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Math 211

Lecture #3

Models of Motion

January 22, 2001

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Interval of Existence

The largest interval over which a solution can exist.

- Example: $y' = 1 + y^2$ with $y(0) = 1$
  - ◇ General solution: $y(t) = \tan(t + C)$
  - ◇ Initial Condition: $y(0) = 1 \Leftrightarrow C = \pi/4$.
- Solution: $y(t) = \tan(t + \pi/4)$ exists and is continuous for $-\pi/2 < t + \pi/4 < \pi/2$ or for $-3\pi/4 < t < \pi/4$.

3

Geometric Interpretation of

$$y' = f(t, y)$$

If $y(t)$ is a solution, and $y(t_0) = y_0$, then

$$y'(t_0) = f(t_0, y(t_0)) = f(t_0, y_0).$$

- The slope to the graph of $y(t)$ at the point (t_0, y_0) is given by $f(t_0, y_0)$.
- Imagine a small line segment attached to each point of the (t, y) plane with the slope $f(t, y)$.

Return

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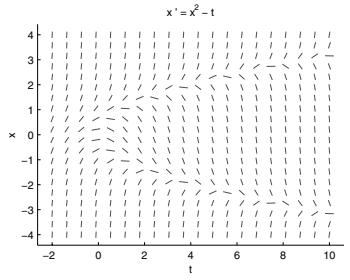
The Direction Field



5

Examples

- $R' = \dfrac{\sin t}{1+R}$ with $R(0) = 1, -2, -1$
- $x' = \dfrac{3t^2 x}{1+2x^2}$ with $x(0) = 1, 0$

Geometric descritption

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Models of Motion

History of models of planetary motion

- Babylonians - 3000 years ago
  - ◇ Initiated the systematic study of astronomy.

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Greeks

- Descriptive model
  - ◇ Geocentric model
  - ◇ Epicycles
- Enabled predictions
- No causal explanation

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Nicholas Copernicus (1543)

- Shifted the center of the universe to the sun.
- Less epicycles required.
- Still descriptive and not causal.
- Major change in human understanding of their place in the universe.

Return Greeks

9

Johann Kepler (1609)

- Based on experimental work of Tycho Brahe.
- Ellipses instead of epicycles.
  - ◇ Sun at a focus of the ellipse.
- Three laws of planetary motion.
- Still descriptive and not causal.

Return Greeks Copernicus

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Isaac Newton

- Three major contributions.
  - ◇ Fundamental theorem of calculus.
    - ⋆ Invention of calculus.
  - ◇ Laws of mechanics.
    - ⋆ Second law — $F = ma$.
  - ◇ Universal law of gravity.
  - ◇ *Principia Mathematica* 1687

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Isaac Newton

- Laws of mechanics and gravitation were based on his own experiments and his understanding of the experiments of others.
- Derived Kepler's three laws of planetary motion.
- Causal explanation.
  - ◇ For any mechanical motion.

Return Greeks Copernicus Kepler Newton 1

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Isaac Newton

- *The Life of Isaac Newton* by Richard Westfall, Cambridge University Press 1993.
- Problems
  - ◇ Force of gravity was action at a distance.
  - ◇ Physical anomalies.

Return

Albert Einstein

- Special theory of relativity – 1905.
- General theory of relativity – 1916.
  - ◇ Gravity is due to curvature of space-time.
  - ◇ Curvature is caused by mass.
  - ◇ Explains action at a distance.
- All known anomalies explained.

Return Newton Problems

Unified Theories

- Four fundamental forces.
  - ◇ Gravity, electromagnetism, strong nuclear, and weak nuclear.
- Last three unified by quantum mechanics.
  - ◇ Quantum chromodynamics.
- Attempts to include gravity.
  - ◇ String theory.

Return

Unified Theories

- String theory.
  - ◇ *The elegant universe : superstrings, hidden dimensions, and the quest for the ultimate theory* by Brian Greene, W.W.Norton, New York 1999.

Unified theories

The Modeling Process

- It is based on experiment and/or observation.
- It is iterative.
  - ◇ For motion we have ≥ 6 iterations.
  - ◇ After each change in the model it must be checked by experimentation and observation.
- It is rare that a model captures all aspects of the phenomenon.