{f} \\ | \\ y \end{array} \quad \begin{array}{c} x \otimes y \\ | \\ \mathbb{1} \end{array} = \begin{array}{c} x \\ | \\ y \end{array}$$

$$\begin{array}{c} x \\ \swarrow \\ y \end{array} \quad \begin{array}{c} y \\ \searrow \\ x \end{array}$$

$$c := \{ c_{x,y} : x \otimes y \xrightarrow{\sim} y \otimes x \}_{x,y \in \mathcal{C}}$$

NAT. ISOM.

SUCH THAT

"BRAID AXIOMS"

$$\begin{array}{c} \text{Braid 1} \\ \text{Diagram 1} \end{array} = \begin{array}{c} \text{Braid 2} \\ \text{Diagram 2} \end{array}$$

$$\begin{array}{c} \text{Braid 3} \\ \text{Diagram 3} \end{array} = \begin{array}{c} \text{Braid 4} \\ \text{Diagram 4} \end{array}$$

$$\begin{array}{ccc} x \otimes y \otimes z & \xrightarrow{c_{x \otimes y, z}} & z \otimes x \otimes y \\ & \searrow \text{id}_x \otimes c_{y,z} & \nearrow c_{x,z} \otimes \text{id}_y \\ & x \otimes z \otimes y & \end{array}$$

$$\begin{array}{ccc} x \otimes y \otimes z & \xrightarrow{c_{x, y \otimes z}} & y \otimes z \otimes x \\ & \searrow c_{x,y} \otimes \text{id}_z & \nearrow \text{id}_y \otimes c_{x,z} \\ & y \otimes x \otimes z & \end{array}$$

$$\begin{array}{c} \text{Braid 5} \\ \text{Diagram 5} \end{array} = \begin{array}{c} \text{Braid 6} \\ \text{Diagram 6} \end{array}$$

$$\begin{array}{c} \text{Braid 7} \\ \text{Diagram 7} \end{array} = \begin{array}{c} \text{Braid 8} \\ \text{Diagram 8} \end{array}$$

"BRAID AXIOMS"

JOYAL-STREET
(1986, 1993)

BRAIDED-TYPE CATEGORIES

BROCHIER (2013)
FIX $(\mathcal{C}, \otimes, \mathbb{1}, c)$

BRAIDED MONOIDAL CATEGORY

$(\mathcal{C}, \otimes, \mathbb{1}, c)$

CATEGORY

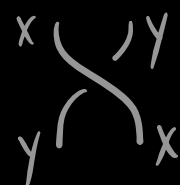
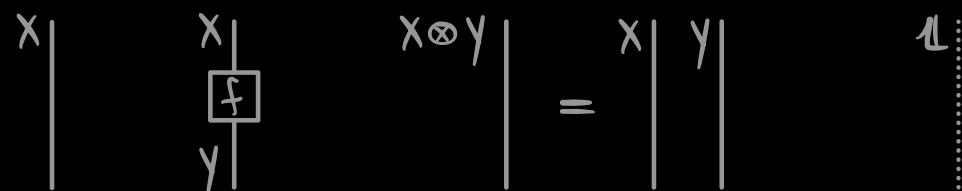
$f: x \rightarrow y$

BIFUNCTOR

$\mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$

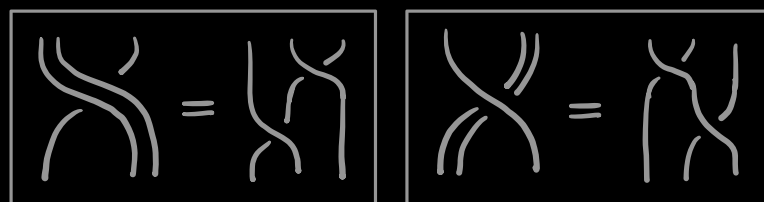
DIST. OBJECT

IN $\mathcal{C}$



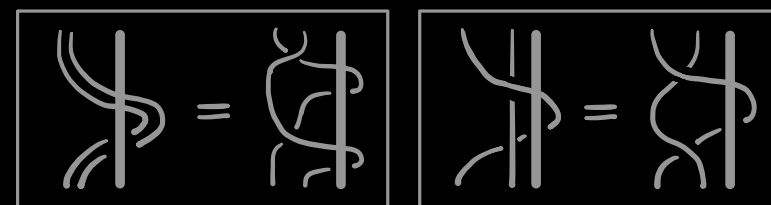
$\mathcal{C} := \{c_{x,y} : x \otimes y \xrightarrow{\sim} y \otimes x\}_{x,y \in \mathcal{C}}$
NAT. ISOM.
SUCH THAT

"BRAID AXIOMS"



BRAIDED $\mathcal{C}$ -MODULE CATEGORY

$(\mathcal{M}, \triangleright, e)$



"BRAID AXIOMS"

JOYAL-STREET
(1986, 1993)

BRAIDED-TYPE CATEGORIES

BROCHIER (2013)
FIX $(\mathcal{C}, \otimes, \mathbb{1}, c)$

BRAIDED MONOIDAL CATEGORY

$(\mathcal{C}, \otimes, \mathbb{1}, c)$

CATEGORY

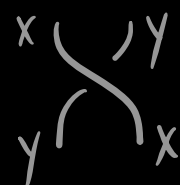
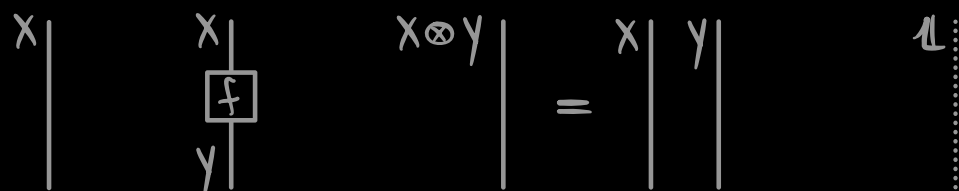
$f: x \rightarrow y$

BIFUNCTOR

$\mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$

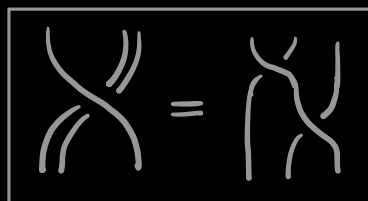
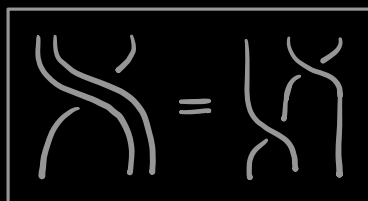
DIST. OBJECT

IN $\mathcal{C}$



$\mathcal{C} := \{ c_{x,y} : x \otimes y \xrightarrow{\sim} y \otimes x \}_{x,y \in \mathcal{C}}$
NAT. ISOM.
SUCH THAT

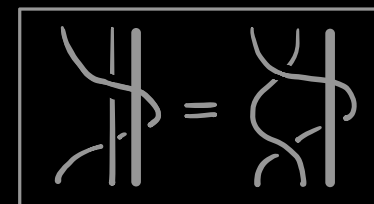
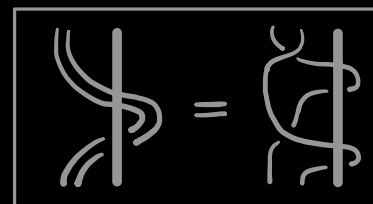
"BRAID AXIOMS"



BRAIDED $\mathcal{C}$ -MODULE CATEGORY

$(\mathcal{M}, \triangleright, e)$

CATEGORY



"BRAID AXIOMS"

JOYAL-STREET
(1986, 1993)

BRAIDED-TYPE CATEGORIES

BROCHIER (2013)
FIX $(\mathcal{C}, \otimes, \mathbb{1}, c)$

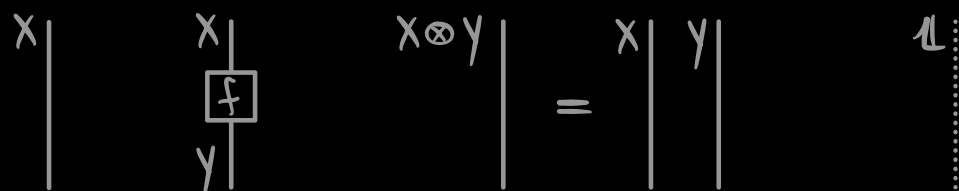
BRAIDED MONOIDAL CATEGORY

$(\mathcal{C}, \otimes, \mathbb{1}, c)$

CATEGORY
 $f: x \rightarrow y$

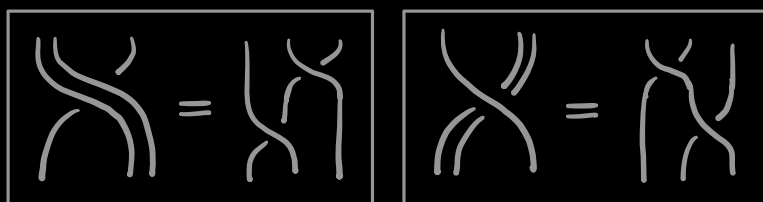
BIFUNCTOR
 $\mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$

DIST. OBJECT
IN $\mathcal{C}$



$\mathcal{C} := \{ c_{x,y} : x \otimes y \xrightarrow{\sim} y \otimes x \}_{x,y \in \mathcal{C}}$
NAT. ISOM. SUCH THAT

"BRAID AXIOMS"

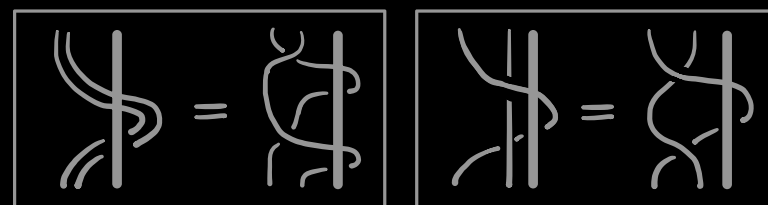


BRAIDED $\mathcal{C}$ -MODULE CATEGORY

$(\mathcal{M}, \triangleright, e)$

CATEGORY

BIFUNCTOR $\mathcal{C} \times \mathcal{M} \rightarrow \mathcal{M}$



"BRAID AXIOMS"

JOYAL-STREET
(1986, 1993)

BRAIDED-TYPE CATEGORIES

BROCHIER (2013)
FIX $(\mathcal{C}, \otimes, \mathbb{1}, c)$

BRAIDED MONOIDAL CATEGORY

$$(\mathcal{C}, \otimes, \mathbb{1}, c)$$

CATEGORY
 $f: x \rightarrow y$

BIFUNCTOR
 $\mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$

DIST. OBJECT
IN $\mathcal{C}$

$$\begin{array}{c} x \\ | \\ \square \\ | \\ y \end{array} = \begin{array}{c} x \otimes y \\ | \\ \square \\ | \\ \mathbb{1} \end{array}$$

$$\mathcal{C} := \{ c_{x,y} : x \otimes y \xrightarrow{\sim} y \otimes x \mid x, y \in \mathcal{C} \}$$

NAT. ISOM. SUCH THAT

"BRAID
AXIOMS"

$$\begin{array}{c} \text{Braid 1} \\ \text{Braid 2} \end{array}$$

BRAIDED $\mathcal{C}$ -MODULE CATEGORY

$$(\mathcal{M}, \triangleright, e)$$

CATEGORY

BIFUNCTOR $\mathcal{C} \times \mathcal{M} \rightarrow \mathcal{M}$

BEHAVES LIKE A MODULE
OVER A MONOID

$$\begin{array}{c} \text{Braid 3} \\ \text{Braid 4} \end{array}$$

"BRAID
AXIOMS"

JOYAL-STREET
(1986, 1993)

BRAIDED-TYPE CATEGORIES

BROCHIER (2013)
FIX $(\mathcal{C}, \otimes, \mathbb{1}, c)$

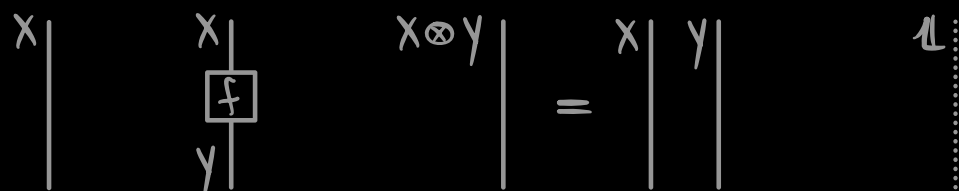
BRAIDED MONOIDAL CATEGORY

$(\mathcal{C}, \otimes, \mathbb{1}, c)$

CATEGORY
 $f: x \rightarrow y$

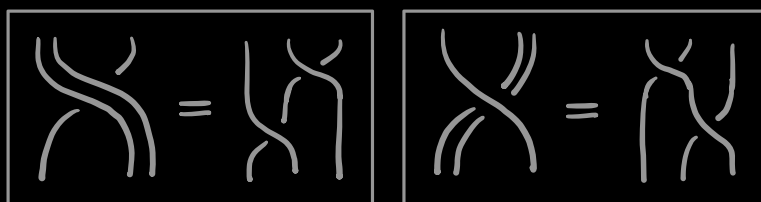
BIFUNCTOR
 $\mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$

DIST. OBJECT
IN $\mathcal{C}$



$c := \{c_{x,y} : x \otimes y \xrightarrow{\sim} y \otimes x\}_{x,y \in \mathcal{C}}$
NAT. ISOM.
SUCH THAT

"BRAID AXIOMS"

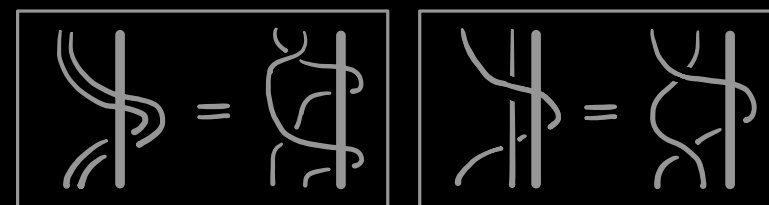


BRAIDED $\mathcal{C}$ -MODULE CATEGORY

$(\mathcal{M}, \triangleright, e)$

CATEGORY

BIFUNCTOR $\mathcal{C} \times \mathcal{M} \rightarrow \mathcal{M}$



"BRAID AXIOMS"

JOYAL-STREET
(1986, 1993)

BRAIDED-TYPE CATEGORIES

BROCHIER (2013)
FIX $(\mathcal{C}, \otimes, \mathbb{1}, c)$

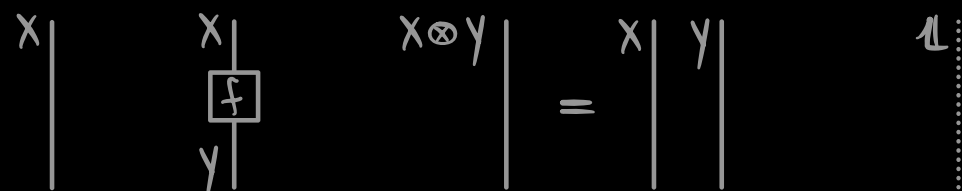
BRAIDED MONOIDAL CATEGORY

$(\mathcal{C}, \otimes, \mathbb{1}, c)$

CATEGORY
 $f: x \rightarrow y$

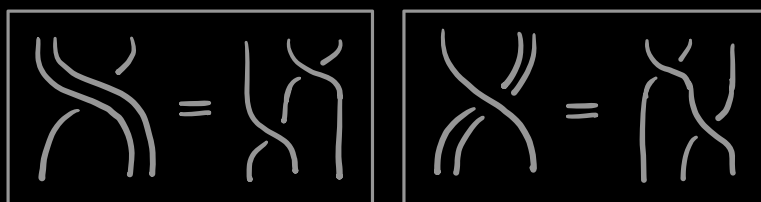
BIFUNCTOR
 $\mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$

DIST. OBJECT
IN $\mathcal{C}$



$c := \{c_{x,y} : x \otimes y \xrightarrow{\sim} y \otimes x\}_{x,y \in \mathcal{C}}$
NAT. ISOM.
SUCH THAT

"BRAID AXIOMS"



BRAIDED $\mathcal{C}$ -MODULE CATEGORY

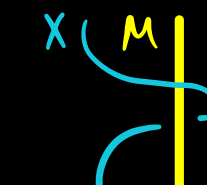
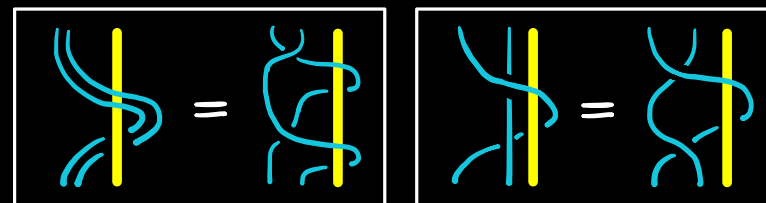
$(\mathcal{M}, \triangleright, e)$

CATEGORY

BIFUNCTOR $\mathcal{C} \times \mathcal{M} \rightarrow \mathcal{M}$



$e := \{e_{x,M} : x \triangleright M \xrightarrow{\sim} M \triangleright x\}_{x \in \mathcal{C}, M \in \mathcal{M}}$
NAT. ISOM.
SUCH THAT



"BRAID AXIOMS"

BRAIDED-TYPE CATEGORIES

BROCHIER (2013)

FIX $(\mathcal{C}, \otimes, \mathbb{1}, c)$

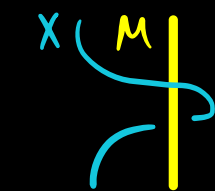
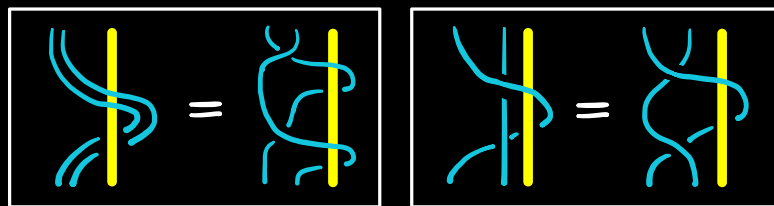
BRAIDED $\mathcal{C}$ -MODULE CATEGORY

$(\mathcal{M}, \triangleright, e)$

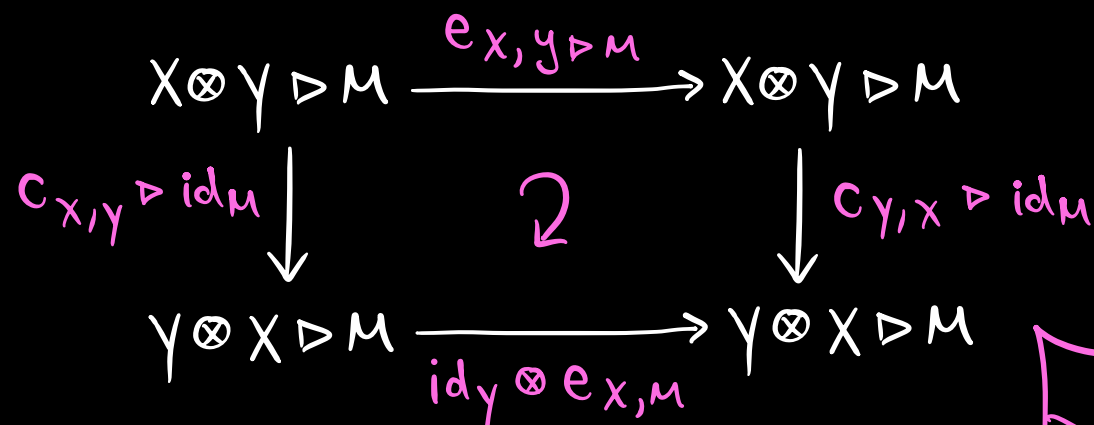
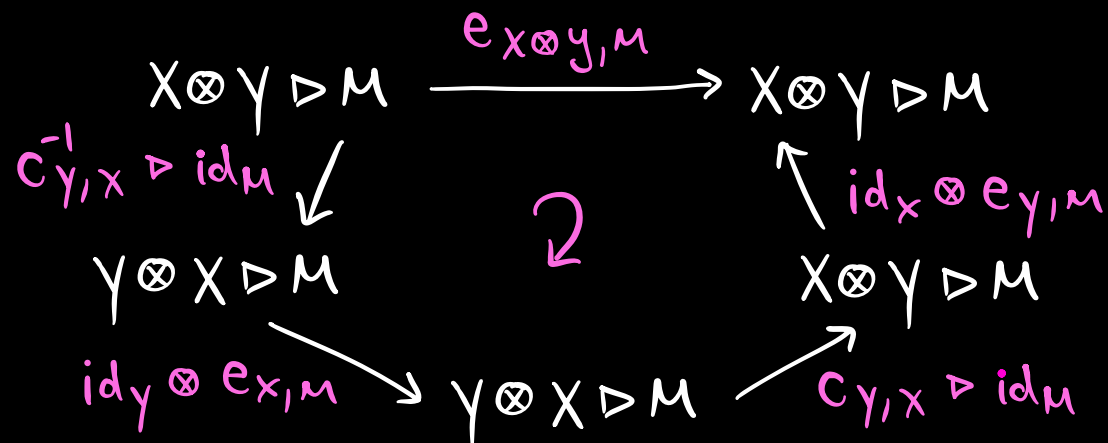
CATEGORY BIFUNCTOR $\mathcal{C} \times \mathcal{M} \rightarrow \mathcal{M}$

$$\begin{array}{c} \mathcal{M} \\ | \end{array} \quad \begin{array}{c} \mathcal{X} \triangleright \mathcal{M} \\ | \end{array} = \begin{array}{c} \mathcal{X} \\ | \end{array} \begin{array}{c} \mathcal{M} \\ | \end{array}$$

