

Math 211

Lecture #14

MATLAB's ODE Solvers

September 27, 2002

Solvers

MATLAB has several solvers.

- ode45
 - ♦ This is the first choice.
- ode23
- Stiff solvers
 - ♦ ode15s

Return

ode45

- Uses variable step size.
 - ♦ Specify error tolerance instead of step size.
 - ♦ MATLAB chooses the step size at each step to achieve the limit on the error.
 - ♦ The default tolerance is good enough for this course.

- Syntax

```
[t,y] = ode45(derfile,[t0,tf],y0);  
plot(t,y)
```

Return

Solvers

Solving Systems

- Example:

$$x' = v$$

$$v' = -9.8 - 0.04v|v|$$

- Change to vector notation. (Use MATLAB vector notation)
 - ♦ $u(1) = x$
 - ♦ $u(2) = v$

Return

Derivative m-file ball.m

```
function upr = ball(t,u)

x = u(1);
v = u(2);
xpr = v;
vpr = -9.8 - 0.04*v*abs(v);
upr = [xpr; vpr];
```

Return

System

Derivative m-file ballshort.m

```
function upr = ballshort(t,u)

upr = zeros(2,1);
upr(1) = u(2);
upr(2) = -9.8 - 0.04*u(2)*abs(u(2));
```

Return

System

ball.m

Computing and Plotting Solutions to Systems

- `[t,u] = ode45('ball', [0,3], [0;50]);`
- `plot(t,u)` – plots all of the components versus t .
- `plot(t,u(:,1))` – first component versus t .
- `plot(u(:,1),u(:,2))` – second component versus the first. This is a *phase plane plot*.
- `plot3(u(:,1),u(:,2),t)` – 3-D plot.

Return

Solving Higher Order Equations

- Reduce to a first order system and solve the system.
- Example: The motion of a pendulum is modeled by

$$\theta'' = -\frac{g}{L} \sin \theta - D\theta'.$$

- Introduce $\omega = \theta'$. Notice

$$\omega' = -\frac{g}{L} \sin \theta - D\omega.$$

Return

Equivalent First Order System

$$\begin{aligned} \theta' &= \omega \\ \omega' &= -\frac{g}{L} \sin \theta - D\omega \end{aligned}$$

- Change to vector notation. (Use MATLAB vector notation)
 - $u(1) = \theta$
 - $u(2) = \omega$

Return

Higher order

Derivative m-file pend.m

```
function upr = pend(t,u)

L= 1;
global D
th = u(1);
om = u(2);
thpr = om;
ompr = -(9.8/L)*sin(th) - D*om;
upr = [thpr; ompr];
```

Pendulum

Short