

# Math 211

Lecture #40

Predator Prey Models

April 27, 2001

## Predator-Prey

Lotka-Volterra system

$$x' = (a - by)x \quad (\text{prey} - \text{fish})$$

$$y' = (-c + dx)y \quad (\text{predator} - \text{sharks})$$

- Equilibrium points
  - ◊  $(0, 0)$  is a saddle.
  - ◊  $(x_0, y_0) = (c/d, a/b)$  is a linear center.

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- The axes are invariant.
- The positive quadrant is invariant.
- The solution curves appear to be closed. Is this actually true?

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- Along the solution curve  $y = y(x)$  we have

$$\frac{dy}{dx} = \frac{y(-c + dx)}{x(a - by)}.$$

- ◊ The solution is

$$H(x, y) = by - a \ln y + dx - c \ln x = C$$

- ◊ This is an implicit equation for the solution curve.  $\Rightarrow$  All solution curves are closed, and represent periodic solutions.

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Predator prey system

### Why Fishing Leads to More Fish

- Compute the averages  $\bar{x}$  and  $\bar{y}$  of the fish & shark populations.

$$\frac{d}{dt} \ln x(t) = \frac{x'}{x} = a - by$$

$$0 = \frac{1}{T} \int_0^T \frac{d}{dt} \ln x(t) dt = a - b\bar{y}.$$

- $\bar{y} = a/b = y_0$ .
- Similarly  $\bar{x} = x_0 = c/d$ .

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Predator prey system

- Assume that fishing does not distinguish between fish and sharks:

$$x' = (a - by)x - ex$$

$$y' = (-c + dx)y - ey$$

- This is the same system with  $a$  replaced by  $a - e$  and  $c$  replaced by  $c + e$ .

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Predator prey system

Averages

- The average populations are

$$\bar{x}_1 = \frac{c+e}{d} \quad \text{and} \quad \bar{y}_1 = \frac{a-e}{b}$$

- Fishing causes the average fish population to increase and the average shark population to decrease.

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### Cottony Cushion Scale Insect & the Ladybird Beetle

- Cottony cushion scale insect accidentally introduced from Australia in 1868.
  - ◊ Threatened the citrus industry.
- Ladybird beetle imported from Australia
  - ◊ Natural predator – reduced the insects to manageable low.

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- DDT kills the scale insect.
  - ◊ Massive spraying ordered.
    - ✱ Despite the warnings of mathematicians and biologists.
  - ◊ The scale insect increased in numbers, as predicted by Volterra.

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