$e := \{ e_{\mathcal{X}, \mathcal{M}} : \mathcal{X} \triangleright \mathcal{M} \xrightarrow{\sim} \mathcal{X} \triangleright \mathcal{M} \}_{\substack{\mathcal{X} \in \mathcal{C} \\ \mathcal{M} \in \mathcal{M}}}$
NAT. ISOM. SUCH THAT



"BRAID AXIOMS"



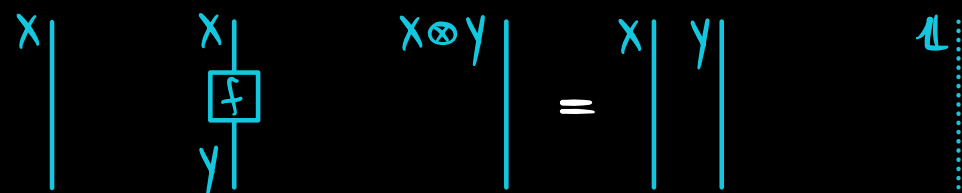
JOYAL-STREET
(1986, 1993)

BRAIDED-TYPE CATEGORIES

BROCHIER (2013)
FIX $(\mathcal{C}, \otimes, \mathbb{1}, c)$

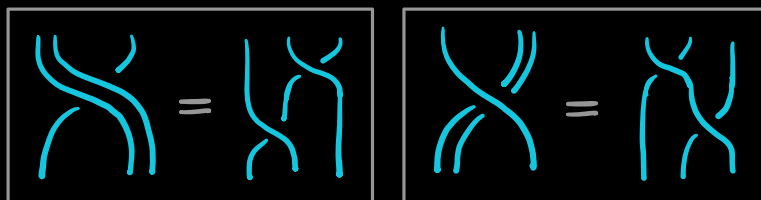
BRAIDED MONOIDAL CATEGORY $(\mathcal{C}, \otimes, \mathbb{1}, c)$

CATEGORY $f: x \rightarrow y$
BIFUNCTOR $\mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$
DIST. OBJECT IN $\mathcal{C}$



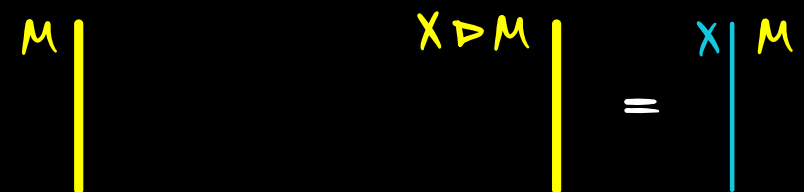
$c := \{c_{x,y} : x \otimes y \xrightarrow{\sim} y \otimes x\}_{x,y \in \mathcal{C}}$
NAT. ISOM.
SUCH THAT

"BRAID AXIOMS"

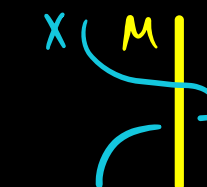
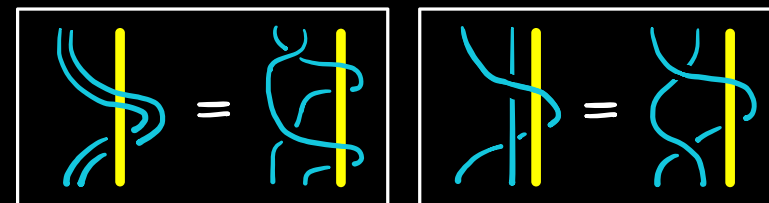


BRAIDED $\mathcal{C}$ -MODULE CATEGORY $(\mathcal{M}, \triangleright, e)$

CATEGORY $x \triangleright m$
BIFUNCTOR $\mathcal{C} \times \mathcal{M} \rightarrow \mathcal{M}$



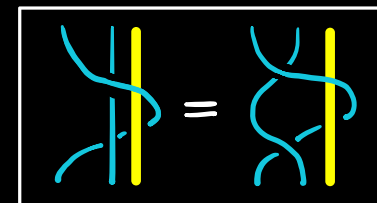
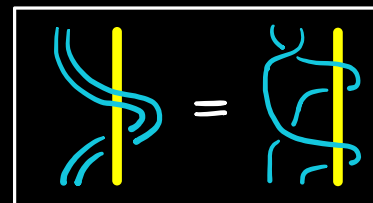
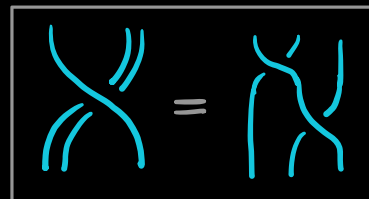
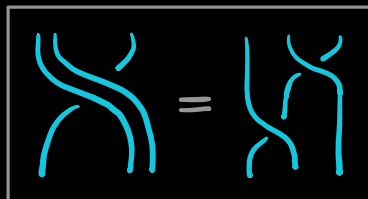
$e := \{e_{x,m} : x \triangleright m \xrightarrow{\sim} x \triangleright m\}_{x \in \mathcal{C}, m \in \mathcal{M}}$
NAT. ISOM.
SUCH THAT



"BRAID AXIOMS"

TIES TO BRAID GROUP REP'NS

"BRAID
AXIOMS"



"BRAID
AXIOMS"

$\equiv \text{Type A} \equiv$

TIES TO BRAID GROUP REP'NS



COXETER GRAPH
OF TYPE A_n

$\equiv \text{TYPE A} \equiv$

TIES TO BRAID GROUP REP'NS



COXETER GRAPH
OF TYPE A_n

B_n^A BRAID GROUP OF TYPE A_n

GENERATORS: $\sigma_1, \dots, \sigma_n$

RELATIONS:
$$\begin{cases} \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1} & \text{FOR } i=1, \dots, n-1 \\ \sigma_i \sigma_j = \sigma_j \sigma_i & \text{FOR } |i-j| \geq 2 \end{cases}$$

$\equiv \text{TYPE A} \equiv$

TIES TO BRAID GROUP REP'NS



COXETER GRAPH
OF TYPE A_n

$B\Gamma_n^A$ BRAID GROUP OF TYPE A_n

GENERATORS: $\sigma_1, \dots, \sigma_n$

RELATIONS:
$$\begin{cases} \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1} \\ \text{FOR } i=1, \dots, n-1 \\ \sigma_i \sigma_j = \sigma_j \sigma_i \text{ FOR } |i-j| \geq 2 \end{cases}$$

FOR $X \in (\mathcal{C}, \otimes, \mathbb{1}, c)$ BR. MONOIDAL CAT.

GET GROUP HOMOM:

$$\rho_n^X : B\Gamma_{n-1}^A \longrightarrow \text{Aut}_{\mathcal{C}}(X^{\otimes n})$$

$\equiv \text{TYPE A} \equiv$

TIES TO BRAID GROUP REP'NS



COXETER GRAPH
OF TYPE A_n

B_n^A BRAID GROUP OF TYPE A_n

GENERATORS: $\sigma_1, \dots, \sigma_n$

RELATIONS:
$$\begin{cases} \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1} \\ \text{FOR } i=1, \dots, n-1 \\ \sigma_i \sigma_j = \sigma_j \sigma_i \text{ FOR } |i-j| \geq 2 \end{cases}$$

FOR $X \in (\mathcal{C}, \otimes, \mathbb{1}, c)$ BR. MONOIDAL CAT.

GET GROUP HOMOM:

$$\rho_n^X : B_{n-1}^A \longrightarrow \text{Aut}_{\mathcal{C}}(X^{\otimes n})$$

$$\sigma_i \longmapsto \text{id}_{X^{\otimes i-1}} \otimes c_{X,X} \otimes \text{id}_{X^{\otimes n-i-1}}$$

$\equiv \text{TYPE A} \equiv$

TIES TO BRAID GROUP REP'NS



COXETER GRAPH
OF TYPE A_n

B_n^A BRAID GROUP OF TYPE A_n

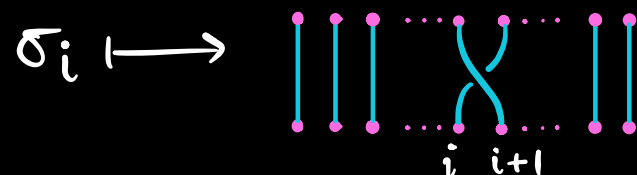
GENERATORS: $\sigma_1, \dots, \sigma_n$

RELATIONS: $\begin{cases} \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1} \\ \text{FOR } i=1, \dots, n-1 \\ \sigma_i \sigma_j = \sigma_j \sigma_i \text{ FOR } |i-j| \geq 2 \end{cases}$

FOR $X \in (\mathcal{C}, \otimes, \mathbb{1}, c)$ BR. MONOIDAL CAT.

GET GROUP HOMOM:

$$\rho_n^X : B_{n-1}^A \longrightarrow \text{Aut}_{\mathcal{C}}(X^{\otimes n})$$



$\equiv \text{TYPE A} \equiv$

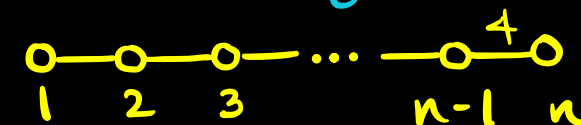
TIES TO BRAID GROUP REP'NS

$\equiv \text{TYPE B}^{\text{C}} \equiv$



COXETER GRAPH
OF TYPE A_n

COXETER GRAPH
OF TYPE B_n



B_n^A BRAID GROUP OF TYPE A_n

GENERATORS: $\sigma_1, \dots, \sigma_n$

RELATIONS: $\begin{cases} \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1} \\ \text{FOR } i=1, \dots, n-1 \\ \sigma_i \sigma_j = \sigma_j \sigma_i \text{ FOR } |i-j| \geq 2 \end{cases}$

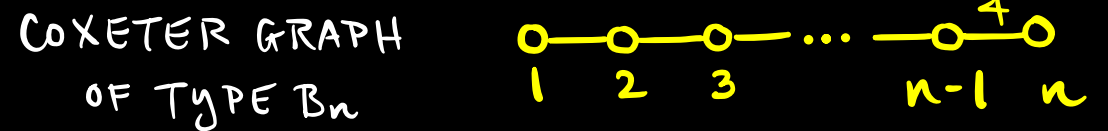
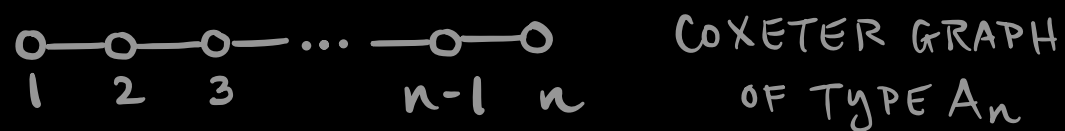
FOR $X \in (\mathcal{C}, \otimes, \mathbb{1}, c)$ BR. MONOIDAL CAT.

GET GROUP HOMOM:

$$\rho_n^X : B_{n-1}^A \longrightarrow \text{Aut}_{\mathcal{C}}(X^{\otimes n})$$

$$\sigma_i \longmapsto \begin{array}{c} \text{Diagram showing a crossing of strands } i \text{ and } i+1 \end{array}$$

$\equiv \text{TYPE A} \equiv$ TIES TO BRAID GROUP REP'NS $\equiv \text{TYPE B} \stackrel{C}{\equiv}$



$B\Gamma_n^A$ BRAID GROUP OF TYPE A_n

GENERATORS: $\sigma_1, \dots, \sigma_n$

RELATIONS: $\begin{cases} \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1} \\ \text{FOR } i=1, \dots, n-1 \\ \sigma_i \sigma_j = \sigma_j \sigma_i \text{ FOR } |i-j| \geq 2 \end{cases}$

$B\Gamma_n^B$ BRAID GROUP OF TYPE B_n

GENERATORS: $\sigma_1, \dots, \sigma_{n-1}, t$

RELATIONS: $\begin{cases} \text{TYPE A RELATIONS FOR } \sigma_i \\ \sigma_i t = t \sigma_i \text{ FOR } i=1, \dots, n-2 \\ \sigma_{n-1} t \sigma_{n-1} t = t \sigma_{n-1} t \sigma_{n-1} \end{cases}$

FOR $X \in (\mathcal{C}, \otimes, \mathbb{1}, c)$ BR. MONOIDAL CAT.

GET GROUP HOMOM:

$$\rho_n^X : B\Gamma_{n-1}^A \longrightarrow \text{Aut}_{\mathcal{C}}(X^{\otimes n})$$

$$\sigma_i \longmapsto \begin{array}{c} \text{Diagram of } \sigma_i \text{ as a braid: two strands crossing, with dots representing other strands.} \\ \text{Label } i \text{ and } i+1 \text{ under the crossing.} \end{array}$$

$\equiv \text{TYPE A} \equiv$

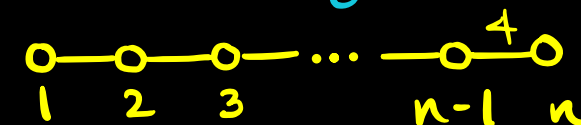
TIES TO BRAID GROUP REP'NS

$\equiv \text{TYPE B}^{\mathbb{C}} \equiv$



COXETER GRAPH
OF TYPE A_n

COXETER GRAPH
OF TYPE B_n



B_n^A BRAID GROUP OF TYPE A_n

GENERATORS: $\sigma_1, \dots, \sigma_n$

RELATIONS:
$$\begin{cases} \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1} \\ \text{FOR } i=1, \dots, n-1 \\ \sigma_i \sigma_j = \sigma_j \sigma_i \text{ FOR } |i-j| \geq 2 \end{cases}$$

B_n^B BRAID GROUP OF TYPE B_n

GENERATORS: $\sigma_1, \dots, \sigma_{n-1}, t$

RELATIONS:
$$\begin{cases} \text{TYPE A RELATIONS FOR } \sigma_i \\ \sigma_i t = t \sigma_i \text{ FOR } i=1, \dots, n-2 \\ \sigma_{n-1} t \sigma_{n-1} t = t \sigma_{n-1} t \sigma_{n-1} \end{cases}$$

FOR $X \in (\mathcal{C}, \otimes, \mathbb{1}, c)$ BR. MONOIDAL CAT.

GET GROUP HOMOM:

$$\rho_n^X: B_{n-1}^A \longrightarrow \text{Aut}_{\mathcal{C}}(X^{\otimes n})$$

$$\sigma_i \longmapsto \begin{array}{c} \text{Diagram of } \sigma_i \text{ as a crossing of strands } i \text{ and } i+1 \end{array}$$

FOR $X \in (\mathcal{C}, \otimes, \mathbb{1}, c) \nsubseteq M \in (\mathcal{M}, \triangleright, e)$ BR MOD CAT.

GET GROUP HOMOM:
$$\rho_n^{X,M}: B_n^B \longrightarrow \text{Aut}_{\mathcal{M}}(X^{\otimes n} \triangleright M)$$

$$\equiv \text{TYPE A} \equiv \boxed{\text{TIES TO BRAID GROUP REP'NS}} \equiv \text{TYPE B} \stackrel{C}{=} \equiv$$



B_n^B BRAID GROUP OF TYPE B_n

GENERATORS: $\sigma_1, \dots, \sigma_{n-1}, t$

RELATIONS: $\left\{ \begin{array}{l} \text{TYPE A RELATIONS FOR } \sigma_i \\ \sigma_i t = t \sigma_i \text{ FOR } i=1, \dots, n-2 \\ \sigma_{n-1} t \sigma_{n-1} t = t \sigma_{n-1} t \sigma_{n-1} \end{array} \right.$

FOR $X \in (\mathcal{C}, \otimes, \mathbb{1}, c)$ & $M \in (\mathcal{M}, \triangleright, e)$ $BR \text{ MOD CAT.}$

GET $\rho_n^{X,M}: Br_n^B \longrightarrow Aut_m(X^{\otimes n} \triangleright M)$

Номом:

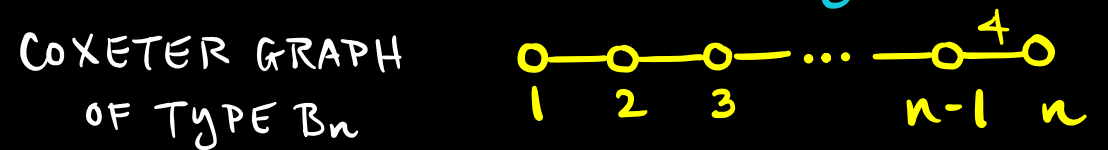
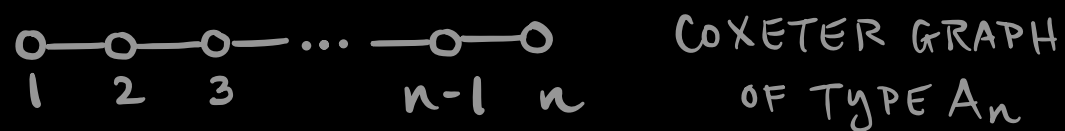
$$\sigma_i \longmapsto \text{id}_X^{\otimes i-1} \otimes C_{X,X} \otimes \text{id}_X^{\otimes n-i-1}$$

$$t \mapsto \text{id}_{X^{\otimes n-1}} \otimes e_{x,n}$$

$\equiv \text{TYPE A} \equiv$

TIES TO BRAID GROUP REP'NS

 $\equiv \text{TYPE B} \stackrel{C}{\equiv}$



$B\Gamma_n^A$
BRAID GROUP OF TYPE A_n

GENERATORS: $\sigma_1, \dots, \sigma_n$

RELATIONS: $\begin{cases} \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1} \\ \text{FOR } i=1, \dots, n-1 \\ \sigma_i \sigma_j = \sigma_j \sigma_i \text{ FOR } |i-j| \geq 2 \end{cases}$

$B\Gamma_n^B$
BRAID GROUP OF TYPE B_n

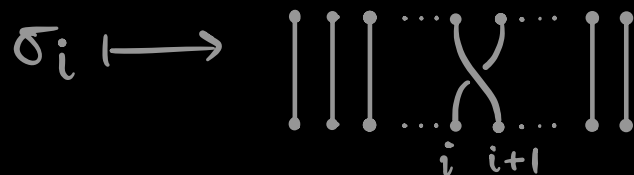
GENERATORS: $\sigma_1, \dots, \sigma_{n-1}, t$

RELATIONS: $\begin{cases} \text{TYPE A RELATIONS FOR } \sigma_i \\ \sigma_i t = t \sigma_i \text{ FOR } i=1, \dots, n-2 \\ \sigma_{n-1} t \sigma_{n-1} t = t \sigma_{n-1} t \sigma_{n-1} \end{cases}$

FOR $X \in (\mathcal{C}, \otimes, \mathbb{1}, c)$ BR. MONOIDAL CAT.

