Math 211

Lecture #42

Immunity to Infectious Deseases

December 5, 2003

Model of the Immune System

- How does the immune system develope immunity to virus caused diseases?
  - Such as flu, the cold, mumps, ...
- *Infectious Diseases of Humans* - Roy M. Anderson & Robert M. May, Oxford University Press 1992
- The model includes the interactions between virus cells and lymphocytes generated by the immune system.
  - $V(t) =$ number of virus cells
  - Two types of lymphocytes, $E_1(t)$ & $E_2(t)$.

Return

The Lymphocytes

- Both types of lymphocytes:
  - are recruited from bone marrow at a constant rate
  - die at a rate proportional to their numbers
  - proliferate due to contact with each other
- The model with no virus present is:

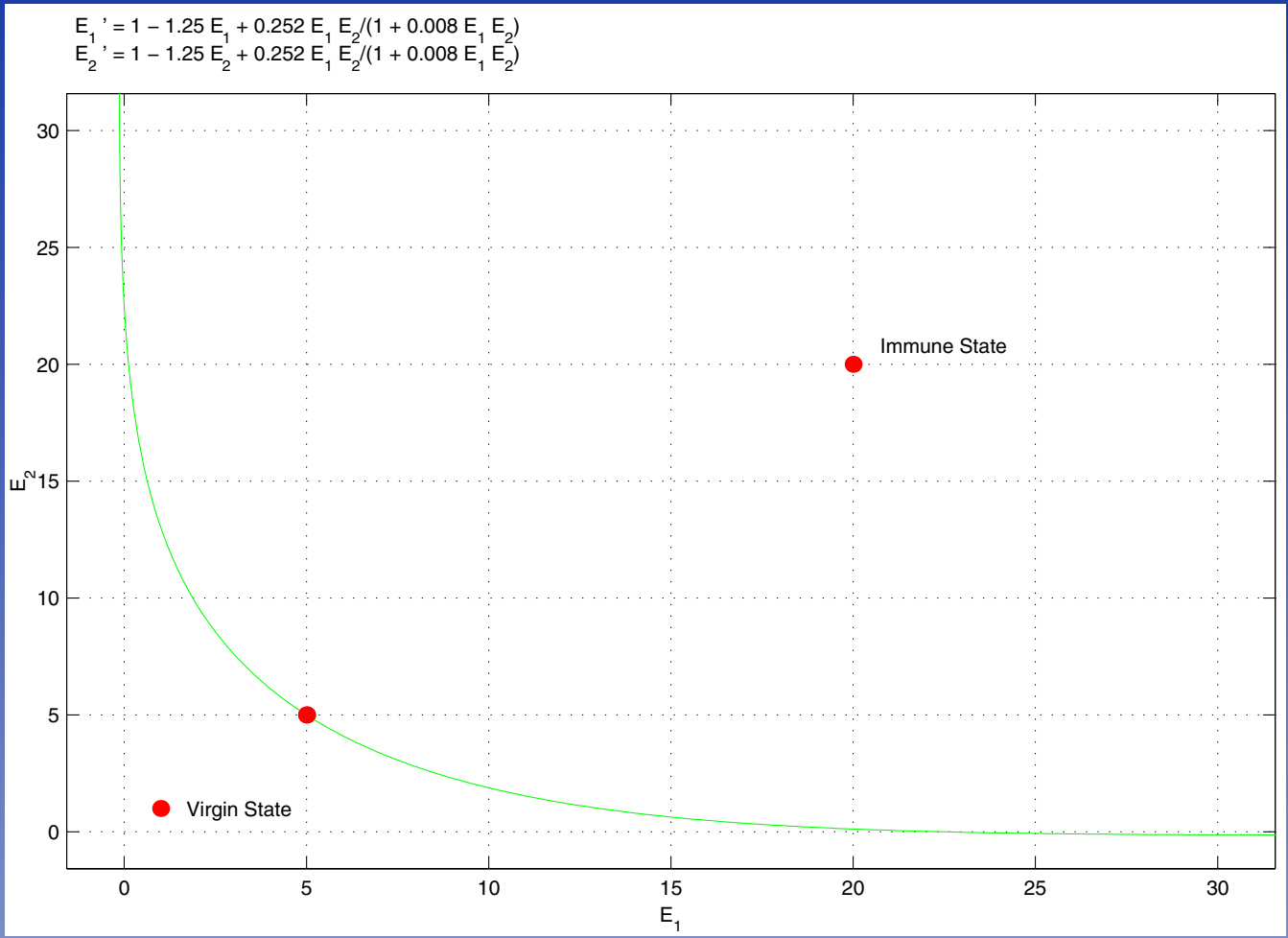
$$E_1' = \Lambda_1 - \mu_1 E_1 + a_1 \frac{E_1 E_2}{1 + b_1 E_1 E_2}$$

$$E_2' = \Lambda_2 - \mu_2 E_2 + a_2 \frac{E_1 E_2}{1 + b_2 E_1 E_2}$$

  - In `pplane6` use parameters $\Lambda_1 = \Lambda_1 = 1$, $\mu_1 = \mu_1 = 1.25$, $a_1 = a_2 = 0.252$, and $b_1 = b_2 = 0.008$.

Return

The Dynamics of the Lymphocytes



Return

Interactions with the Virus

- Virus cells have an intrinsic growth rate r.
- Lymphocytes of type E_1:
  - kill virus because of contacts with them
  - proliferate because of contacts with virus
- Lymphocytes of type E_2:
  - do not directly interact with the virus
  - regulate cells of type E_1

Return

The Model With the Virus Present

$$E_1' = \Lambda_1 - \mu_1 E_1 + a_1 \frac{E_1 E_2}{1 + b_1 E_1 E_2} + KV E_1$$

$$E_2' = \Lambda_2 - \mu_2 E_2 + a_2 \frac{E_1 E_2}{1 + b_2 E_1 E_2}$$

$$V' = rV - kV E_1$$

Analysis of the System

- There are three realistic equilibrium points

$$\begin{pmatrix} E_1 \\ E_2 \\ V \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 5 \\ 5 \\ 0 \end{pmatrix}, \quad \& \quad \begin{pmatrix} 20 \\ 20 \\ 0 \end{pmatrix}$$

- The first two are unstable. The third is asymptotically stable.
- What is the global behavior?
  - The best we can do is to compute with `ode45`. (Use $K = 0.5$, $k = 0.01$ and $r = 0.1$.)