

Math 211

Lecture #1

Introduction

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Welcome to Math 211

Math 211 Section 1 – John C. Polking

Herman Brown 402

713-348-4841

polking@rice.edu

Office Hours: 1:30 – 2:30 TWTh
and by appointment.

Ordinary Differential Equations with Linear Algebra

- Applications & modeling.
 - ◇ Mechanics, electric circuits, population genetics epidemiology, pollution, pharmacology, personal finance, etc.
- Analytic solutions.
- Numerical solutions.
- Qualitative analysis.
 - ◇ Properties of solutions without knowing what they are.

Math 211 Web Page

Official source of information about the course.

<http://www.owlnet.rice.edu/~math211/> .

What Is a Derivative?

- The rate of change of a function.
- The slope of the tangent line to the graph of a function.
- The best linear approximation to the function.
- The limit of difference quotients.
- Rules and tables that allow computation.

What Is an Integral?

- The area under the graph of a function.
- An anti-derivative.
- Rules and tables for computing.

Differential Equations

$$y' = f(t, y) \quad y' = 2ty$$

- t is the *independent variable*.
- y is the *unknown function*.
- This equation is of order 1.

Equations and Solutions

$$y' = f(t, y) \quad y' = 2ty$$

A solution is a function $y(t)$, defined for t in an interval, which is differentiable at each point and satisfies

$$y'(t) = f(t, y(t))$$

for every point t in the interval.

- Example $y'(t) = 2ty(t)$.

Example: $y' = 2ty$

Claim: $y(t) = e^{t^2}$ is a solution.

- Verify by substitution.
 - ◇ Left hand side: $y'(t) = 2te^{t^2}$
 - ◇ Right hand side: $2ty(t) = 2te^{t^2}$
- Therefore $y' = 2ty$.
- Verification by substitution is always available.

Types of Solutions

For the equation $y' = 2ty$

- $y(t) = \frac{1}{2}e^{t^2}$ is a solution. It is a *particular solution*.
- $y(t) = Ce^{t^2}$ is a solution for any constant C .
This is a *general solution*.

General solutions contain arbitrary constants.

Particular solutions do not.

Initial Value Problem

A differential equation & an initial condition.

- Example: $y' = -2ty$ and $y(0) = 4$.
- General solution: $y(t) = Ce^{-t^2}$.
- Plug in the initial condition:

$$y(0) = 4,$$

$$Ce^0 = 4,$$

$$C = 4$$

Solution to the IVP: $y(t) = 4e^{-t^2}$.

Normal Form of an Equation

$$y' = f(t, y)$$

Example: $(1 + t^2)y' + y^2 = t^3$

- Solve for y' to put into normal form:

$$y' = \frac{t^3 - y^2}{1 + t^2}$$