GET GROUP HOMOM:

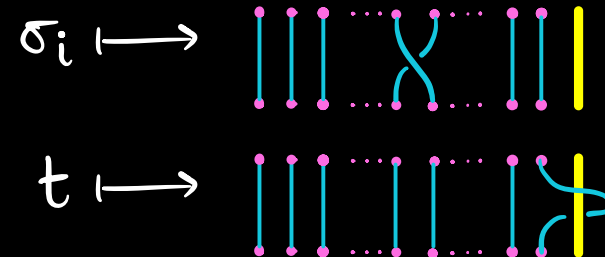
$\rho_n^X: B\Gamma_{n-1}^A \longrightarrow \text{Aut}_{\mathcal{C}}(X^{\otimes n})$



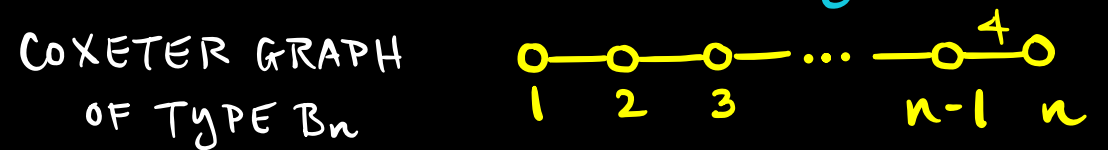
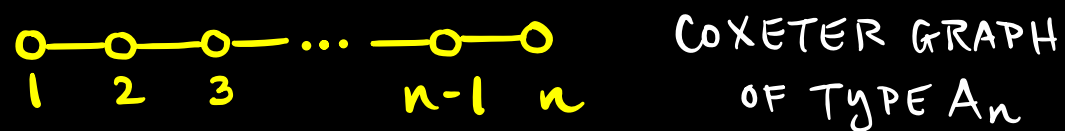
FOR $X \in (\mathcal{C}, \otimes, \mathbb{1}, c) \nmid M \in (\mathcal{M}, \triangleright, e)$ BR MOD CAT.

GET GROUP HOMOM: $\rho_n^{X,M}: B\Gamma_n^B \longrightarrow \text{Aut}_{\mathcal{M}}(X^{\otimes n} \triangleright M)$

GET GROUP HOMOM:



$\equiv \text{TYPE A} \equiv$ TIES TO BRAID GROUP REP'NS $\equiv \text{TYPE B} \stackrel{C}{\equiv}$



B_n^A BRAID GROUP OF TYPE A_n

GENERATORS: $\sigma_1, \dots, \sigma_n$

RELATIONS: $\begin{cases} \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1} \\ \text{FOR } i=1, \dots, n-1 \\ \sigma_i \sigma_j = \sigma_j \sigma_i \text{ FOR } |i-j| \geq 2 \end{cases}$

B_n^B BRAID GROUP OF TYPE B_n

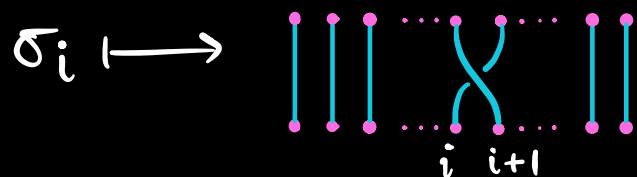
GENERATORS: $\sigma_1, \dots, \sigma_{n-1}, t$

RELATIONS: $\begin{cases} \text{TYPE A RELATIONS FOR } \sigma_i \\ \sigma_i t = t \sigma_i \text{ FOR } i=1, \dots, n-2 \\ \sigma_{n-1} t \sigma_{n-1} t = t \sigma_{n-1} t \sigma_{n-1} \end{cases}$

FOR $X \in (\mathcal{C}, \otimes, \mathbb{1}, c)$ BR. MONOIDAL CAT.

GET GROUP HOMOM:

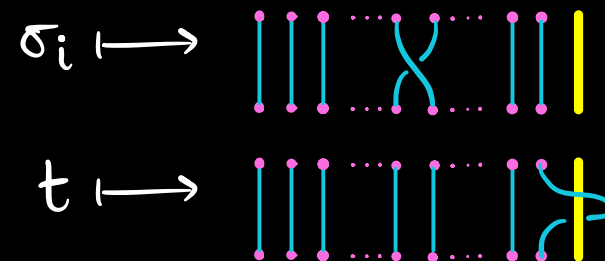
$$\rho_n^X: B_{n-1}^A \longrightarrow \text{Aut}_{\mathcal{C}}(X^{\otimes n})$$



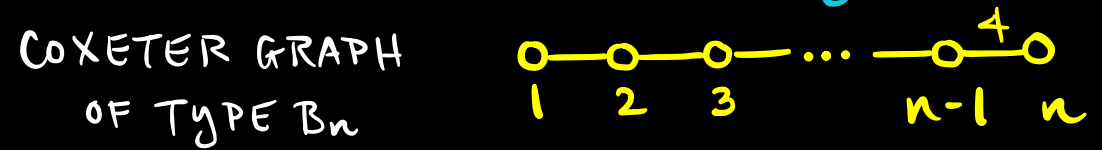
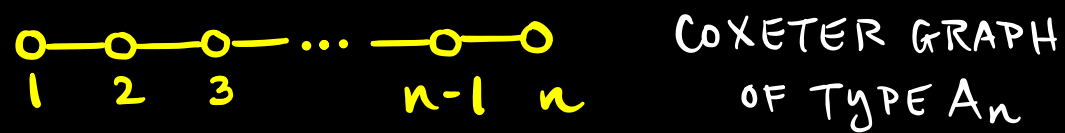
FOR $X \in (\mathcal{C}, \otimes, \mathbb{1}, c) \nmid M \in (\mathcal{M}, \triangleright, e)$ BR MOD CAT.

GET GROUP HOMOM: $\rho_n^{X,M}: B_n^B \longrightarrow \text{Aut}_{\mathcal{M}}(X^{\otimes n} \triangleright M)$

GET GROUP HOMOM:



$\equiv \text{TYPE A} \equiv$ TIES TO BRAID GROUP REP'NS $\equiv \text{TYPE B} \stackrel{C}{\equiv}$



$B\Gamma_n^A$ BRAID GROUP OF TYPE A_n

GENERATORS: $\sigma_1, \dots, \sigma_n$

RELATIONS: $\left\{ \begin{array}{l} \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1} \\ \sigma_i \sigma_j = \sigma_j \sigma_i \text{ for } |i-j| \geq 2 \end{array} \right.$ FOR $i=1, \dots, n-1$

qYBE

$B\Gamma_n^B$ BRAID GROUP OF TYPE B_n

GENERATORS: $\sigma_1, \dots, \sigma_{n-1}, t$

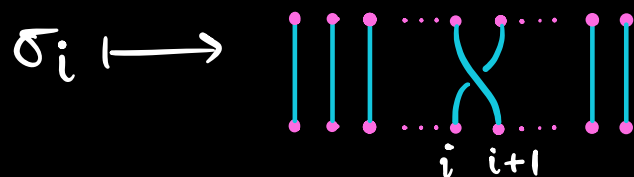
RELATIONS: $\left\{ \begin{array}{l} \text{TYPE A RELATIONS FOR } \sigma_i \\ \sigma_i t = t \sigma_i \text{ for } i=1, \dots, n-2 \\ \sigma_{n-1} t \sigma_{n-1} t = t \sigma_{n-1} t \sigma_{n-1} \end{array} \right.$

qRE

FOR $X \in (\mathcal{C}, \otimes, \mathbb{1}, c)$ BR. MONOIDAL CAT.

GET GROUP HOMOM:

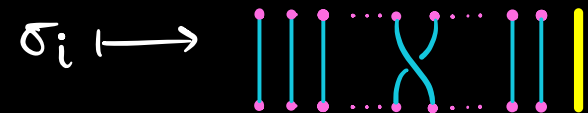
$$\rho_n^X: B\Gamma_{n-1}^A \longrightarrow \text{Aut}_{\mathcal{C}}(X^{\otimes n})$$



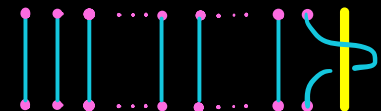
FOR $X \in (\mathcal{C}, \otimes, \mathbb{1}, c) \nmid M \in (\mathcal{M}, \triangleright, e)$ BR MOD CAT.

GET GROUP HOMOM: $\rho_n^{X,M}: B\Gamma_n^B \longrightarrow \text{Aut}_{\mathcal{M}}(X^{\otimes n} \triangleright M)$

$\sigma_i \mapsto$



$t \mapsto$



≡ TYPE A ≡

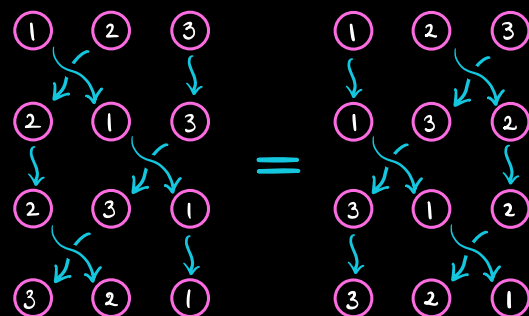
SUMMARY ... SO FAR

≡ TYPE B^{=C} ≡

≡ TYPE A ≡

SUMMARY ... SO FAR

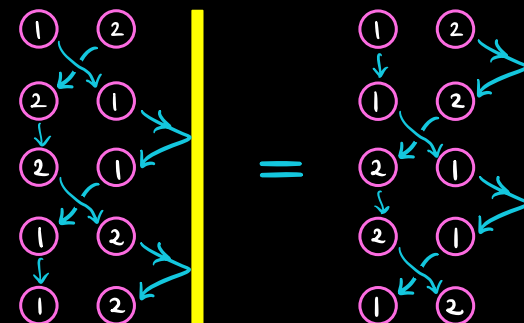
≡ TYPE B^C ≡



$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)

≡ WANT SOLUTIONS ≡



$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

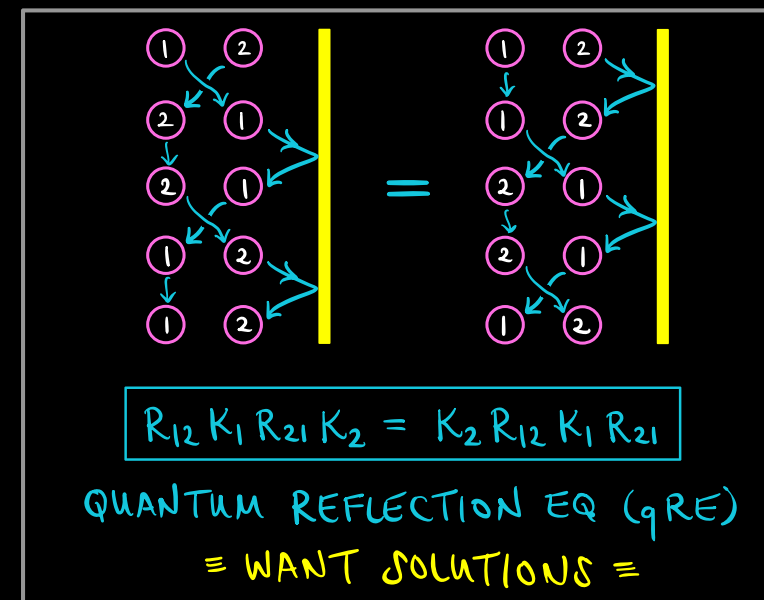
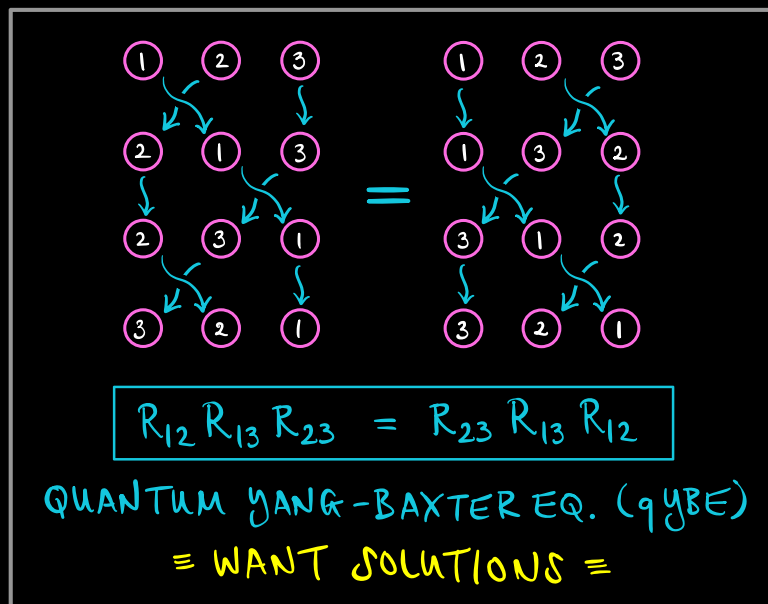
QUANTUM REFLECTION EQ (qRE)

≡ WANT SOLUTIONS ≡

$\equiv \text{Type A} \equiv$

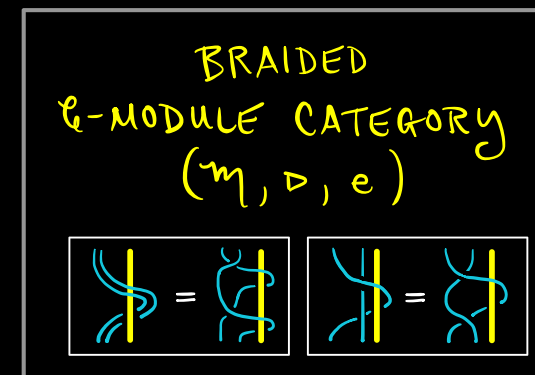
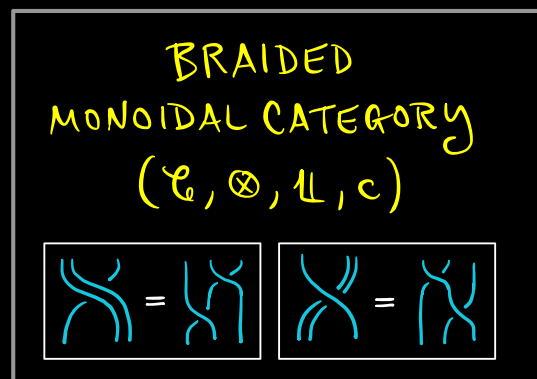
SUMMARY ... SO FAR

$\equiv \text{Type B}^c \equiv$



OBJECTS

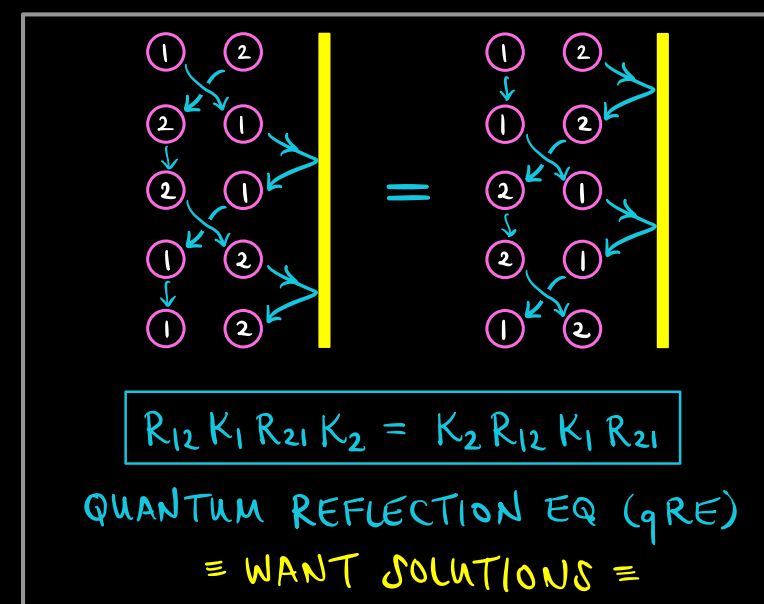
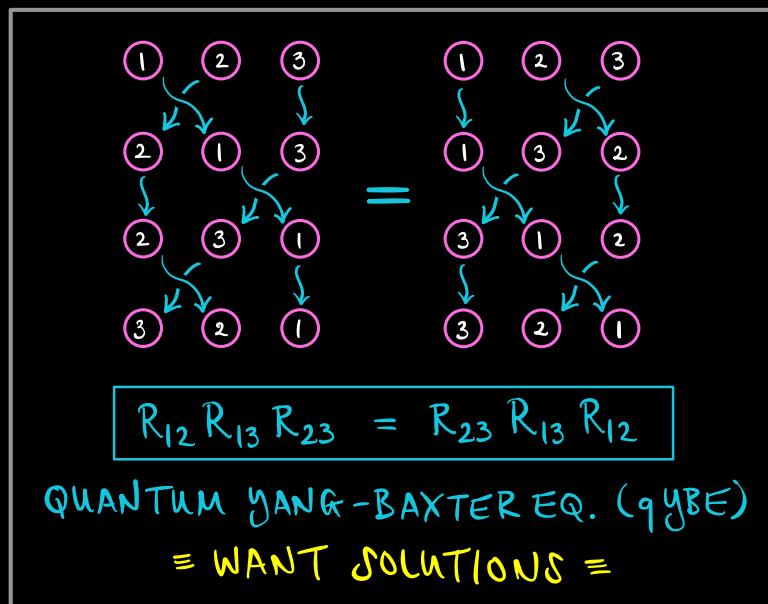
OBJECTS



$\equiv \text{TYPE A} \equiv$

SUMMARY ... SO FAR

$\equiv \text{TYPE B} \stackrel{C}{=} \equiv$

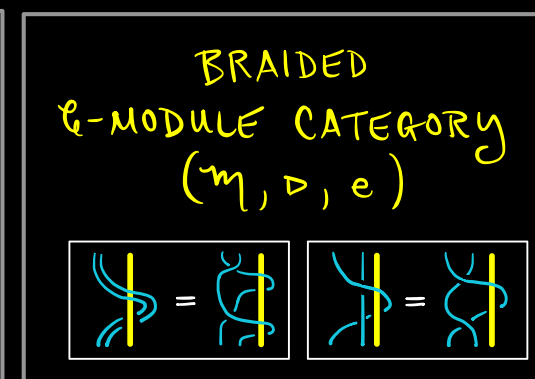
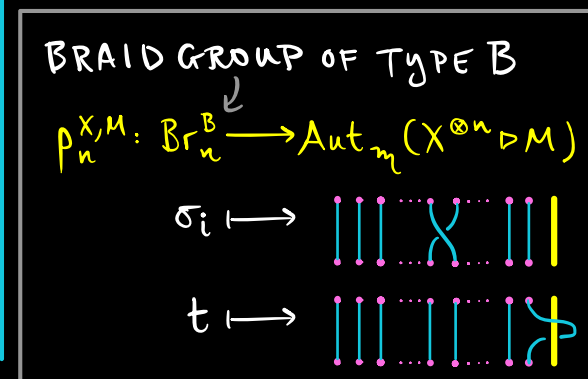
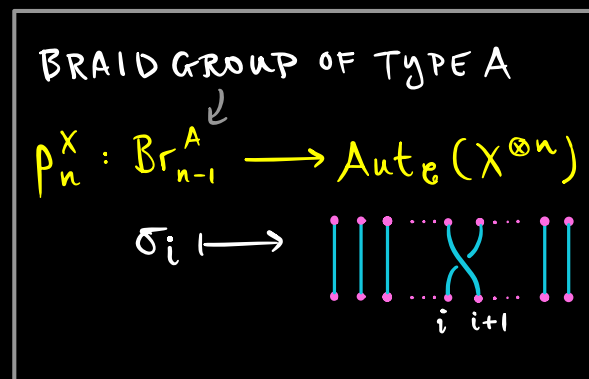
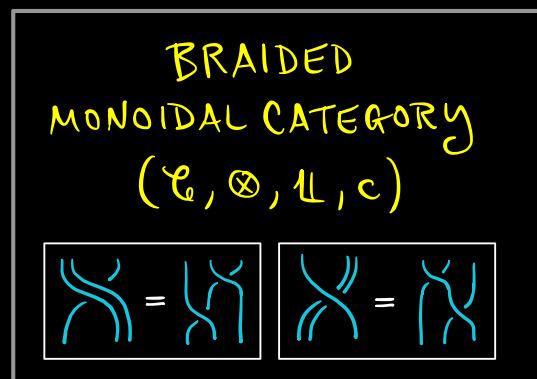


