# Math 211

Lecture #1

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# Welcome to Math 211

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# Ordinary Differential Equations with Linear Algebra

- Applications & modeling.
  - Mechanics, electric circuits, population genetics epidemiology, pollution, pharmacology, personal finance, etc.
- Analytic solutions.
- Numerical solutions.
- Qualitative analysis.
  - Properties of solutions without knowing what they are.

# Math 211 Web Page

Official source of information about the course.

http://www.owlnet.rice.edu/~math211/ .

# What Is a Derivative?

- The rate of change of a function.
- The slope of the tangent line to the graph of a function.
- The best linear approximation to the function.
- The limit of difference quotients.
- Rules and tables that allow computation.

# What Is an Integral?

- The area under the graph of a function.
- An anti-derivative.
- Rules and tables for computing.

# Differential Equations

$$y' = f(t, y) \qquad y' = 2ty$$

- $t$ is the *independent variable*.
- $y$ is the *unknown function*.
- This equation is of order 1.

# Equations and Solutions

$$y' = f(t, y) \qquad y' = 2ty$$

A solution is a function y(t), defined for t in an interval, which is differentiable at each point and satisfies

$$y'(t) = f(t, y(t))$$

for every point $t$ in the interval.

Example $y'(t) = 2ty(t)$.

# Example: $y' = 2ty$

Claim: $y(t) = e^{t^2}$ is a solution.

Verify by substitution.

Left hand side: $y'(t) = 2te^{t^2}$

Right hand side: $2ty(t) = 2te^{t^2}$

Therefore $y' = 2ty$.

# Types of Solutions

For the equation $y' = 2ty$

$y(t) = \frac{1}{2}e^{t^2}$ is a solution. It is a *particular solution.*

$y(t) = Ce^{t^2}$ is a solutionfor any constant $C$.

This is a *general solution.*

General solutions contain arbitrary constants.

Particular solutions do not.

# Initial Value Problem

Consists of a differential equation and an initial condition.

E.g., $y' = -2ty \quad$ and $\quad y(0) = 4$.

General solution: $\quad y(t) = Ce^{-t^2}$.

Initial condition:

$$y(0) = 4,$$

$$Ce^0 = 4,$$

$$C = 4$$

Solution to the IVP: $\quad y(t) = 4e^{-t^2}$.

# Normal Form of an Equation

$$y' = f(t, y)$$

Example:

$$(1 + t^2)y' + y^2 = t^3$$

Solve for $y'$ to put into normal form:

$$y' = \frac{t^3 - y^2}{1 + t^2}$$

# Interval of Existence

The largest interval over which a solution can exist.

Example: $y' = 1 + y^2 \quad$ with $\quad y(0) = 1$

General solution: $y(t) = \tan(t + C)$

Initial Condition: $y(0) = 1 \Leftrightarrow C = \pi/4$.

Solution: $y(t) = \tan(t + \pi/4)$

$y(t)$ exists and is continuous for

$-\pi/2 < t + \pi/4 < \pi/2$

or for $-3\pi/4 < t < \pi/4$.

# Geometric Interpretation of $y' = f(t, y)$

If $y(t)$ is a solution, and $y(t_0) = y_0$, then

$$y'(t_0) = f(t_0, y(t_0)) = f(t_0, y_0).$$

The slope to the graph of $y(t)$ at the point $(t_0, y_0)$ is given by $f(t_0, y_0)$.
Imagine a small line segment attached to each point of the $(t, y)$ plane with the slope $f(t, y)$.

# The Direction Field