

Math 211

Lecture #3

Models of Motion

January 22, 2001

Interval of Existence

The largest interval over which a solution can exist.

- Example: $y' = 1 + y^2$ with $y(0) = 1$
 - ◇ General solution: $y(t) = \tan(t + C)$
 - ◇ Initial Condition: $y(0) = 1 \Leftrightarrow C = \pi/4$.
- Solution: $y(t) = \tan(t + \pi/4)$ exists and is continuous for $-\pi/2 < t + \pi/4 < \pi/2$ or for $-3\pi/4 < t < \pi/4$.

Geometric Interpretation of

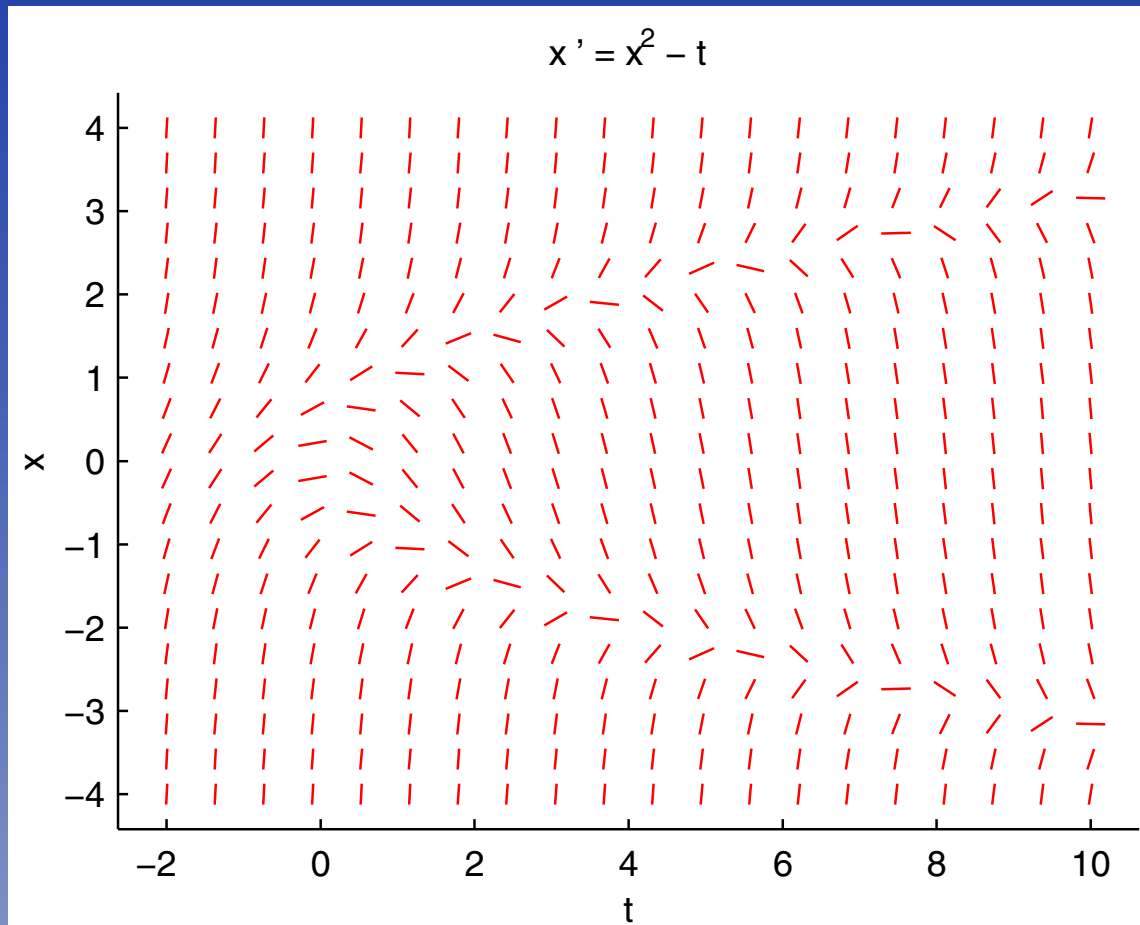
$$y' = f(t, y)$$

If $y(t)$ is a solution, and $y(t_0) = y_0$, then

$$y'(t_0) = f(t_0, y(t_0)) = f(t_0, y_0).$$

- The slope to the graph of $y(t)$ at the point (t_0, y_0) is given by $f(t_0, y_0)$.
- Imagine a small line segment attached to each point of the (t, y) plane with the slope $f(t, y)$.

The Direction Field



Examples

- $R' = \frac{\sin t}{1 + R}$ with $R(0) = 1, -2, -1$
- $x' = \frac{3t^2 x}{1 + 2x^2}$ with $x(0) = 1, 0$

Models of Motion

History of models of planetary motion

- Babylonians - 3000 years ago
 - ◇ Initiated the systematic study of astronomy.

Greeks

- Descriptive model
 - ◇ Geocentric model
 - ◇ Epicycles
- Enabled predictions
- No causal explanation

Nicholas Copernicus (1543)

- Shifted the center of the universe to the sun.
- Less epicycles required.
- Still descriptive and not causal.
- Major change in human understanding of their place in the universe.

Johann Kepler (1609)

- Based on experimental work of Tycho Brahe.
- Ellipses instead of epicycles.
 - ◇ Sun at a focus of the ellipse.
- Three laws of planetary motion.
- Still descriptive and not causal.

Isaac Newton

- Three major contributions.
 - ◇ Fundamental theorem of calculus.
 - ★ Invention of calculus.
 - ◇ Laws of mechanics.
 - ★ Second law — $F = ma$.
 - ◇ Universal law of gravity.
 - ◇ *Principia Mathematica* 1687

Isaac Newton

- Laws of mechanics and gravitation were based on his own experiments and his understanding of the experiments of others.
- Derived Kepler's three laws of planetary motion.
- Causal explanation.
 - ◇ For any mechanical motion.

Isaac Newton

- *The Life of Isaac Newton* by Richard Westfall, Cambridge University Press 1993.
- Problems
 - ◇ Force of gravity was action at a distance.
 - ◇ Physical anomalies.

Albert Einstein

- Special theory of relativity – 1905.
- General theory of relativity – 1916.
 - ◇ Gravity is due to curvature of space-time.
 - ◇ Curvature is caused by mass.
 - ◇ Explains action at a distance.
- All known anomalies explained.

Unified Theories

- Four fundamental forces.
 - ◇ Gravity, electromagnetism, strong nuclear, and weak nuclear.
- Last three unified by quantum mechanics.
 - ◇ Quantum chromodynamics.
- Attempts to include gravity.
 - ◇ String theory.

Unified Theories

- String theory.
 - ◇ *The elegant universe : superstrings, hidden dimensions, and the quest for the ultimate theory* by Brian Greene, W.W.Norton, New York 1999.

The Modeling Process

- It is based on experiment and/or observation.
- It is iterative.
 - ◇ For motion we have ≥ 6 iterations.
 - ◇ After each change in the model it must be checked by experimentation and observation.
- It is rare that a model captures all aspects of the phenomenon.