OBJECTS

OBJECTS

OBJECTS

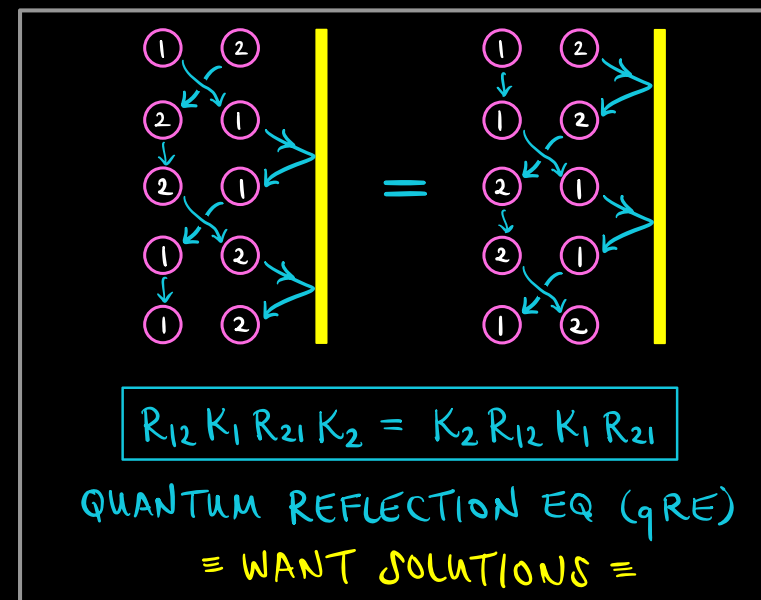
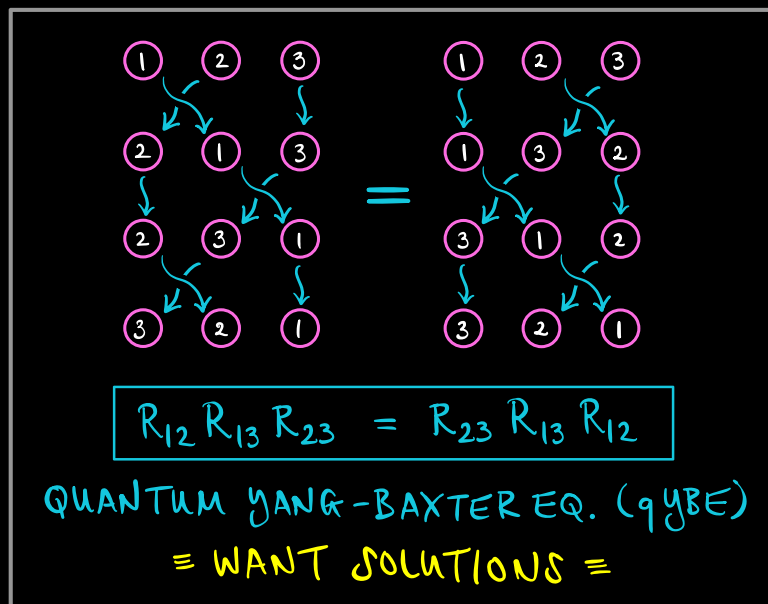
OBJECTS



$\equiv \text{Type A} \equiv$

SUMMARY ... SO FAR

$\equiv \text{Type B} \stackrel{C}{\equiv}$



OBJECTS

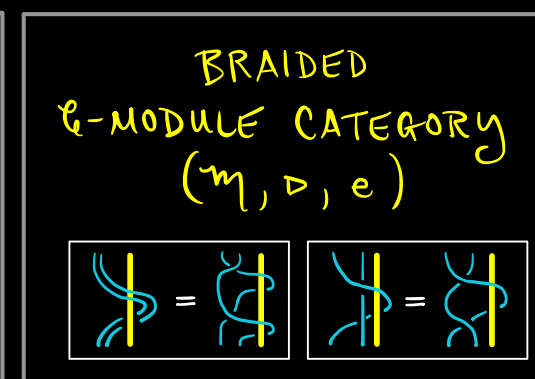
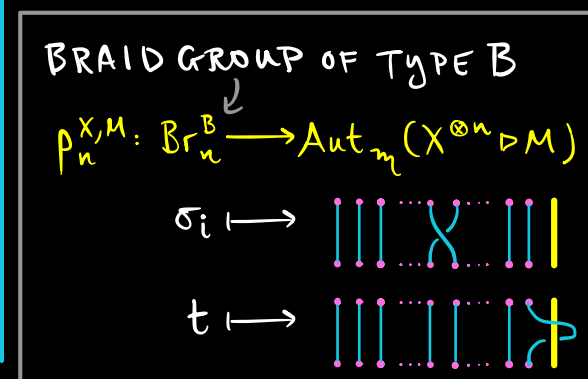
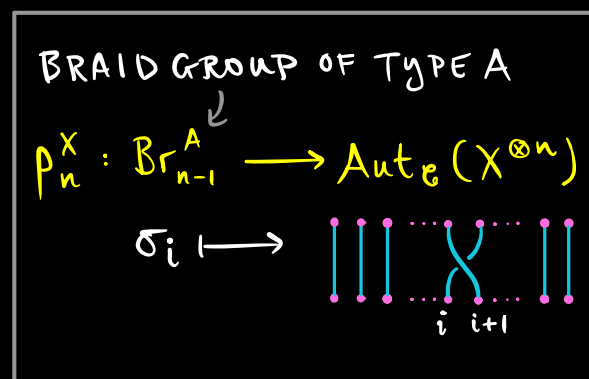
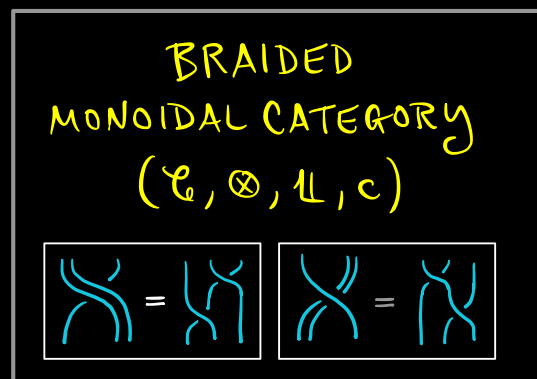
OBJECTS

FOR GENERAL
REPS OF Br^A

FOR GENERAL
REPS OF Br^B

OBJECTS

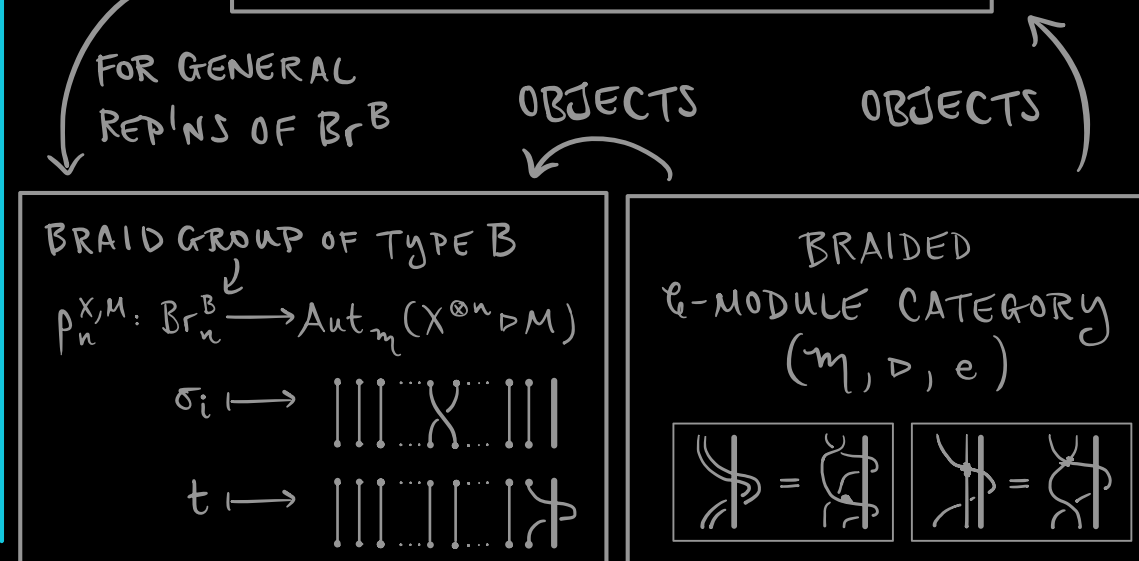
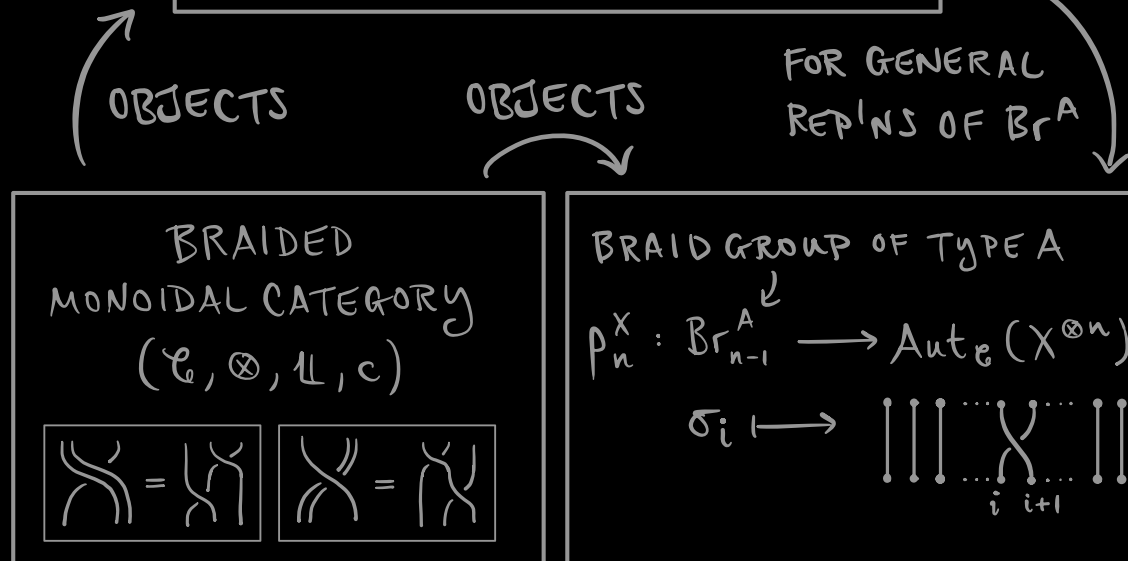
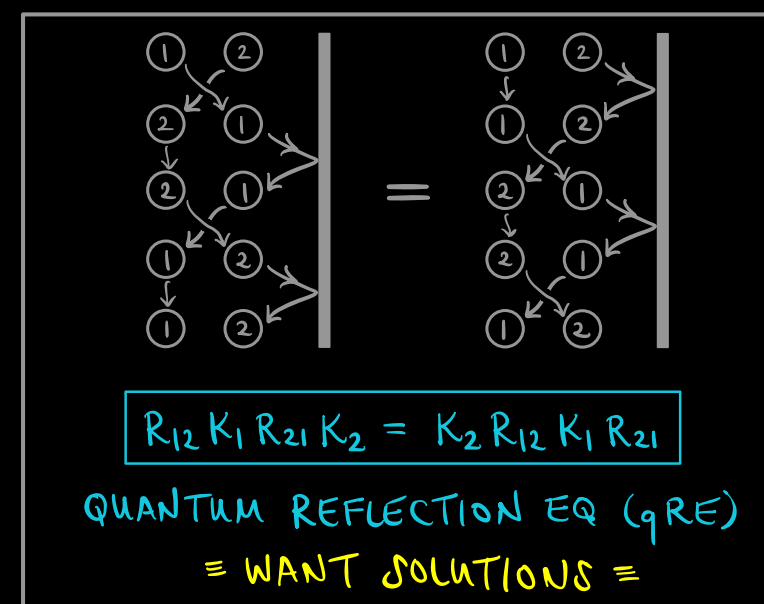
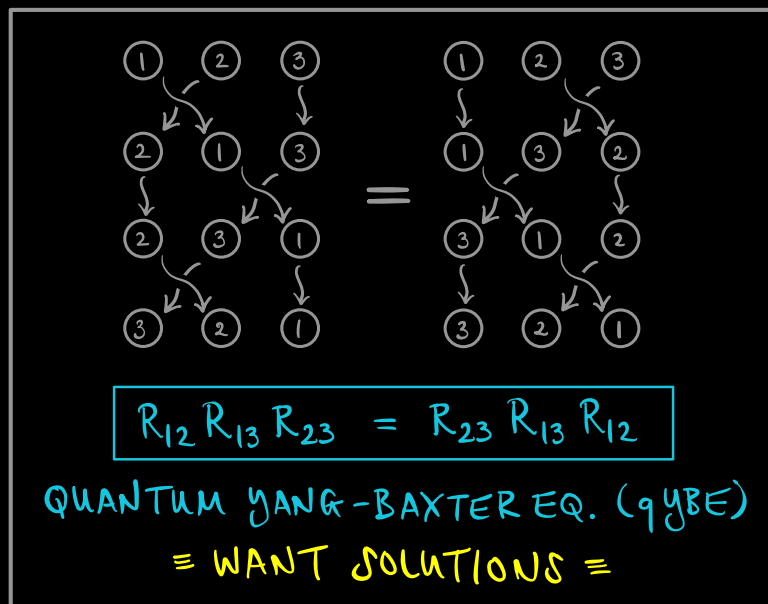
OBJECTS



$\equiv \text{TYPE A} \equiv$

HOPF-TYPE SOLUTIONS

$\equiv \text{TYPE B} \stackrel{C}{=} \equiv$



≡ TYPE A ≡

HOPF-TYPE SOLUTIONS

≡ TYPE B^C ≡

TAKE A QUASITRIANGULAR HOPF ALG. (H, R)

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)

≡ WANT SOLUTIONS ≡

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)

≡ WANT SOLUTIONS ≡

≡ TYPE A ≡

HOPF-TYPE SOLUTIONS

≡ TYPE B^C ≡

TAKE A QUASITRIANGULAR HOPF ALG. (H, R)

$\mathcal{C} := H\text{-Mod}$

MONOIDAL CATEGORY

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)

≡ WANT SOLUTIONS ≡

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)

≡ WANT SOLUTIONS ≡

≡ TYPE A ≡

HOPF-TYPE SOLUTIONS

≡ TYPE B^C ≡

TAKE A QUASITRIANGULAR HOPF ALG. (H, R)

INV. ELT. OF $H \otimes H$ + AXIOMS ↑

$\mathcal{C} := H\text{-Mod}$

MONOIDAL CATEGORY

"R-MATRIX" R

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)

≡ WANT SOLUTIONS ≡

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)

≡ WANT SOLUTIONS ≡

≡ TYPE A ≡

HOPF-TYPE SOLUTIONS

≡ TYPE B^C ≡

TAKE A QUASITRIANGULAR HOPF ALG. (H, R)

INV. ELT. OF $H \otimes H$ + AXIOMS 

$\mathcal{C} := H\text{-Mod}$ BRAIDED MONOIDAL CATEGORY

BRAIDING $C \longleftrightarrow$ "R-MATRIX" R

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

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$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

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≡ WANT SOLUTIONS ≡

≡ TYPE A ≡

HOPF-TYPE SOLUTIONS

≡ TYPE B^C ≡

TAKE A QUASITRIANGULAR HOPF ALG. (H, R)

INV. ELT. OF $H \otimes H$ + AXIOMS ↗

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QUANTUM YANG-BAXTER EQ. (qYBE)

≡ WANT SOLUTIONS ≡ ✓

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)

≡ WANT SOLUTIONS ≡

ON "SAMENESS" OF SOLUTIONS OF qYBE

= BRAIDED MORITA EQUIVALENCE:

$$(H, R)\text{-Mod} \simeq (H', R')\text{-Mod}$$

AS BRAIDED MONOIDAL CATEGORIES

$\equiv \text{TYPE A} \equiv$

HOPF-TYPE SOLUTIONS

$\equiv \text{TYPE B}^{\text{C}} \equiv$

THEOREM [SHIMIZU, 2010] TAKE F.D.
QUASITRIANGULAR HOPF ALGAS. $(H, R) \ncong (H', R')$.

$(H, R) \ncong (H', R')$ BR. MORITA EQUIV.

$\Rightarrow \rho_n^H \simeq \rho_n^{H'}$ AS REP'NS OF B_{n-1}^A

$$[\rho_n^H: B_{n-1}^A \rightarrow \text{Aut}_{H\text{-MOD}}(H^{\otimes n})]$$

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (QRE)
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ON "SAMENESS" OF SOLUTIONS OF QYBE

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AS BRAIDED MONOIDAL CATEGORIES

≡ TYPE A ≡

HOPF-TYPE SOLUTIONS

≡ TYPE B^C ≡

THEOREM [SHIMIZU, 2010] TAKE F.D.
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$(H, R) \nmid (H', R')$ BR. MORITA EQUIV.

$\Rightarrow \rho_n^H \simeq \rho_n^{H'}$ AS REP'NS OF B_{n-1}^A

EXAMPLE TAKE FINITE GROUPS G, G' .

$\leadsto (kG, R=e \otimes e)$ IS A QUASID HOPF ALG.

