Math 211

Lecture #1

Introduction

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Welcome to Math 211

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Ordinary Differential Equations with Linear Algebra

There are four themes to the course:

- Applications & modeling.
  - Mechanics, electric circuits, population genetics epidemiology, pollution, pharmacology, personal finance, etc.
- Analytic solutions.
  - Solutions which are given by an explicit formula.

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- Numerical solutions.
  - Approximate solutions computed at a discrete set of points.
- Qualitative analysis.
  - Properties of solutions without knowing a formula for the solution.

Math 211 Web Pages

- Official source of information about the course. http://www.owlnet.rice.edu/˜math211/ .
- Source for the slides for section 3. http://math.rice.edu/˜polking/slidesf01.html .

What Is a Derivative?

- The rate of change of a function.
- The slope of the tangent line to the graph of a function.
- The best linear approximation to the function.
- The limit of difference quotients.
- Rules and tables that allow computation.

What Is an Integral?

- The area under the graph of a function.
- An anti-derivative.
- Rules and tables for computing.

Differential Equations

An equation involving an unknown function and one or more of its derivatives, in addition to the independent variable.

- Example: $y' = 2ty$
- General equation: $y' = f(t, y)$
- t is the *independent variable*.
- $y = y(t)$ is the *unknown function*.
- $y' = 2ty$ is of *order 1*.

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Equations and Solutions

$$y' = f(t, y) \qquad y' = 2ty$$

A *solution* is a function $y(t)$, defined for t in an interval, which is differentiable at each point and satisfies

$$y'(t) = f(t, y(t))$$

for every point t in the interval.

- What is a function?
- An ODE is a function generator.

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Example: $y' = 2ty$

Claim: $y(t) = e^{t^2}$ is a solution.

- Verify by substitution.
  - Left-hand side: $y'(t) = 2te^{t^2}$
  - Right-hand side: $2ty(t) = 2te^{t^2}$
- Therefore $y'(t) = 2ty(t)$, if $y(t) = e^{t^2}$.
- Verification by substitution is always available.

Is $y(t) = e^t$ a solution to the equation $y' = 2ty$?

- Check by substitution.
  - Left-hand side: $y'(t) = e^t$
  - Right-hand side: $2ty(t) = 2te^t$
- Therefore $y'(t) \neq 2ty(t)$, if $y(t) = e^t$.
- $y(t) = e^t$ is *not* a solution to the equation $y' = 2ty$.

Types of Solutions

For the equation $y' = 2ty$

- $y(t) = \frac{1}{2}e^{t^2}$ is a solution. It is a *particular solution*.
- $y(t) = Ce^{t^2}$ is a solution for any constant C. This is a *general solution*.

General solutions contain arbitrary constants. Particular solutions do not.

Initial Value Problem (IVP)

A differential equation & an initial condition.

- Example: $y' = -2ty$ with $y(0) = 4$.
- General solution: $y(t) = Ce^{-t^2}$.
- Plug in the initial condition:

$$\begin{aligned} y(0) &= 4, \\ Ce^0 &= 4, \\ C &= 4 \end{aligned}$$

Solution to the IVP: $y(t) = 4e^{-t^2}$.

Normal Form of an Equation

$$y' = f(t, y)$$

Example: $(1 + t^2)y' + y^2 = t^3$

- This equation is not in normal form.
- Solve for y' to put into normal form:

$$y' = \frac{t^3 - y^2}{1 + t^2}$$