Math 211

The Predator-Prey System

Predator-Prey System

Lotka-Volterra system with prey x and predator y

$$x' = (a - by)x$$

$$y' = (-c + dx)y$$

- Equilibrium points: $(0,0)$ is a saddle, and $(x_0, y_0) = (c/d, a/b)$ is a linear center.
- The axes are invariant.
- The positive quadrant is invariant.
- Are the solution curves really closed?

Solutions are Periodic

Along the solution curve $y = y(x)$ we have

$$\frac{dy}{dx} = \frac{y(-c + dx)}{x(a - by)}.$$

- The solution is

$$H(x, y) = by - a \ln y + dx - c \ln x = C$$

- This is an implicit equation for the solution curve. $\Rightarrow$ All solution curves are closed, and represent periodic solutions.

Why Fishing Leads to More Fish

Compute the average of the fish & shark populations over the period T of the solution.

$$\frac{d}{dt}\ln x(t) = \frac{x'}{x} = a - by$$

$$0 = \frac{1}{T}\int_0^T \frac{d}{dt}\ln x(t)\,dt = a - b\overline{y}.$$

- So $\overline{y} = a/b = y_0$.
- Similarly $\overline{x} = x_0 = c/d$.

- If fishing extracts a fixed percentage of the fish and sharks, and does not distinquish we get the system

$$x' = (a - by)x - ex$$

$$y' = (-c + dx)y - ey$$

- This is the same system with a replaced by $a - e$ and c replaced by $c + e$.
- The average populations are

$$\overline{x} = \frac{c + e}{d} \quad \text{and} \quad \overline{y} = \frac{a - e}{b}$$

- Fishing causes the average fish population to increase and the average shark population to decrease.

Cottony Cushion Scale Insect & the Ladybird Beetle

- Insect accidentally introduced from Australia in 1868.
  - It threatened the citrus industry.
- Ladybird beetle imported from Australia
  - Natural predator – reduced the insects to manageable low.
- DDT kills the scale insect.
  - Massive spraying ordered, despite warnings.
  - The insect increased in numbers, as predicted by Volterra.