NOW $\text{trace}(\rho_n^G(e_{B_{n-1}^A})) = |G|^n$

$\therefore (kG, e \otimes e) \simeq (kG', e' \otimes e')$ BR. MOR. EQ

$\Rightarrow |G| = |G'|$

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)
≡ WANT SOLUTIONS ≡

≡ TYPE A ≡

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THEOREM [SHIMIZU, 2010] TAKE F.D.
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QUANTUM REFLECTION EQ (qRE)
≡ WANT SOLUTIONS ≡

≡ TYPE A ≡

HOPF-TYPE SOLUTIONS

≡ TYPE B^C ≡

TAKE A QUASITRIANGULAR HOPF ALG. (H, R)

INV. ELT. OF $H \otimes H$ + AXIOMS 

$\mathcal{C} := H\text{-Mod}$ BRAIDED MONOIDAL CATEGORY

BRAIDING $C \longleftrightarrow$ "R-MATRIX" R

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)

≡ WANT SOLUTIONS ≡ 

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

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ON "SAMENESS" OF SOLUTIONS OF qYBE

= BRAIDED MORITA EQUIVALENCE:

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AS BRAIDED MONOIDAL CATEGORIES

$\equiv \text{TYPE A} \equiv$

HOPF-TYPE SOLUTIONS

$\equiv \text{TYPE B}^{\text{C}} \equiv$

TAKE A QUASITRIANGULAR HOPF ALG. (H, R)
INV. ELT. OF $H \otimes H$ + AXIOMS $\uparrow$

$\mathcal{C} := H\text{-Mod}$ BRAIDED MONOIDAL CATEGORY

BRAIDING $C \longleftrightarrow$ "R-MATRIX" R

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)
 $\equiv$ WANT SOLUTIONS $\equiv \checkmark$

FIX A QUASIA HOPF ALG. (H, R)

$\nmid$ TAKE A QUASIA H -COMODULE ALG. (A, K)

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)
 $\equiv$ WANT SOLUTIONS $\equiv$

ON "SAMENESS" OF SOLUTIONS OF qYBE

= BRAIDED MORITA EQUIVALENCE:

$$(H, R)\text{-Mod} \simeq (H', R')\text{-Mod}$$

AS BRAIDED MONOIDAL CATEGORIES

$\equiv \text{TYPE A} \equiv$

HOPF-TYPE SOLUTIONS

$\equiv \text{TYPE B}^{\text{C}} \equiv$

TAKE A QUASITRIANGULAR HOPF ALG. (H, R)
INV. ELT. OF $H \otimes H$ + AXIOMS $\uparrow$

$\mathcal{C} := H\text{-Mod}$ BRAIDED MONOIDAL CATEGORY

BRAIDING $C \longleftrightarrow$ "R-MATRIX" R

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QUANTUM YANG-BAXTER EQ. (qYBE)
 $\equiv$ WANT SOLUTIONS $\equiv \checkmark$

FIX A QUASIA HOPF ALG. (H, R)

$\nmid$ TAKE A QUASIA H -COMODULE ALG. (A, K)

$\mathcal{M} := A\text{-Mod}$

$\mathcal{C}$ -MODULE CATEGORY

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)
 $\equiv$ WANT SOLUTIONS $\equiv$

ON "SAMENESS" OF SOLUTIONS OF qYBE

$=$ BRAIDED MORITA EQUIVALENCE:

$$(H, R)\text{-Mod} \simeq (H', R')\text{-Mod}$$

AS BRAIDED MONOIDAL CATEGORIES

≡ TYPE A ≡

HOPF-TYPE SOLUTIONS

≡ TYPE B^C ≡

TAKE A QUASITRIANGULAR HOPF ALG. (H, R)
INV. ELT. OF $H \otimes H$ + AXIOMS ↑

$\mathcal{C} := H\text{-Mod}$ BRAIDED MONOIDAL CATEGORY

BRAIDING $C \longleftrightarrow$ "R-MATRIX" R

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)
≡ WANT SOLUTIONS ≡ ✓

FIX A QUASIA HOPF ALG. (H, R)

≠ TAKE A QUASIA H -COMODULE ALG. (A, K)
[KOLB, 2020] INV. ELT. OF $H \otimes A$ + AXIOMS ↑

$\mathcal{M} := A\text{-Mod}$

$\mathcal{C}$ -MODULE CATEGORY

"K-MATRIX" K

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)
≡ WANT SOLUTIONS ≡

ON "SAMENESS" OF SOLUTIONS OF qYBE

= BRAIDED MORITA EQUIVALENCE:

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AS BRAIDED MONOIDAL CATEGORIES

$\equiv \text{TYPE A} \equiv$

HOPF-TYPE SOLUTIONS

$\equiv \text{TYPE B} \stackrel{C}{\equiv}$

TAKE A QUASITRIANGULAR HOPF ALG. (H, R)
INV. ELT. OF $H \otimes H$ + AXIOMS $\uparrow$

$\mathcal{C} := H\text{-Mod}$ BRAIDED MONOIDAL CATEGORY

BRAIDING $c \longleftrightarrow$ "R-MATRIX" R

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)
 $\equiv$ WANT SOLUTIONS $\equiv \checkmark$

FIX A QUASIA HOPF ALG. (H, R)

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[KOLB, 2020] INV. ELT. OF $H \otimes A$ + AXIOMS $\uparrow$

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BRAIDING $e \longleftrightarrow$ "K-MATRIX" K

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)
 $\equiv$ WANT SOLUTIONS $\equiv \checkmark$

ON "SAMENESS" OF SOLUTIONS OF qYBE

$=$ BRAIDED MORITA EQUIVALENCE:

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AS BRAIDED MONOIDAL CATEGORIES

$\equiv \text{TYPE A} \equiv$

HOPF-TYPE SOLUTIONS

$\equiv \text{TYPE B} \stackrel{C}{\equiv}$

TAKE A QUASITRIANGULAR HOPF ALG. (H, R)
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[KOLB, 2020] INV. ELT. OF $H \otimes A$ + AXIOMS $\uparrow$

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QUANTUM REFLECTION EQ (qRE)
 $\equiv$ WANT SOLUTIONS $\equiv \checkmark$

ON "SAMENESS" OF SOLUTIONS OF qYBE

$=$ BRAIDED MORITA EQUIVALENCE:

$$(H, R)\text{-Mod} \simeq (H', R')\text{-Mod}$$

AS BRAIDED MONOIDAL CATEGORIES

ON "SAMENESS" OF SOLUTIONS OF qRE

$=$ BRAIDED MORITA EQUIVALENCE:

$$(A, K)\text{-Mod} \simeq (A', K')\text{-Mod}$$

AS BRAIDED $\mathcal{C}$ -MODULE CATEGORIES

≡ TYPE A ≡

HOPF-TYPE SOLUTIONS

≡ TYPE B^C ≡

THEOREM [SHIMIZU, 2010] TAKE F.D.
QUASIA HOPF ALGS. (H, R) & (H', R') .

(H, R) & (H', R') BR. MORITA EQUIV.

⇒ $p_n^H \simeq p_n^{H'}$ AS REP'NS OF B_{n-1}^A

$$[p_n^H: B_{n-1}^A \rightarrow \text{Aut}_{H\text{-Mod}}(H^{\otimes n})]$$

ON "SAMENESS" OF SOLUTIONS OF qYBE

= BRAIDED MORITA EQUIVALENCE:

$$(H, R)\text{-Mod} \simeq (H', R')\text{-Mod}$$

AS BRAIDED MONOIDAL CATEGORIES

FIX A QUASIA HOPF ALG. (H, R)

& TAKE A QUASIA H-COMODULE ALG. (A, K)

[KOLB, 2020] INV. ELT. OF $H \otimes A$ + AXIOMS

$\mathcal{M} := A\text{-Mod}$ BRAIDED $\mathcal{C}$ -MODULE CATEGORY

BRAIDING $e \longleftrightarrow$ "K-MATRIX" K

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)

≡ WANT SOLUTIONS ≡ ✓

ON "SAMENESS" OF SOLUTIONS OF qRE

= BRAIDED MORITA EQUIVALENCE:

$$(A, K)\text{-Mod} \simeq (A', K')\text{-Mod}$$

AS BRAIDED $\mathcal{C}$ -MODULE CATEGORIES

≡ TYPE A ≡

HOPF-TYPE SOLUTIONS

≡ TYPE B^C ≡

THEOREM [SHIMIZU, 2010] TAKE F.D.
QUASIA HOPF ALGS. $(H, R) \nmid (H', R')$.

$(H, R) \nmid (H', R')$ BR. MORITA EQUIV.

$\Rightarrow \rho_n^H \simeq \rho_n^{H'}$ AS REP'NS OF $B\Gamma_{n-1}^A$

$$[\rho_n^H: B\Gamma_{n-1}^A \rightarrow \text{Aut}_{H\text{-Mod}}(H^{\otimes n})]$$

ON "SAMENESS" OF SOLUTIONS OF qYBE

= BRAIDED MORITA EQUIVALENCE:

$$(H, R)\text{-Mod} \simeq (H', R')\text{-Mod}$$

AS BRAIDED MONOIDAL CATEGORIES

FIX A QUASIA HOPF ALG. (H, R) COMOD ALGS

THEOREM [MÜLLER-W, 2026] ↙ "IN" H

TAKE F.D. QUASIA COIDEAL SUBALGS OF (H, R) :

$$(A, K) \nmid (A', K')$$

THAT ARE BRAIDED MORITA EQUIV.

$\Rightarrow \rho_n^{H,A} \simeq \rho_n^{H,A'}$ AS REP'NS OF $B\Gamma_n^B$

$$[\rho_n^{H,A}: B\Gamma_n^B \rightarrow \text{Aut}_{A\text{-Mod}}(H^{\otimes n} \otimes A)]$$

ON "SAMENESS" OF SOLUTIONS OF qRE

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THEOREM [SHIMIZU, 2010] TAKE F.D.
QUASIA HOPF ALGS. $(H, R) \ncong (H', R')$.

$(H, R) \ncong (H', R')$ BR. MORITA EQUIV.

$\Rightarrow \rho_n^H \simeq \rho_n^{H'}$ AS REP'NS OF $B\Gamma_{n-1}^A$

EXAMPLE TAKE FINITE GROUPS G, G' .

$\leadsto (kG, R=e \otimes e)$ IS A QUASIA HOPF ALG

NOW $\text{trace}(\rho_n^G(e_{B\Gamma_{n-1}^A})) = |G|^n$

$\therefore (kG, e \otimes e) \simeq (kG', e' \otimes e')$ BR. MOR. EQ

$\Rightarrow |G| = |G'|$

FIX A QUASIA HOPF ALG. (H, R) COMOD ALGS

THEOREM [MÜLLER-W, 2026] $\swarrow$ "IN" H

TAKE F.D. QUASIA COIDEAL SUBALGS OF (H, R) :

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ON "SAMENESS" OF SOLUTIONS OF qRE

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THEOREM [MÜLLER-W, 2026] $\swarrow$ "IN" H

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THAT ARE BRAIDED MORITA EQUIV.

$\Rightarrow \rho_n^{H,A} \simeq \rho_n^{H,A'}$ AS REP'NS OF B_n^B

EXAMPLE TAKE A FINITE GROUP G
WITH SUBGROUPS L, L' .

LIKEWISE...

$(kL, e \otimes e) \simeq (kL', e \otimes e)$ BR. MOR. EQ

$\Rightarrow |L| = |L'|$

$\equiv \text{Type A} \equiv$

SUMMARY ... SO FAR

$\equiv \text{Type B} \stackrel{C}{\equiv}$

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

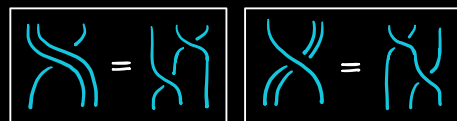
QUANTUM YANG-BAXTER EQ. (qYBE)
 $\equiv \text{WANT SOLUTIONS} \equiv$

OBJECTS

OBJECTS

FOR GENERAL
 REPS OF Br^A

BRAIDED
 MONOIDAL CATEGORY
 $(\mathcal{C}, \otimes, \mathbb{1}, c)$



BRAID GROUP OF TYPE A

$$\rho_n^X: Br_{n-1}^A \rightarrow \text{Aut}_{\mathcal{C}}(X^{\otimes n})$$

$$\sigma_i \mapsto \begin{array}{c} \text{Diagram of a crossing between strands } i \text{ and } i+1 \end{array}$$

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)
 $\equiv \text{WANT SOLUTIONS} \equiv$

FOR GENERAL
 REPS OF Br^B

OBJECTS

OBJECTS

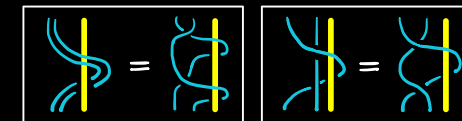
BRAID GROUP OF TYPE B

$$\rho_n^{X,M}: Br_n^B \rightarrow \text{Aut}_M(X^{\otimes n} \triangleright M)$$

$$\sigma_i \mapsto \begin{array}{c} \text{Diagram of a crossing between strands } i \text{ and } i+1 \end{array}$$

$$t \mapsto \begin{array}{c} \text{Diagram of a twist on strand } i \end{array}$$

BRAIDED
 $\mathcal{C}$ -MODULE CATEGORY
 $(\mathcal{M}, \triangleright, e)$



$\equiv \text{TYPE A} \equiv$

SUMMARY ... SO FAR

$\equiv \text{TYPE B}^{\mathbb{C}} \equiv$

HOPF CASE:
QUASIA HOPF ALGS
... UP TO BR. MORITA EQUIV

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

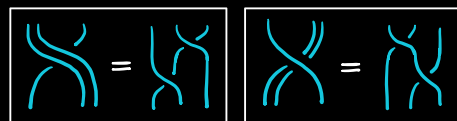
QUANTUM YANG-BAXTER EQ. (qYBE)
 $\equiv \text{WANT SOLUTIONS} \equiv$

OBJECTS

OBJECTS

FOR GENERAL
REP'NS OF Br^A

BRAIDED
MONOIDAL CATEGORY
($\mathcal{C}, \otimes, \mathbb{1}, c$)



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HOPF CASE:
QUASIA COMODULE ALGS
... UP TO BR. MORITA EQUIV

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)
 $\equiv \text{WANT SOLUTIONS} \equiv$

FOR GENERAL
REP'NS OF Br^B

OBJECTS

OBJECTS

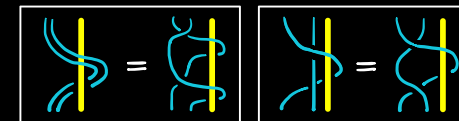
BRAID GROUP OF TYPE B

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BRAIDED
 $\mathcal{C}$ -MODULE CATEGORY
($\mathcal{M}, \triangleright, e$)



$\equiv \text{TYPE A} \equiv$

SUMMARY ... SO FAR

$\equiv \text{TYPE B}^{\mathbb{C}} \equiv$

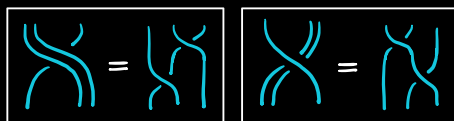
$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)

$\equiv \text{WANT SOLUTIONS} \equiv$

OBJECTS

BRAIDED
MONOIDAL CATEGORY
 $(\mathcal{C}, \otimes, 1, c)$



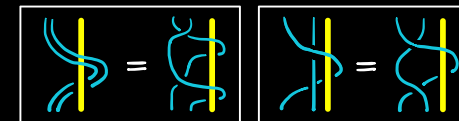
$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)

$\equiv \text{WANT SOLUTIONS} \equiv$

OBJECTS

BRAIDED
 $\mathbb{C}$ -MODULE CATEGORY
 $(\mathcal{M}, \triangleright, e)$



$\equiv \text{Type A} \equiv$

CATEGORICAL CENTERS

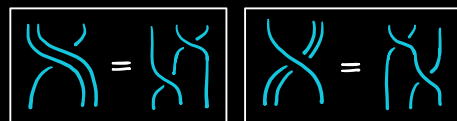
$\equiv \text{Type B}^{\mathbb{C}} \equiv$

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)
 $\equiv \text{WANT SOLUTIONS} \equiv$

OBJECTS

BRAIDED
MONOIDAL CATEGORY
 $(\mathcal{C}, \otimes, 1, c)$



• CHELSEA WALTON (RICE UNIV.) •

LET'S
CONSTRUCT
THESE

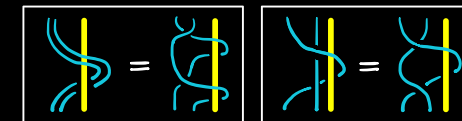


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QUANTUM REFLECTION EQ (qRE)
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OBJECTS

BRAIDED
 $\mathcal{C}$ -MODULE CATEGORY
 $(\mathcal{M}, \triangleright, e)$



• INTL CT CONF., JULY 2026 •

LET'S
CONSTRUCT
THESE



• VENTURING... BOUNDARY w/ BRAIDED MOD. CATS •

DRINFELD (1987)
JOYAL-STREET (1991)
MAJID (1991)

KASSEL
(1995)

CATEGORICAL CENTERS

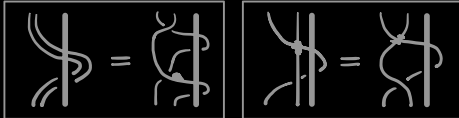
$(\mathcal{C}, \otimes, \mathbb{1}, c)$



• CHELSEA WALTON (RICE UNIV) •

• VENTURING... BOUNDARY w/ BRAIDED MOD. CATS •

$(\mathcal{M}, \triangleright, e)$



• INTL CT CONF., JULY 2026 •

DRINFELD (1987)
JOYAL-STREET (1991)
MAJID (1991)

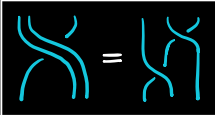
KASSEL
(1995)

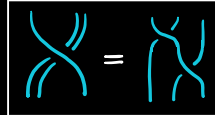
CATEGORICAL CENTERS

TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

$\leadsto$ DRINFELD CENTER $Z(\mathcal{C})$:

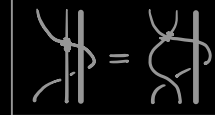
$(\mathcal{C}, \otimes, \mathbb{1}, c)$





$(\mathcal{M}, \triangleright, e)$





DRINFELD (1987)
JOYAL-STREET (1991)
MAJID (1991)

KASSEL
(1995)

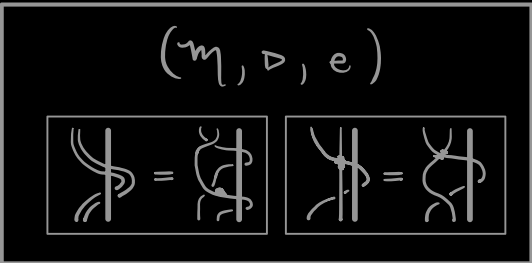
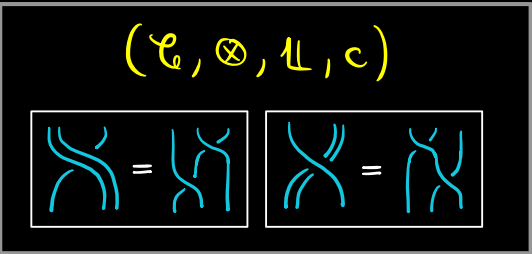
CATEGORICAL CENTERS

TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

$\rightarrow$ DRINFELD CENTER $Z(\mathcal{C})$:

OBJS: $(V \in \mathcal{C}, c^V := \{c^V_x : X \otimes V \xrightarrow{\sim} V \otimes X\}_{X \in \mathcal{C}})$

WITH $c^V_{x \otimes y}$ SATISFYING...



