

Math 211

Review for the
First Exam

September 28, 2003

Themes of the Course

- Modeling.
 - ♦ One dimensional motion, mixing problems, population growth, and personal finance.
- Exact solution methods.
 - ♦ Substitution, separable equations, and linear equations.
- Numerical solution methods.
 - ♦ Euler's method, Runge-Kutta methods, dfield6.
- Qualitative analysis.
 - ♦ Existence and uniqueness, continuity in initial conditions, sensitivity to initial conditions, and autonomous equations.

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Separable Equations

$$\frac{dy}{dt} = g(y)h(t)$$

The three step solution process:

1. Separate the variables. $\frac{dy}{g(y)} = h(t) dt$ if $g(y) \neq 0$.
2. Integrate both sides. $\int \frac{dy}{g(y)} = \int h(t) dt$
3. Solve for $y(t)$.

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Linear Equations — Variation of Parameters

To solve $x' = a(t)x + f(t)$:

1. Solve the homogeneous equation $x'_0 = ax_0$.
2. Find v such that $x = vx_0$ is a solution by substituting into the equation.
3. Write down the general solution, $x(t) = v(t)x_0(t)$.

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[Integrating Factor](#)

Linear Equations — Integrating Factor

To solve $x' = a(t)x + f(t)$:

1. Rewrite as $x' - ax = f$.
2. Multiply by the integrating factor

$$u(t) = e^{-\int a(t) dt}.$$

Equation becomes $[ux] = ux' - aux = uf$.

3. Integrate: $u(t)x(t) = \int u(t)f(t) dt + C$.
4. Solve for $x(t)$.

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[Variation of Parameters](#)

Existence Theorem

Theorem: Suppose the function $f(t, y)$ is defined and continuous in the rectangle R in the ty -plane. Then given any point $(t_0, y_0) \in R$, the initial value problem

$$y' = f(t, y) \quad \text{with} \quad y(t_0) = y_0$$

has a solution $y(t)$ defined in an interval containing t_0 .

Furthermore the solution will be defined at least until the solution curve $t \rightarrow (t, y(t))$ leaves the rectangle R .

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[Uniqueness](#)

Uniqueness Theorem

Theorem: Suppose the function $f(t, y)$ and its partial derivative $\partial f / \partial y$ are continuous in the rectangle R in the ty -plane. Suppose that $(t_0, x_0) \in R$. Suppose that

$$x' = f(t, x) \quad \text{and} \quad y' = f(t, y),$$

and that

$$x(t_0) = y(t_0) = x_0.$$

Then as long as $(t, x(t))$ and $(t, y(t))$ stay in R we have

$$x(t) = y(t).$$

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[Existence](#)

Autonomous Equation $y' = f(y)$

1. Graph $y \rightarrow f(y)$.
2. Find the equilibrium points where $f(y) = 0$.
3. Determine the behavior between eq. pts.
4. Analyze the equilibrium points.
5. Transfer the phase line to ty -space.
6. Plot the equilibrium solutions.
7. Plot other solutions approximately.

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