Math 211

Lecture #40

Predator Prey Models

April 27, 2001

Predator-Prey

Lotka-Volterra system

$$x' = (a - by)x \quad \text{(prey – fish)}$$

$$y' = (-c + dx)y \quad \text{(predator – sharks)}$$

- Equilibrium points
  - ◇ $(0, 0)$ is a saddle.
  - ◇ $(x_0, y_0) = (c/d, a/b)$ is a linear center.

- The axes are invariant.
- The positive quadrant is invariant.
- The solution curves appear to be closed. Is this actually true?

- Along the solution curve $y = y(x)$ we have

$$\frac{dy}{dx} = \frac{y(-c + dx)}{x(a - by)}.$$

  - ◇ The solution is

$$H(x, y) = by - a \ln y + dx - c \ln x = C$$

  - ◇ This is an implicit equation for the solution curve. $\Rightarrow$ All solution curves are closed, and represent periodic solutions.

Why Fishing Leads to More Fish

- Compute the averages $\overline{x}$ and $\overline{y}$ of the fish & shark populations.

$$\frac{d}{dt}\ln x(t) = \frac{x'}{x} = a - by$$

$$0 = \frac{1}{T}\int_0^T \frac{d}{dt}\ln x(t)\,dt = a - b\overline{y}.$$

- $\overline{y} = a/b = y_0.$
- Similarly $\overline{x} = x_0 = c/d.$

- Assume that fishing does not distinquish between fish and sharks:

$$x' = (a - by)x - ex$$

$$y' = (-c + dx)y - ey$$

- This is the same system with a replaced by $a - e$ and c replaced by $c + e$.

- The average populations are

$$\overline{x}_1 = \frac{c+e}{d} \quad \text{and} \quad \overline{y}_1 = \frac{a-e}{b}$$

- Fishing causes the average fish population to increase and the average shark population to decrease.

Cottony Cushion Scale Insect & the Ladybird Beetle

- Cottony cushion scale insect accidentally introduced from Australia in 1868.
  - ◇ Threatened the citrus industry.
- Ladybird beetle imported from Australia
  - ◇ Natural predator – reduced the insects to manageable low.

- DDT kills the scale insect.
  - ◇ Massive spraying ordered.
    - ⋆ Despite the warnings of mathematicians and biologists.
  - ◇ The scale insect increased in numbers, as predicted by Volterra.