DRINFELD (1987)
JOYAL-STREET (1991)
MAJID (1991)

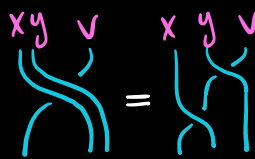
KASSEL
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CATEGORICAL CENTERS

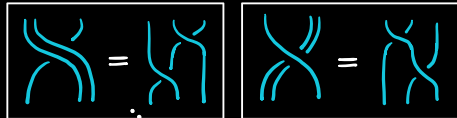
TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

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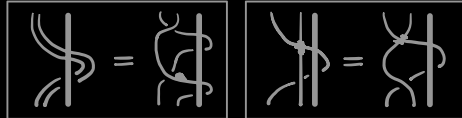
OBJS: $(V \in \mathcal{C}, c^V := \{c^V_x : X \otimes V \xrightarrow{\sim} V \otimes X\}_{X \in \mathcal{C}})$

WITH $c^V_{x \otimes y}$ SATISFYING 

$(\mathcal{C}, \otimes, \mathbb{1}, c)$



$(\mathcal{M}, \triangleright, e)$



DRINFELD (1987)
JOYAL-STREET (1991)
MAJID (1991)

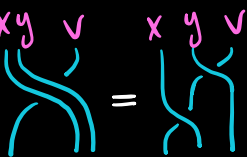
KASSEL
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CATEGORICAL CENTERS

TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

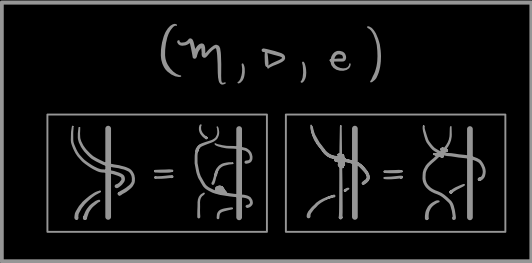
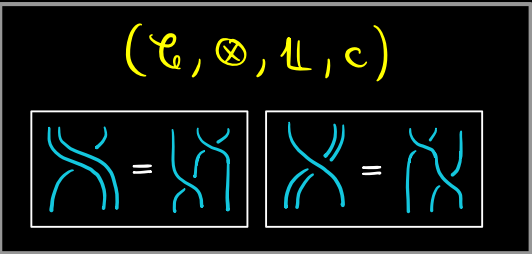
$\rightarrow$ DRINFELD CENTER $Z(\mathcal{C})$:

OBJS: $(V \in \mathcal{C}, c^V := \{c_x^V : X \otimes V \xrightarrow{\sim} V \otimes X\}_{X \in \mathcal{C}})$

WITH $c_{x \otimes y}^V$ SATISFYING 

$\otimes : (V, c^V) \otimes (W, c^W) := (V \otimes W, c^{V \otimes W})$

WITH $c_x^{V \otimes W} := \dots$



DRINFELD (1987)
JOYAL-STREET (1991)
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CATEGORICAL CENTERS

TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

→ DRINFELD CENTER $Z(\mathcal{C})$:

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WITH $c_x^{V \otimes W} := \begin{array}{c} x \quad V \quad W \\ \text{diagram} \end{array} = \begin{array}{c} x \quad V \quad W \\ \text{diagram} \end{array}$

$(\mathcal{C}, \otimes, \mathbb{1}, c)$

$(\mathcal{M}, \triangleright, e)$

DRINFELD (1987)
JOYAL-STREET (1991)
MAJID (1991)

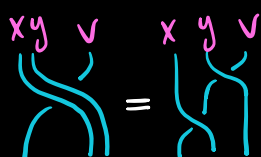
KASSEL
(1995)

CATEGORICAL CENTERS

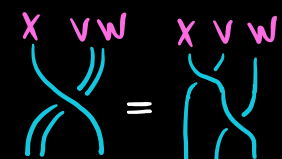
TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

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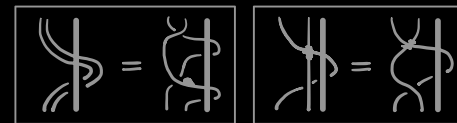
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WITH $c_x^{V \otimes W} :=$ 

$c : (V, c^V) \otimes (W, c^W) \xrightarrow{c_V^W} (W, c^W) \otimes (V, c^V)$

$\equiv$ BRAIDED MONOIDAL CATEGORY $\equiv$

$$(\mathcal{M}, \triangleright, e)$$


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JOYAL-STREET (1991)
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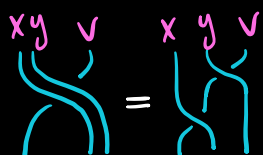
CATEGORICAL CENTERS

LAUGWITZ-W
-YAKIMOV (2025)

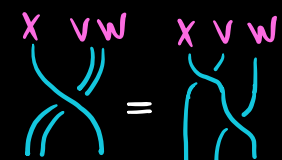
TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

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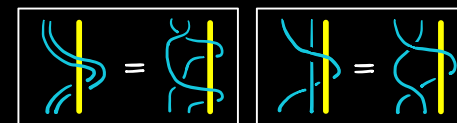
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CATEGORICAL CENTERS

LAUGWITZ-W
-YAKIMOV (2025)

TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

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$c : (V, c^V) \otimes (W, c^W) \xrightarrow{c_V^W} (W, c^W) \otimes (V, c^V)$

$\equiv$ BRAIDED MONOIDAL CATEGORY $\equiv$

FIX A BRAIDED MONOIDAL CAT $\mathcal{C}$

TAKE A $\mathcal{C}$ -MODULE CAT $(\mathcal{M}, \triangleright)$

$\rightarrow$ REFLECTIVE CENTER $\mathcal{E}_{\mathcal{C}}(\mathcal{M})$:

$(\mathcal{M}, \triangleright, e)$

$$\begin{array}{|c|} \hline \begin{array}{c} \text{blue} \quad \text{yellow} \\ \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} = \begin{array}{c} \text{blue} \quad \text{yellow} \\ \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \end{array} \quad \begin{array}{|c|} \hline \begin{array}{c} \text{blue} \quad \text{yellow} \\ \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} = \begin{array}{c} \text{blue} \quad \text{yellow} \\ \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \end{array}$$

DRINFELD (1987)
JOYAL-STREET (1991)
MAJID (1991)

KASSEL
(1995)

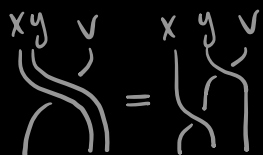
CATEGORICAL CENTERS

LAUGWITZ-W
-YAKIMOV (2025)

TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

$\rightarrow$ DRINFELD CENTER $Z(\mathcal{C})$:

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$\otimes : (V, c^V) \otimes (W, c^W) := (V \otimes W, c^{V \otimes W})$

WITH $c_x^{V \otimes W} := \img alt="Diagram showing a braid of two strands labeled x and y, with a vertical line labeled V \otimes W passing through them. The diagram is equal to a similar braid with the strands labeled y and x." data-bbox="370 601 476 692"/>$

$c : (V, c^V) \otimes (W, c^W) \xrightarrow{c_V^W} (W, c^W) \otimes (V, c^V)$

$\equiv$ BRAIDED MONOIDAL CATEGORY $\equiv$

FIX A BRAIDED MONOIDAL CAT $\mathcal{C}$

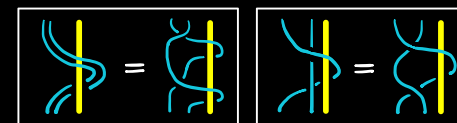
TAKE A $\mathcal{C}$ -MODULE CAT $(\mathcal{M}, \triangleright)$

$\rightarrow$ REFLECTIVE CENTER $\mathcal{E}_{\mathcal{C}}(\mathcal{M})$:

OBJS: $(M \in \mathcal{M}, e^M := \{e_x^M : X \triangleright M \xrightarrow{\sim} X \triangleright M\}_{X \in \mathcal{C}})$

WITH $e_{x \otimes y}^M$ SATISFYING ...

$(\mathcal{M}, \triangleright, e)$



DRINFELD (1987)
JOYAL-STREET (1991)
MAJID (1991)

KASSEL
(1995)

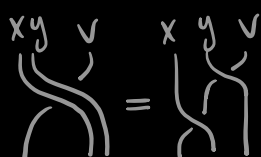
CATEGORICAL CENTERS

LAUGWITZ-W
-YAKIMOV (2025)

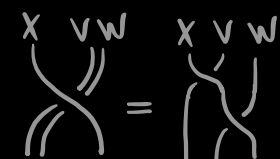
TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

$\rightarrow$ DRINFELD CENTER $Z(\mathcal{C})$:

OBJS: $(V \in \mathcal{C}, c^V := \{c_x^V : X \otimes V \xrightarrow{\sim} V \otimes X\}_{X \in \mathcal{C}})$

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$\otimes : (V, c^V) \otimes (W, c^W) := (V \otimes W, c^{V \otimes W})$

WITH $c_x^{V \otimes W} :=$ 

$c : (V, c^V) \otimes (W, c^W) \xrightarrow{c_V^W} (W, c^W) \otimes (V, c^V)$

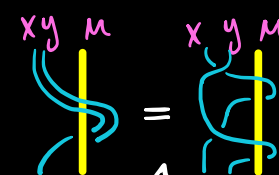
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FIX A BRAIDED MONOIDAL CAT $\mathcal{C}$

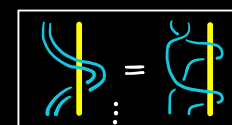
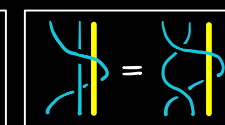
TAKE A $\mathcal{C}$ -MODULE CAT $(\mathcal{M}, \triangleright)$

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$(\mathcal{M}, \triangleright, e)$

DRINFELD (1987)
JOYAL-STREET (1991)
MAJID (1991)

KASSEL
(1995)

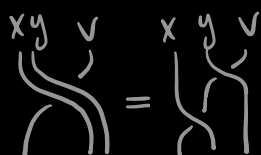
CATEGORICAL CENTERS

LAUGWITZ-W
-YAKIMOV (2025)

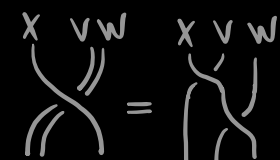
TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

$\rightarrow$ DRINFELD CENTER $\mathcal{Z}(\mathcal{C})$:

OBJS: $(V \in \mathcal{C}, c^V := \{c_x^V : X \otimes V \xrightarrow{\sim} V \otimes X\}_{X \in \mathcal{C}})$

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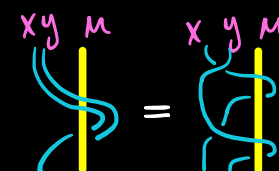
$\equiv$ BRAIDED MONOIDAL CATEGORY $\equiv$

FIX A BRAIDED MONOIDAL CAT $\mathcal{C}$

TAKE A $\mathcal{C}$ -MODULE CAT $(\mathcal{M}, \triangleright)$

$\rightarrow$ REFLECTIVE CENTER $\mathcal{E}_{\mathcal{C}}(\mathcal{M})$:

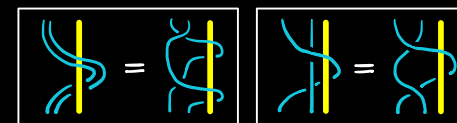
OBJS: $(M \in \mathcal{M}, e^M := \{e_x^M : X \triangleright M \xrightarrow{\sim} X \triangleright M\}_{X \in \mathcal{C}})$

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WITH $e_x^{Y \triangleright M} := \dots$

$(\mathcal{M}, \triangleright, e)$



DRINFELD (1987)
JOYAL-STREET (1991)
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KASSEL
(1995)

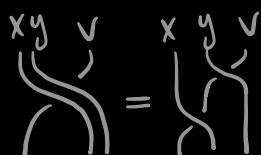
CATEGORICAL CENTERS

LAUGWITZ-W
-YAKIMOV (2025)

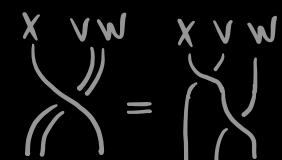
TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

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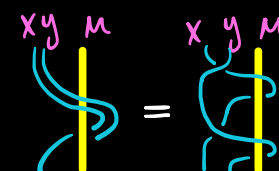
$\equiv$ BRAIDED MONOIDAL CATEGORY $\equiv$

FIX A BRAIDED MONOIDAL CAT $\mathcal{C}$

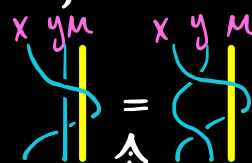
TAKE A $\mathcal{C}$ -MODULE CAT $(\mathcal{M}, \triangleright)$

$\rightarrow$ REFLECTIVE CENTER $\mathcal{E}_{\mathcal{C}}(\mathcal{M})$:

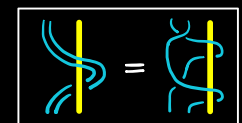
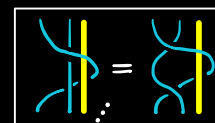
OBJS: $(M \in \mathcal{M}, e^M := \{e_x^M : X \triangleright M \xrightarrow{\sim} X \triangleright M\}_{X \in \mathcal{C}})$

WITH $e_{x \otimes y}^M$ SATISFYING 

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$(\mathcal{M}, \triangleright, e)$

 $=$ 

DRINFELD (1987)
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MAJID (1991)

KASSEL
(1995)

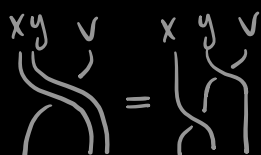
CATEGORICAL CENTERS

LAUGWITZ-W
-YAKIMOV (2025)

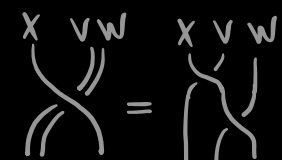
TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

$\rightarrow$ DRINFELD CENTER $Z(\mathcal{C})$:

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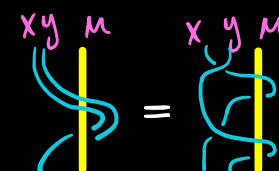
$\equiv$ BRAIDED MONOIDAL CATEGORY $\equiv$

FIX A BRAIDED MONOIDAL CAT $\mathcal{C}$

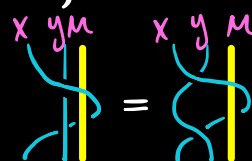
TAKE A $\mathcal{C}$ -MODULE CAT $(\mathcal{M}, \triangleright)$

$\rightarrow$ REFLECTIVE CENTER $\mathcal{E}_{\mathcal{C}}(\mathcal{M})$:

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$\equiv$ BRAIDED $\mathcal{C}$ -MODULE CATEGORY $\equiv$

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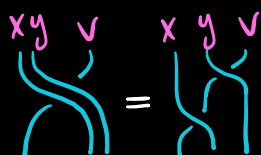
CATEGORICAL CENTERS

LAUGWITZ-W
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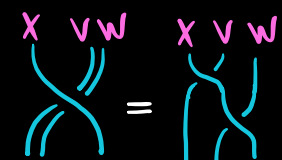
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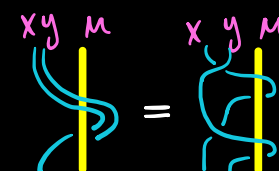
= BRAIDED MONOIDAL CATEGORY =

FIX A BRAIDED MONOIDAL CAT $\mathcal{C}$

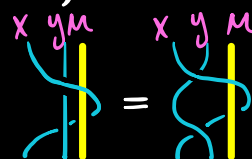
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= BRAIDED $\mathcal{C}$ -MODULE CATEGORY =

DRINFELD (1987)
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CATEGORICAL CENTERS

LAUGWITZ-W
-YAKIMOV (2025)

TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

$\rightarrow$ DRINFELD CENTER $Z(\mathcal{C})$:

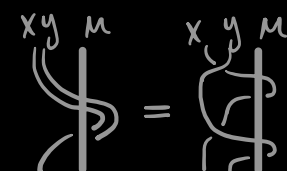
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FIX A BRAIDED MONOIDAL CAT $\mathcal{C}$

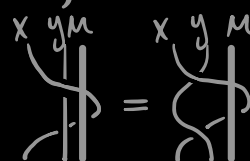
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CATEGORICAL CENTERS

LAUGWITZ-W
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TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

$\rightarrow$ DRINFELD CENTER $\mathcal{Z}(\mathcal{C})$:

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HOPF CASE: TAKE F.D. HOPF ALG H .

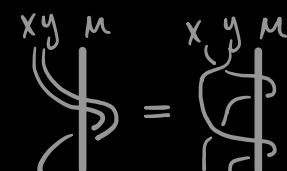
$\rightarrow \boxed{\mathcal{Z}(H\text{-Mod}) \stackrel{\text{BR} \otimes}{\simeq} ???\text{-Mod}}$

FIX A BRAIDED MONOIDAL CAT $\mathcal{C}$

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WITH $e_x^{y \triangleright M} := \img alt="Diagrammatic equation for e_x^{y \triangleright M}. On the left, a box labeled 'x' and 'y' is connected to a box labeled 'M' by a vertical line. On the right, a box labeled 'x' and 'y' is connected to a box labeled 'M' by a vertical line. The two diagrams are shown to be equal." data-bbox="810 589 903 680"/>$

$e : y \triangleright (M, e^M) \xrightarrow{e_y^M} y \triangleright (M, e^M)$

$\equiv$ BRAIDED $\mathcal{C}$ -MODULE CATEGORY $\equiv$

DRINFELD (1987)
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CATEGORICAL CENTERS

LAUGWITZ-W
-YAKIMOV (2025)

TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

$\rightarrow$ DRINFELD CENTER $\mathcal{Z}(\mathcal{C})$:
 $\equiv$ BRAIDED MONOIDAL CATEGORY $\equiv$

HOPF CASE: TAKE F.D. HOPF ALG H .

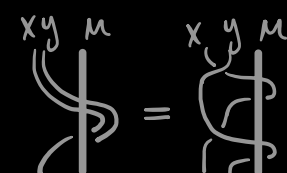
$\rightarrow \boxed{\mathcal{Z}(H\text{-Mod}) \stackrel{\text{BR} \otimes}{\simeq} D(H)\text{-Mod}}$
(QUASI Δ) DRINFELD DOUBLE OF H
($= H^* \otimes H$ AS A VS)

FIX A BRAIDED MONOIDAL CAT $\mathcal{C}$

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CATEGORICAL CENTERS

LAUGWITZ-W
-YAKIMOV (2025)

TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

→ DRINFELD CENTER $Z(\mathcal{C})$:
= BRAIDED MONOIDAL CATEGORY =

HOPF CASE: TAKE F.D. HOPF ALG H .

→ $Z(H\text{-Mod}) \stackrel{\text{BR} \otimes}{\simeq} D(H)\text{-Mod}$
(QUASI Δ) DRINFELD DOUBLE OF H
(= $H^* \otimes H$ AS A VS)

SHIMIZU: H, H' F.D. HOPF ALGS.

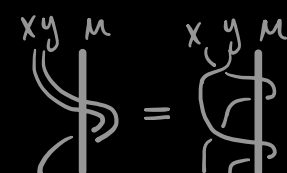
$H\text{-Mod} \simeq H'\text{-Mod} \Rightarrow \rho_n^{D(H)} \simeq \rho_n^{D(H')}$
AS MONOIDAL CATS AS REPS OF Br_{n-1}^A

FIX A BRAIDED MONOIDAL CAT $\mathcal{C}$

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$e : y \triangleright (M, e^M) \xrightarrow{e_y^M} y \triangleright (M, e^M)$
= BRAIDED $\mathcal{C}$ -MODULE CATEGORY =

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(1995)

CATEGORICAL CENTERS

LAUGWITZ-W
-YAKIMOV (2025)

TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

$\rightarrow$ DRINFELD CENTER $\mathcal{Z}(\mathcal{C})$:
 $\equiv$ BRAIDED MONOIDAL CATEGORY $\equiv$

HOPF CASE: TAKE F.D. HOPF ALG H .

$\rightarrow \boxed{\mathcal{Z}(H\text{-Mod}) \stackrel{\text{BR} \otimes}{\simeq} D(H)\text{-Mod}}$
 $\uparrow$
(QUASI Δ) DRINFELD DOUBLE OF H
($= H^* \otimes H$ AS A VS)

SHIMIZU: H, H' F.D. HOPF ALGS.

$H\text{-Mod} \simeq H'\text{-Mod} \Rightarrow \rho_n^{D(H)} \simeq \rho_n^{D(H')}$
AS MONOIDAL CATS AS REPS OF Br_{n-1}^A

FIX A BRAIDED MONOIDAL CAT $\mathcal{C}$

TAKE A $\mathcal{C}$ -MODULE CAT $(\mathcal{M}, \triangleright)$

$\rightarrow$ REFLECTIVE CENTER $\mathcal{E}_{\mathcal{C}}(\mathcal{M})$:
 $\equiv$ BRAIDED $\mathcal{C}$ -MODULE CATEGORY $\equiv$

DRINFELD (1987)
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(1995)

CATEGORICAL CENTERS

LAUGWITZ-W
-YAKIMOV (2025)

TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

→ DRINFELD CENTER $\mathcal{Z}(\mathcal{C})$:
= BRAIDED MONOIDAL CATEGORY =

HOPF CASE: TAKE F.D. HOPF ALG H .

→ $\mathcal{Z}(H\text{-Mod}) \stackrel{\text{BR} \otimes}{\simeq} D(H)\text{-Mod}$
(QUASI Δ) DRINFELD DOUBLE OF H
(= $H^* \otimes H$ AS A VS)

SHIMIZU: H, H' F.D. HOPF ALGS.

$H\text{-Mod} \simeq H'\text{-Mod} \Rightarrow \rho_n^{D(H)} \simeq \rho_n^{D(H')}$
AS MONOIDAL CATS AS REPS OF Br_{n-1}^A

FIX A BRAIDED MONOIDAL CAT $\mathcal{C}$

TAKE A $\mathcal{C}$ -MODULE CAT $(\mathcal{M}, \triangleright)$

→ REFLECTIVE CENTER $\mathcal{E}_{\mathcal{C}}(\mathcal{M})$:
= BRAIDED $\mathcal{C}$ -MODULE CATEGORY =

HOPF CASE: FIX A F.D. QUASI Δ HOPF ALG H
& TAKE A F.D. H -COMOD ALG A

→ $\mathcal{E}_{H\text{-Mod}}(A\text{-Mod}) \stackrel{\text{BR MOD}}{\simeq} ???\text{-Mod}$

DRINFELD (1987)
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CATEGORICAL CENTERS

LAUGWITZ-W
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TAKE A MONOIDAL CAT $(\mathcal{C}, \otimes, \mathbb{1})$

→ DRINFELD CENTER $\mathcal{Z}(\mathcal{C})$:
= BRAIDED MONOIDAL CATEGORY =

HOPF CASE: TAKE F.D. HOPF ALG H .

→ $\mathcal{Z}(H\text{-Mod}) \stackrel{\text{BR} \otimes}{\simeq} D(H)\text{-Mod}$
 $(\underline{\text{QUASI}} \Delta)$ DRINFELD DOUBLE OF H
 $(= H^* \otimes H \text{ AS } A \text{ VS})$

SHIMIZU: H, H' F.D. HOPF ALGS.

$H\text{-Mod} \simeq H'\text{-Mod} \Rightarrow \rho_n^{D(H)} \simeq \rho_n^{D(H')}$
 AS MONOIDAL CATS AS REPS OF Br_{n-1}^A

FIX A BRAIDED MONOIDAL CAT $\mathcal{C}$

TAKE A $\mathcal{C}$ -MODULE CAT $(\mathcal{M}, \triangleright)$

→ REFLECTIVE CENTER $\mathcal{E}_{\mathcal{C}}(\mathcal{M})$:
= BRAIDED $\mathcal{C}$ -MODULE CATEGORY =

HOPF CASE: FIX A F.D. QUASI Δ HOPF ALG H
 & TAKE A F.D. H -COMOD ALG A

→ $\mathcal{E}_{H\text{-Mod}}(A\text{-Mod}) \stackrel{\text{BR MOD}}{\simeq} R_H(A)\text{-Mod}$
 $(\underline{\text{QUASI}} \Delta)$ REFLECTIVE ALG OF A W.R.T. H
 $(= H^* \otimes A \text{ AS } A \text{ VS})$

DRINFELD (1987)
JOYAL-STREET (1991)
MAJID (1991)

KASSEL
(1995)

CATEGORICAL CENTERS

LAUGWITZ-W
-YAKIMOV (2025)

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• CHELSEA WALTON (RICE UNIV) •

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• VENTURING... BOUNDARY W/ BRAIDED MOD. CATS •

• INTL CT CONF., JULY 2026 •

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• INTL CT CONF., JULY 2026 •

$\equiv \text{Type A} \equiv$

SUMMARY ... SO FAR

$\equiv \text{Type B}^c \equiv$

HOPF CASE:
QUASIA HOPF ALGS
... UP TO BR. MORITA EQUIV

HOPF CASE:
QUASIA COMODULE ALGS
... UP TO BR. MORITA EQUIV

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)
 $\equiv \text{WANT SOLUTIONS} \equiv$

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)
 $\equiv \text{WANT SOLUTIONS} \equiv$

OBJECTS

OBJECTS

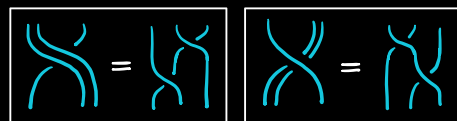
FOR GENERAL
REP'NS OF Br^A

FOR GENERAL
REP'NS OF Br^B

OBJECTS

OBJECTS

BRAIDED
MONOIDAL CATEGORY
($\mathcal{C}, \otimes, \mathbb{1}, c$)



BRAID GROUP OF TYPE A

$$\rho_n^X: Br_{n-1}^A \rightarrow \text{Aut}_{\mathcal{C}}(X^{\otimes n})$$

$$\sigma_i \mapsto \text{diagram of two strands crossing at positions } i \text{ and } i+1$$

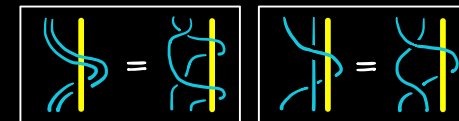
BRAID GROUP OF TYPE B

$$\rho_n^{X,M}: Br_n^B \rightarrow \text{Aut}_{\mathcal{M}}(X^{\otimes n} \triangleright M)$$

$$\sigma_i \mapsto \text{diagram of two strands crossing with a blue 'X'}$$

$$t \mapsto \text{diagram of a strand with a blue 't' label}$$

BRAIDED
 $\mathcal{C}$ -MODULE CATEGORY
($\mathcal{M}, \triangleright, e$)



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SUMMARY ... SO FAR

$\equiv \text{Type B}^c \equiv$

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QUASIA HOPF ALGS
... UP TO BR. MORITA EQUIV

CATEG. CONSTR'N:
DRINFELD
CENTER

CATEG. CONSTR'N:
REFLECTIVE
CENTER

HOPF CASE:
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OBJECTS

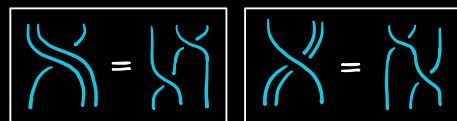
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REP'NS OF Br^A

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OBJECTS

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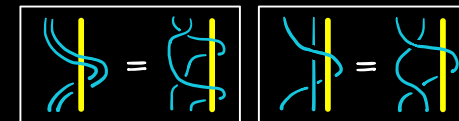
$$\sigma_i \longmapsto \begin{array}{c} \text{diagram of } \sigma_i \text{ as a braid} \\ i \quad i+1 \end{array}$$

BRAID GROUP OF TYPE B

$$\rho_n^{X,M} : Br_n^B \longrightarrow \text{Aut}_{\mathcal{M}}(X^{\otimes n} \triangleright M)$$

$$\begin{array}{l} \sigma_i \longmapsto \text{diagram of } \sigma_i \text{ as a braid} \\ t \longmapsto \text{diagram of } t \text{ as a braid} \end{array}$$

BRAIDED
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$\equiv \text{Type A} \equiv$

SUMMARY ... SO FAR

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DRINFELD
DOUBLE

REFLECTIVE
ALGS.

HOPF CASE:

QUASIA HOPF ALGS

... UP TO BR. MORITA EQUIV

CATEG. CONSTR'N:

DRINFELD
CENTER

CATEG. CONSTR'N:

REFLECTIVE
CENTER

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OBJECTS

OBJECTS

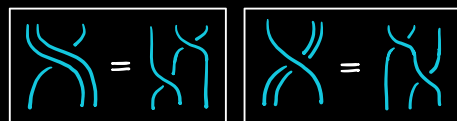
FOR GENERAL
REP'NS OF Br^A

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OBJECTS

OBJECTS

BRAIDED
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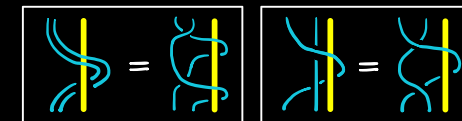
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BRAIDED
 $\mathcal{C}$ -MODULE CATEGORY
($\mathcal{M}, \triangleright, e$)



≡ TYPE A ≡

ON NONDEGENERACY

≡ TYPE B^C ≡

DRINFELD
DOUBLE

HOPF CASE:

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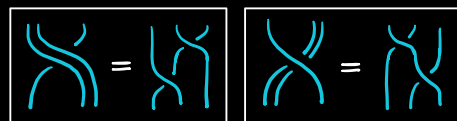
... UP TO BR. MORITA EQUIV

TOWARDS "MODULARITY"

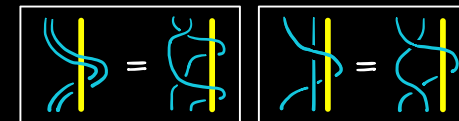
... USED IN APPLICATIONS [3D-TQFTS, TQC, ...]

≡ SEE AGUSTINA'S TALK NEXT ≡

BRAIDED
MONOIDAL CATEGORY
($\mathcal{C}, \otimes, \mathbb{1}, c$)



BRAIDED
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SHIMIZU (2019)

BUILDING ON SEVERAL WORKS
IN THE SEMISIMPLE CASE

ON NONDEGENERACY

THEOREM: GIVEN A BRAIDED MONOIDAL CAT.

$(\mathcal{C}, \otimes, \mathbb{1}, c)$ THAT IS "FINITE TENSOR"

[LIKE $H\text{-FdMod}$ FOR H F.D. QUASITRIANGULAR HOPF ALG.]

TFAE:

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TFAE:

(1)

(2)

$$(3) \mathcal{I}_2(\mathcal{C}) := \left\{ x \in \mathcal{C} : \sum_{x \otimes y}^{\otimes} = \prod_{x \otimes y}^{\otimes} \forall y \in \mathcal{C} \right\} \simeq \text{Vec}$$

$\equiv \mathcal{C} \text{ HAS TRIVIAL SYM. CENTER}$

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TFAE:

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Ex. $\mathcal{D}(H)\text{-FdMod}$, $u_q(\mathfrak{g})\text{-FdMod}$

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ON NONDEGENERACY

W-YADAV (2025)

FIX BRAIDED FINITE TENSOR CAT. $\mathcal{C}$

THEOREM: GIVEN A BRAIDED $\mathcal{C}$ -MODULE CAT.

$(\mathcal{M}, \triangleright, e)$ THAT IS "EXACT".

GET:

(1)

$\Updownarrow$

(2)

$\Downarrow$

(3)

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TFAE: FACTORIZABLE?

TFAE:

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(1) END $\int_{M \in \mathcal{M}} \underline{\text{Hom}}(M, M) \in \text{Alg}(\mathcal{C})$

$\Updownarrow$

HAS A NONDEGENERATE PAIRING

(2) $\mathcal{E}_{\mathcal{C}}(\mathcal{M}) \simeq \mathcal{M} \boxtimes \text{Func}(\mathcal{M}, \mathcal{M})$

$\Downarrow$

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$\equiv \mathcal{M}$ HAS TRIVIAL SYM. CENTER

Ex. $R_{\#}(A)\text{-FdMod}$, QUANTUM GROUP EXS?

SHIMIZU (2019)

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(3) $\mathcal{Z}_2(\mathcal{C}) := \left\{ x \in \mathcal{C} : \begin{array}{c} x \ y \\ \text{---} \\ x \ y \end{array} = \begin{array}{c} x \ y \\ \text{---} \\ x \ y \end{array} \forall y \in \mathcal{C} \right\} \simeq \text{Vec}$
 $\equiv \mathcal{C}$ HAS TRIVIAL SYM. CENTER

Ex. $\mathcal{D}(H)\text{-FdMod}$, $u_q(\mathfrak{g})\text{-FdMod}$

FIX BRAIDED FINITE TENSOR CAT. $\mathcal{C}$

THEOREM: GIVEN A BRAIDED $\mathcal{C}$ -MODULE CAT.
 $(\mathcal{M}, \triangleright, e)$ THAT IS "EXACT".

GET:

(1) END $\int_{M \in \mathcal{M}} \underline{\text{Hom}}(M, M) \in \text{Alg}(\mathcal{C})$
 $\Downarrow$ HAS A NONDEGENERATE PAIRING

(2) $\mathcal{E}_{\mathcal{C}}(\mathcal{M}) \simeq \mathcal{M} \boxtimes \text{Func}(\mathcal{M}, \mathcal{M})$
 $\Downarrow \equiv \mathcal{M}$ IS FACTORIZABLE

(3) $\mathcal{Z}_2(\mathcal{M}) := \left\{ x \in \mathcal{C} : \begin{array}{c} x \ M \\ \text{---} \\ x \ M \end{array} = \begin{array}{c} x \ M \\ \text{---} \\ x \ M \end{array} \forall M \in \mathcal{M} \right\} \simeq \text{Vec}$
 $\equiv \mathcal{M}$ HAS TRIVIAL SYM. CENTER

Ex. $R_{\#}(A)\text{-FdMod}$, QUANTUM GROUP EXS?

$\equiv \text{Type A} \equiv$

SUMMARY

$\equiv \text{Type B}^c \equiv$

DRINFELD
DOUBLE

CATEG. CONSTR'N:
DRINFELD CENTER

HOPF CASE:

QUASIA HOPF ALGS

... UP TO BR. MORITA EQUIV

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

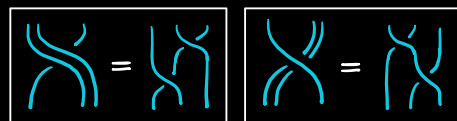
QUANTUM YANG-BAXTER EQ. (qYBE)
 $\equiv \text{WANT SOLUTIONS} \equiv$

OBJECTS

OBJECTS

FOR GENERAL
REP'NS OF Br^A

BRAIDED
MONOIDAL CATEGORY
 $(\mathcal{C}, \otimes, \mathbb{1}, c)$



BRAID GROUP OF TYPE A

$$\rho_n^X: Br_{n-1}^A \rightarrow \text{Aut}_{\mathcal{C}}(X^{\otimes n})$$

$$\sigma_i \mapsto \begin{array}{c} \text{diagram of } \sigma_i \text{ in } Br_n^A \end{array}$$

CATEG. CONSTR'N:
REFLECTIVE CENTER

REFLECTIVE
ALGS.

HOPF CASE:

QUASIA COMODULE ALGS

... UP TO BR. MORITA EQUIV

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

QUANTUM REFLECTION EQ (qRE)
 $\equiv \text{WANT SOLUTIONS} \equiv$

FOR GENERAL
REP'NS OF Br^B

OBJECTS

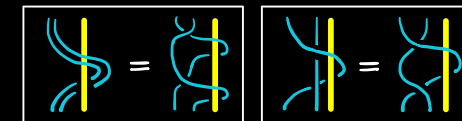
OBJECTS

BRAID GROUP OF TYPE B

$$\rho_n^{X,M}: Br_n^B \rightarrow \text{Aut}_M(X^{\otimes n} \triangleright M)$$

$$\begin{array}{l} \sigma_i \mapsto \text{diagram of } \sigma_i \text{ in } Br_n^B \\ t \mapsto \text{diagram of } t \text{ in } Br_n^B \end{array}$$

BRAIDED
 $\mathcal{C}$ -MODULE CATEGORY
 $(\mathcal{M}, \triangleright, e)$



$\equiv \text{Type A} \equiv$

SUMMARY

$\equiv \text{Type B}^c \equiv$

DRINFELD DOUBLE
 CATEG. CONSTR'N:
 DRINFELD CENTER
 HOPF CASE: $\swarrow$
 QUASIA HOPF ALGS
 ... UP TO BR. MORITA EQUIV
 NONDEGENERACY

REFLECTIVE ALGS.
 CATEG. CONSTR'N:
 REFLECTIVE CENTER
 HOPF CASE: $\swarrow$
 QUASIA COMODULE ALGS
 ... UP TO BR. MORITA EQUIV
 NONDEGENERACY

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

QUANTUM YANG-BAXTER EQ. (qYBE)
 $\equiv \text{WANT SOLUTIONS} \equiv$

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QUANTUM REFLECTION EQ (qRE)
 $\equiv \text{WANT SOLUTIONS} \equiv$

OBJECTS

OBJECTS

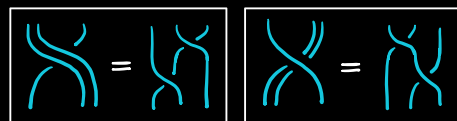
FOR GENERAL
 REP'NS OF Br^A

FOR GENERAL
 REP'NS OF Br^B

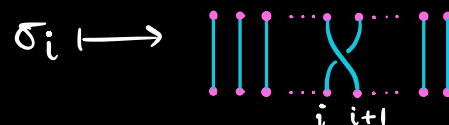
OBJECTS

OBJECTS

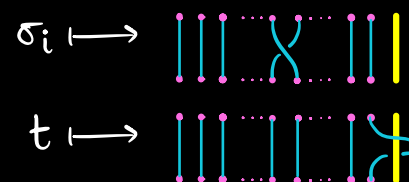
BRAIDED
 MONOIDAL CATEGORY
 $(\mathcal{C}, \otimes, \mathbb{1}, c)$



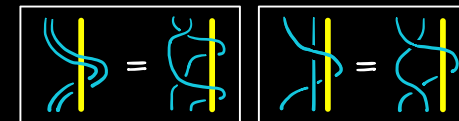
BRAID GROUP OF TYPE A
 $\rho_n^X: \text{Br}_{n-1}^A \rightarrow \text{Aut}_{\mathcal{C}}(X^{\otimes n})$



BRAID GROUP OF TYPE B
 $\rho_n^{X,M}: \text{Br}_n^B \rightarrow \text{Aut}_{\mathcal{M}}(X^{\otimes n} \triangleright M)$



BRAIDED
 $\mathcal{C}$ -MODULE CATEGORY
 $(\mathcal{M}, \triangleright, e)$



$\equiv \text{TYPE A} \equiv$

QUANTUM
YANG-BAXTER
EQ. (qYBE)

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

BRAIDED
MONOIDAL CAT
($\mathcal{C}, \otimes, \mathbb{1}, c$)

$$\rho_n^X: \text{Br}_{n-1}^A \curvearrowright \text{Aut}_e(X^{\otimes n})$$

$\sigma_i \mapsto$

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$\equiv \text{SPECULATIONS} \equiv$

• VENTURING... BOUNDARY w/ BRAIDED MOD. CATS •

$\equiv \text{TYPE B}^c \equiv$

QUANTUM
REFLECTION
EQ (qRE)

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

BRAIDED
 $\mathcal{C}$ -MODULE CAT
($\mathcal{M}, \triangleright, e$)

$$\rho_n^{X,M}: \text{Br}_n^B \curvearrowright \text{Aut}_{\mathcal{M}}(X^{\otimes n} \triangleright M)$$

$\sigma_i \mapsto$

$t \mapsto$

• INTL CT CONF., JULY 2024 •

≡ TYPE A ≡

QUANTUM
YANG-BAXTER
EQ. (qYBE)

$$R_{12} R_{13} R_{23} \\ = R_{23} R_{13} R_{12}$$

BRAIDED
MONOIDAL CAT
($\mathcal{C}, \otimes, \mathbb{1}, c$)

$$\begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} = \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array}$$

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$$\rho_n^X: Br_{n-1}^A \curvearrowright \\ Aut_e(X^{\otimes n})$$

$$\sigma_i \mapsto \begin{array}{c} \vdots \quad \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \quad \vdots \end{array} \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \begin{array}{c} \vdots \quad \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \quad \vdots \end{array}$$

$i \quad i+1$

• CHELSEA WALTON (RICE UNIV.) •

≡ SPECULATIONS ≡

THERE ARE LOADS OF COXETER TYPES ...

≡ TYPE B^c ≡

QUANTUM
REFLECTION
EQ (qRE)

$$R_{12} K_1 R_{21} K_2 \\ = K_2 R_{12} K_1 R_{21}$$

BRAIDED
 $\mathcal{C}$ -MODULE CAT
($\mathcal{M}, \triangleright, e$)

$$\begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} = \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} \diagup \quad \diagdown \\ \diagup \quad \diagdown \end{array} = \begin{array}{c} \diagdown \quad \diagup \\ \diagdown \quad \diagup \end{array}$$

$$\rho_n^{X,M}: Br_n^B \curvearrowright \\ Aut_{\mathcal{M}}(X^{\otimes n} \triangleright M)$$

$$\sigma_i \mapsto \begin{array}{c} \vdots \quad \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \quad \vdots \end{array} \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \begin{array}{c} \vdots \quad \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \quad \vdots \end{array}$$

$$t \mapsto \begin{array}{c} \vdots \quad \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \quad \vdots \end{array} \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array}$$

• INTL CT CONF., JULY 2026 •

• VENTURING... BOUNDARY w/ BRAIDED MOD. CATS •

≡ TYPE A ≡

QUANTUM
YANG-BAXTER
EQ. (qYBE)

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

BRAIDED
MONOIDAL CAT
($\mathcal{C}, \otimes, \mathbb{1}, c$)

$$\text{Crossing} = \text{Braid}$$

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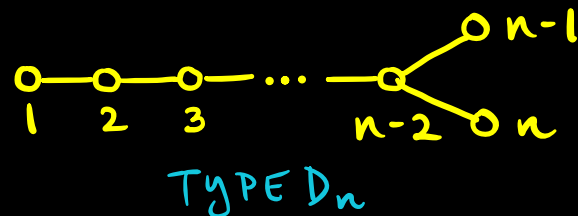
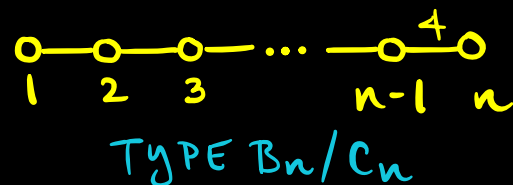
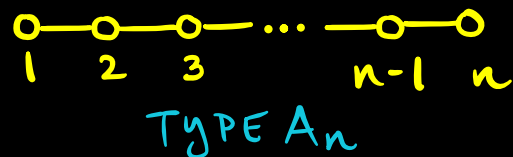
$$\rho_n^X: Br_{n-1}^A \hookrightarrow \text{Aut}_{\mathcal{C}}(X^{\otimes n})$$

$$\sigma_i \mapsto \text{Braid diagram with crossing at } i, i+1$$

≡ SPECULATIONS ≡

THERE ARE LOADS OF COXETER TYPES ...

COXETER GRAPHS



E_6 E_7 E_8 F_4 G_2

H_3 H_4 $I_2(m)$

AFFINE TYPES...

≡ TYPE B^C ≡

QUANTUM
REFLECTION
EQ (qRE)

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

BRAIDED
 $\mathcal{C}$ -MODULE CAT
($\mathcal{M}, \triangleright, e$)

$$\text{Crossing} = \text{Braid}$$

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$$\rho_n^{X,M}: Br_n^B \hookrightarrow \text{Aut}_{\mathcal{M}}(X^{\otimes n} \triangleright M)$$

$$\sigma_i \mapsto \text{Braid diagram with crossing at } i$$

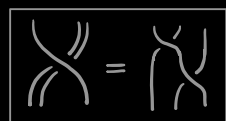
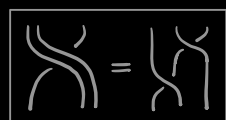
$$t \mapsto \text{Braid diagram with crossing at } t$$

≡ TYPE A ≡

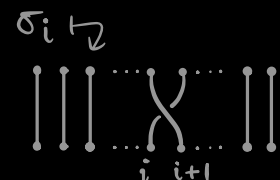
QUANTUM
YANG-BAXTER
EQ. (qYBE)

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BRAIDED
MONOIDAL CAT
($\mathcal{C}, \otimes, \mathbb{1}, c$)



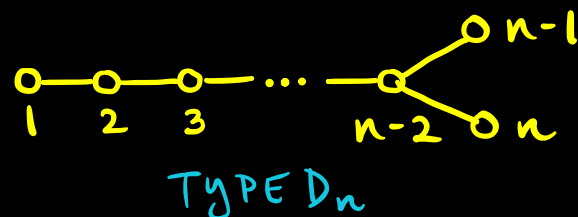
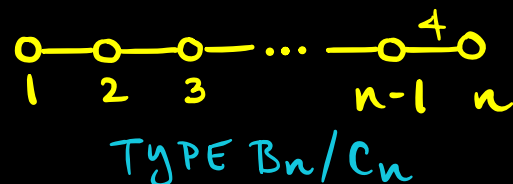
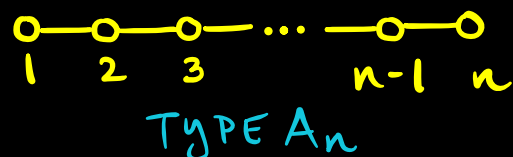
$$\rho_n^X: Br_{n-1}^A \hookrightarrow \text{Aut}_{\mathcal{C}}(X^{\otimes n})$$



≡ SPECULATIONS ≡

THERE ARE LOADS OF COXETER TYPES...

COXETER GRAPHS



$E_6 \ E_7 \ E_8 \ F_4 \ G_2$

$H_3 \ H_4 \ I_2(m)$

AFFINE TYPES...

COXETER GROUPS
IN TYPE X_n

$$Br_n^X: \text{GENS: } \theta_1, \dots, \theta_n$$

RELATIONS:

$$\theta_i \theta_j \theta_i \dots = \theta_j \theta_i \theta_j \dots$$

a_{ij} FACTORS

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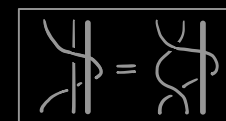
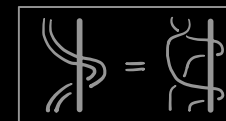
FOR (a_{ij}) COXETER
MATRIX OF TYPE X_n

≡ TYPE B^C ≡

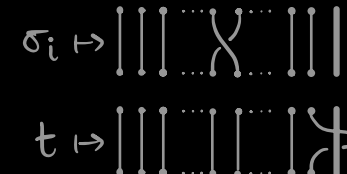
QUANTUM
REFLECTION
EQ (qRE)

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

BRAIDED
 $\mathcal{C}$ -MODULE CAT
($\mathcal{M}, \triangleright, e$)



$$\rho_n^{X,M}: Br_n^B \hookrightarrow \text{Aut}_{\mathcal{M}}(X^{\otimes n} \triangleright M)$$

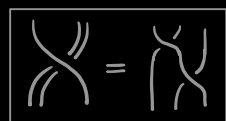
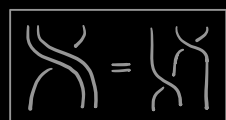


≡ TYPE A ≡

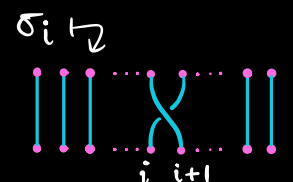
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BRAIDED
MONOIDAL CAT
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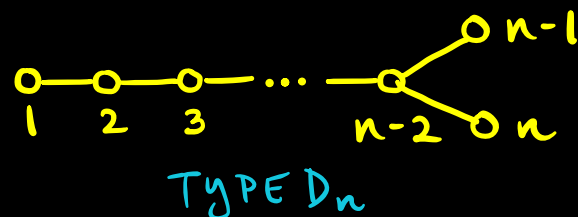
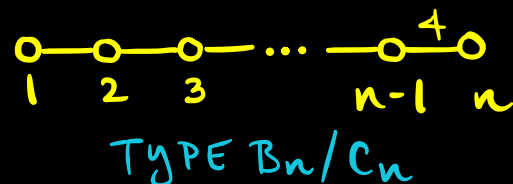
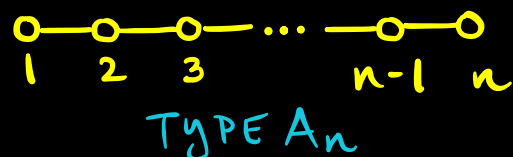
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≡ SPECULATIONS ≡

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COXETER GRAPHS



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AFFINE TYPES...

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RELATIONS:

$$\theta_i \theta_j \theta_i \dots = \theta_j \theta_i \theta_j \dots$$

a_{ij} FACTORS

a_{ij} FACTORS

FOR (a_{ij}) COXETER
MATRIX OF TYPE X_n

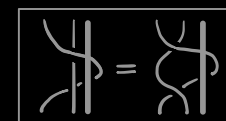
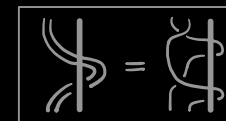
CAN WE REPRESENT
THESE GROUPS W/
STRING MODELS??

≡ TYPE B^{=C} ≡

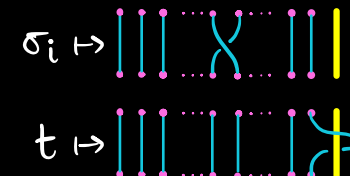
QUANTUM
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BRAIDED
 $\mathcal{C}$ -MODULE CAT
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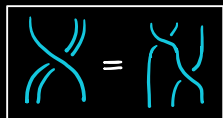
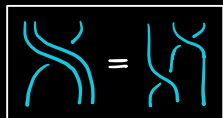


≡ TYPE A ≡

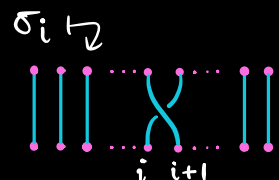
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YANG-BAXTER
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BRAIDED
MONOIDAL CAT
($\mathcal{C}, \otimes, \mathbb{1}, c$)



$\rho_n^X: Br_n^A \hookrightarrow \text{Aut}_{\mathcal{C}}(X^{\otimes n})$



≡ SPECULATIONS ≡

THERE ARE LOADS OF COXETER TYPES ...

Br_n^X : GENS: $\theta_1, \dots, \theta_n$

RELATIONS:

$$\underbrace{\theta_i \theta_j \theta_i \dots}_{a_{ij} \text{ FACTORS}} = \underbrace{\theta_j \theta_i \theta_j \dots}_{a_{ij} \text{ FACTORS}}$$

FOR (a_{ij}) COXETER
MATRIX OF TYPE X_n

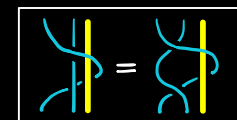
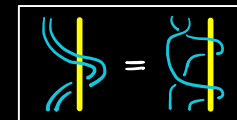
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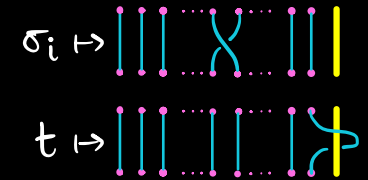
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BRAIDED
 $\mathcal{C}$ -MODULE CAT
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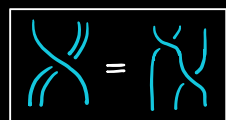
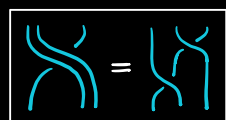


≡ TYPE A ≡

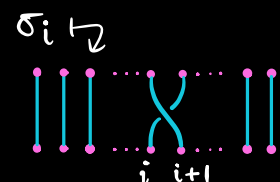
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BRAIDED
MONOIDAL CAT
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$$\rho_n^X: Br_{n-1}^A \hookrightarrow \text{Aut}_e(X^{\otimes n})$$



≡ SPECULATIONS ≡

THERE ARE LOADS OF COXETER TYPES ...

IS THERE A
BRAIDED
CATEGORICAL
STRUCTURE
"OF TYPE X"
WHOSE
OBJECTS YIELD
REPS OF Br_n^X ??

Br_n^X : GENS: $\theta_1, \dots, \theta_n$

RELATIONS:

$$\underbrace{\theta_i \theta_j \theta_i \dots}_{a_{ij} \text{ FACTORS}} = \underbrace{\theta_j \theta_i \theta_j \dots}_{a_{ij} \text{ FACTORS}}$$

FOR (a_{ij}) COXETER
MATRIX OF TYPE X_n

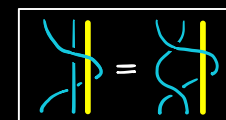
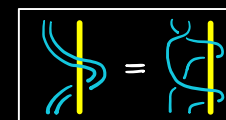
CAN WE REPRESENT
THESE GROUPS W/
STRING MODELS??

≡ TYPE B^C ≡

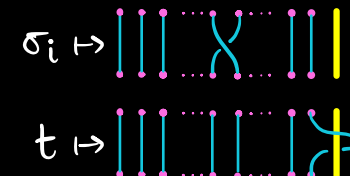
QUANTUM
REFLECTION
EQ (qRE)

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

BRAIDED
 $\mathcal{C}$ -MODULE CAT
($\mathcal{M}, \triangleright, e$)



$$\rho_n^{X,M}: Br_n^B \hookrightarrow \text{Aut}_{\mathcal{M}}(X^{\otimes n} \triangleright M)$$

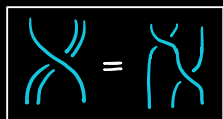
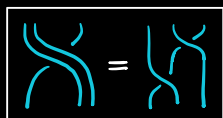


≡ TYPE A ≡

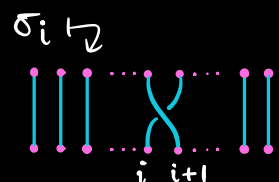
QUANTUM
YANG-BAXTER
EQ. (qYBE)

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

BRAIDED
MONOIDAL CAT
($\mathcal{C}, \otimes, \mathbb{1}, c$)



$$\rho_n^X: Br_{n-1}^A \curvearrowright Aut_e(X^{\otimes n})$$



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≡ SPECULATIONS ≡

THERE ARE LOADS OF COXETER TYPES ...

IS THERE A
BRAIDED
CATEGORICAL
STRUCTURE
"OF TYPE X"
WHOSE
OBJECTS YIELD
REPIQS OF Br_n^X ??

BRAID GROUP OF
COXETER TYPE X

CAN WE REPRESENT
THESE GROUPS W/
STRING MODELS??

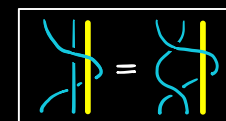
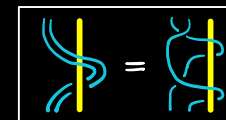
• VENTURING... BOUNDARY w/ BRAIDED MOD. CATS •

≡ TYPE B^{=C} ≡

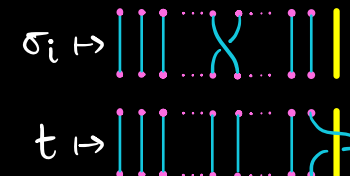
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BRAIDED
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$$\rho_n^{X,M}: Br_n^B \curvearrowright Aut_{\mathcal{M}}(X^{\otimes n} \triangleright M)$$



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$\equiv \text{TYPE A} \equiv$

QUANTUM
YANG-BAXTER
EQ. (qYBE)

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

BRAIDED
MONOIDAL CAT
($\mathcal{C}, \otimes, \mathbb{1}, c$)

$$\text{Diagram 1} = \text{Diagram 2}$$

$$\text{Diagram 3} = \text{Diagram 4}$$

$$\rho_n^X: Br_{n-1}^A \curvearrowright \text{Aut}_e(X^{\otimes n})$$

$$\sigma_i \mapsto \text{Diagram with crossing at } i, i+1$$

$\equiv \text{SPECULATIONS} \equiv$

CAN WE USE OBJECTS IN
BRAIDED CAT. STRUCTURES
"OF TYPE X" TO MODEL
PARTICLE SCATTERING
IN PHYSICAL SYSTEMS
W/ OBSTRUCTIONS??

IS THERE A
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CATEGORICAL
STRUCTURE
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WHOSE
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BRAID GROUP OF
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CAN WE REPRESENT
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$\equiv \text{TYPE B}^c \equiv$

QUANTUM
REFLECTION
EQ (qRE)

$$R_{12} K_1 R_{21} K_2 = K_2 R_{12} K_1 R_{21}$$

BRAIDED
 $\mathcal{E}$ -MODULE CAT
($\mathcal{M}, \triangleright, e$)

$$\text{Diagram 5} = \text{Diagram 6}$$

$$\text{Diagram 7} = \text{Diagram 8}$$

$$\rho_n^{X,M}: Br_n^B \curvearrowright \text{Aut}_m(X^{\otimes n} \triangleright M)$$

$$\sigma_i \mapsto \text{Diagram with crossing}, t \mapsto \text{Diagram with twist}$$

$\equiv \text{TYPE A} \equiv$

QUANTUM
YANG-BAXTER
EQ. (qYBE)

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

BRAIDED
MONOIDAL CAT
($\mathcal{C}, \otimes, \mathbb{1}, c$)

$$\text{Diagram 1} = \text{Diagram 2}$$

$$\text{Diagram 3} = \text{Diagram 4}$$

$$\rho_n^X: Br_{n-1}^A \curvearrowright Aut_e(X^{\otimes n})$$

$$\sigma_i \mapsto \text{Diagram 5}$$

AND IS THERE 1 EQUATION TO RULE THEM ALL??



IS THERE A
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CAN WE USE OBJECTS IN
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BRAIDED
 $\mathcal{E}$ -MODULE CAT
($\mathcal{M}, \triangleright, e$)

$$\text{Diagram 6} = \text{Diagram 7}$$

$$\text{Diagram 8} = \text{Diagram 9}$$

$$\rho_n^{X,M}: Br_n^B \curvearrowright Aut_{\mathcal{M}}(X^{\otimes n} \triangleright M)$$

$$\sigma_i \mapsto \text{Diagram 10}$$

$$t \mapsto \text{Diagram 11}$$

≡ TYPE A ≡

QUANTUM
YANG-BAXTER
EQ. (qYBE)

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$$

BRAIDED
MONOIDAL CAT
($\mathcal{C}, \otimes, \mathbb{1}, c$)

$$\text{Crossing} = \text{Braid}$$

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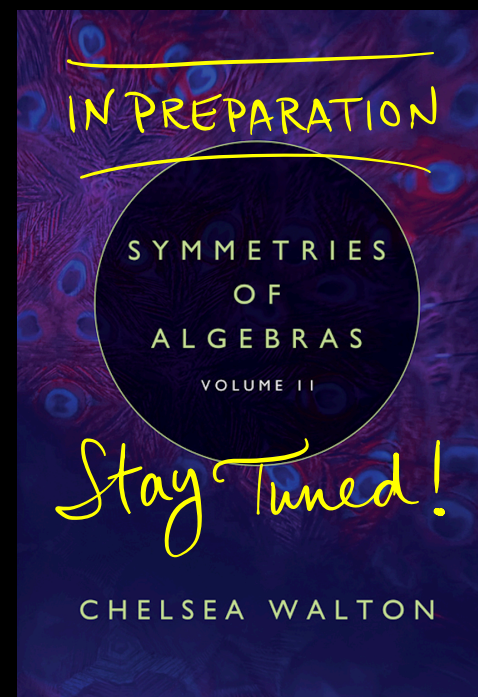
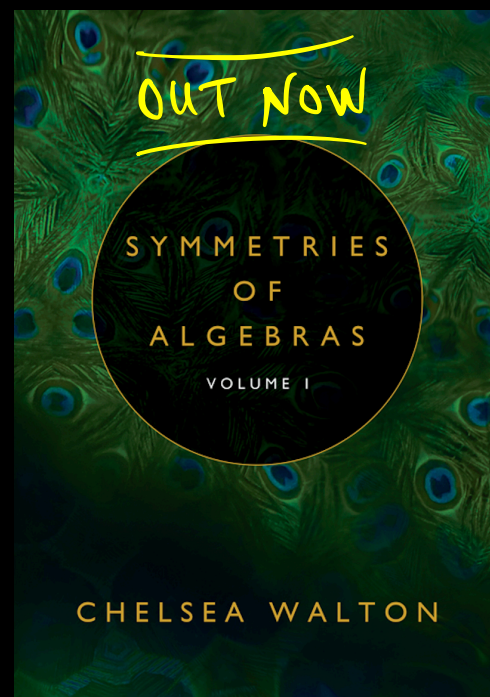
$$\rho_n^X : Br_{n-1}^A \hookrightarrow \text{Aut}_{\mathcal{C}}(X^{\otimes n})$$

$$\sigma_i \mapsto \text{Diagram with crossing at } i \text{ and } i+1$$

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Thanks for listening!!

NOTLAW @ RICE.EDU



• VENTURING... BOUNDARY w/ BRAIDED MOD. CATS •

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$$\rho_n^{X,M} : Br_n^B \hookrightarrow \text{Aut}_{\mathcal{M}}(X^{\otimes n} \triangleright M)$$

$$\sigma_i \mapsto \text{Diagram with crossing at } i$$

$$t \mapsto \text{Diagram with crossing at } t$$

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