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Invariants in Low-Dimensional Topology and Knot Theory

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Abstract. This meeting concentrated on topological invariants in low dimensional topology and knot theory. We include both three and four dimensional manifolds in our phrase "low dimensional topology". The intent of the conference was to understand the reach of knot theoretic invariants into four dimensions, including results in Khovanov homology, variants of Floer homology and quandle cohomology and to understand relationships among categorification, topological quantum field theories and four dimensional manifold invariants as in particular Seiberg-Witten invariants.

Mathematics Subject Classification (2000): 55-xx, 57-xx.

Introduction by the Organisers

The purpose of this conference was to bring together people working in low-dimensional topology, both in knot theory and 3-manifold theory and in 4-manifold theory.

Here is a short comment on the combinatorial topology side of the topics. In 1969 John H. Conway published a version of the Alexander polynomial that involves nothing more than a recursion on diagrams controlled by a "skein formula" that expresses the difference between the polynomial for a knot with a given crossing, the same diagram with a switched crossing and the same diagram with the crossing replaced by connecting arcs that do not cross (a smoothing of the crossing). This remarkable reformulation of the Alexander polynomial remained a mystery for some years. In the late 1970's people became interested in this relation again and, among others, Kauffman wrote a paper explaining the skein relation approach of Conway in terms of the Seifert pairing of the knot. In the early 1980's Kauffman found another model of the Alexander-Conway Polynomial as state summation related to Alexander's original definition using a determinant of a matrix associated with the link diagram. Then in 1983, Vaughan Jones found a new and powerful polynomial invariant of knots and links that was quite different from the Alexander polynomial, but also satisfied a skein relation. This discovery of the Jones polynomial quickly led to a number of other skein-type invariants – the Homflypt polynomial and a two-variable Kauffman polynomial. Also Kauffman found a state sum model for the original Jones polynomial. After this initial combinatorial revolution in the knot theory, there came a big influx of algebra, first via von Neuman algebras and the Temrperley Lieb algebra from Jones himself, then Hecke algebras and quantum groups (deformations of classical Lie algebras) and Hopf algebras with the work of Reshetikhin and Turaev. Then quantum field theory entered the picture with the work of Edward Witten and this led to the development of new invariants of three-manifolds, the formulation of Vassiliev invariants, work of Birman, Lin and Bar Natan and a mix of research problems that continues to the present day. In the 1990's Kauffman and Goussarov, Polyak and Viro introduced virtual knot theory a generalization of classical knot theory to knots and links in thickend surfaces that has a simple diagrammatic extension from classical knot diagrams. Virtual knot theory continues in a very active way to the present day with contributions from many people and a first book on the subject by Manturov and Ilyutko, containing significant recent advances by Manturov and collaborators. In 1999 Misha Khovanov discovered an extension of the Kauffman bracket state sum model for the Jones polynomial to a graded homology theory such that the coefficients of the Jones polynomial become Euler characteristics of graded parts of the homology. The Khovanov homology of a knot is more powerful than the Jones polynomial of that knot and in fact it was shown in 2008 that the Khovanov homology detects the unknot, a property that is still unknown for the Jones polynomial. This "categorification" of the Jones polynomial was followed by a quite different categorification of the Alexander-Conway polynomial in the work of Oszváth and Szábo, and this work led to astonishing results such as a homological method to find the Seifert genus of a knot and, in both cases of these theories, a bridge between three manifolds and four manifolds. This sketch indicates the background of our conference on the side of combinatorial topology.

There were 51 participants, and 42 speakers among them. Participants without talks presented their results in various private communications during the discussion time or in the evening at the workshops or in an unofficial manner.

Several talks were organized for the whole audience; the other talks were held in two parallel sessions.

Nevertheless, all participants could share their results with everyone in formal or less formal workshops organized every day in the evening time. Research reports of the majority of participants were posted on the wall as well as on the conference webpage.

The main topics of the conference were:

- Recognition of the Unknot
- Virtual Knot Theory and Parity Theory
- Cobordisms and Concordance of Knots
- Finite-type invariants
- Heegaard-Floer Homology
- Exotic structures and Corks in 4-manifold Theory
- Seiberg-Witten Invariants, Gauge Theory
- 2-knots and their diagrams
- Khovanov homology theory
- Braid Theory
- Unknotting numbers and related topics in classical and virtual knot theory
- Quandles and Related Structures in Knot Theory
- Knot Mutations
- Knots and DNA
- Contact topology
- Topological Methods in Combinatorial Group Theory
- Calabi-Yau Manifolds
- Fibred Manifolds

In addition to the talks, three workshops were organized during the conference. A workshop on Virtual Knot Theory and parity in Low-Dimensional Topology was organized by V.O.Manturov. It was devoted to further applications of parity theory as well as to various unsolved problems, and others contributed to the discussion.

One workshop was devoted to the result of Chad Musick on the recognition of the unknot in a polynomial time.

One workshop was organized by Scott Carter on various algebraic generalizations of quandles possessing distributitivty and associativity properties leading to invariants of knots, 2-knots and trivalent graphs.

Workshop: Invariants in Low-Dimensional Topology and Knot Theory

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Abstracts

Virtual Knot Theory - Problems and Ideas

Louis H. Kauffman

1. Introduction

Virtual knot theory [3, 2] is a generalization of classical knot theory that can be described in a number of ways. One way to describe it is to say that virtual knot theory uses diagrams just like the diagrams for classical knot theory except that there is a new type of crossing, called *virtual*, that is neither an under-crossing nor an overcrossing. The virtual crossing behaves in the following way: any arc in the diagram with a consecutive sequence of virtual crossings can be excised from the diagram and another arc can be placed transversely to the remaining part of the diagram so that all the crossings introduced in the new arc are also taken to be virtual. Otherwise, one applies the usual Reidemister moves to the classical crossings in the diagram. This diagrammatic definition of virtual knot theory is equivalent to saying that virtual knot theory is the set of equivalence classes of oriented Gauss codes modulo the abstract Reidemeister moves on these codes. Since not all codes are realizable in the plane, a diagram for a virtual knot or link may contain virtual crossings in the same way that an attempt to embed a non-planar graph into the plane leads to extra crossings of the edges of the graph with itself. In this view, the virtual crossings are artifacts of the attempt to embed a non-planar code into the plane.

Another equivalent definition of virtual knot theory is that a virtual knot is represented by an embedding of a circle (or circles) into a thickened oriented surface of some genus g. Two such embeddings are equivalent if there is an orientation preserving homeomorphism of the surface that carries one embedding to the other, *or* if one can perform a 1-handle surgery in the complement of the knot in the thickened surface, retaining connectiviity and changing the genus. We say that virtual knots are knots in thickened surfaces taken up to handle stabilization. From this point of view, one would like to find the least genus surface in which such a knot could be represented. Kuperberg [4] proved that the embedding type of the knot in its minimal genus surface is uniquely determined and so there is a definite topological interpretation for the virual knot as its least genus embedding. It follows from this Theorem of Kuperberg that virtual knot theory embeds in classical knot theory (this was proved earlier by a more algebraic argument [3, 2]).

In this talk we describe a polynomial invariant of virtual knots and links that has the advantage of being quite easy to calculate, and it can detect features that the Jones polynomial cannot see.

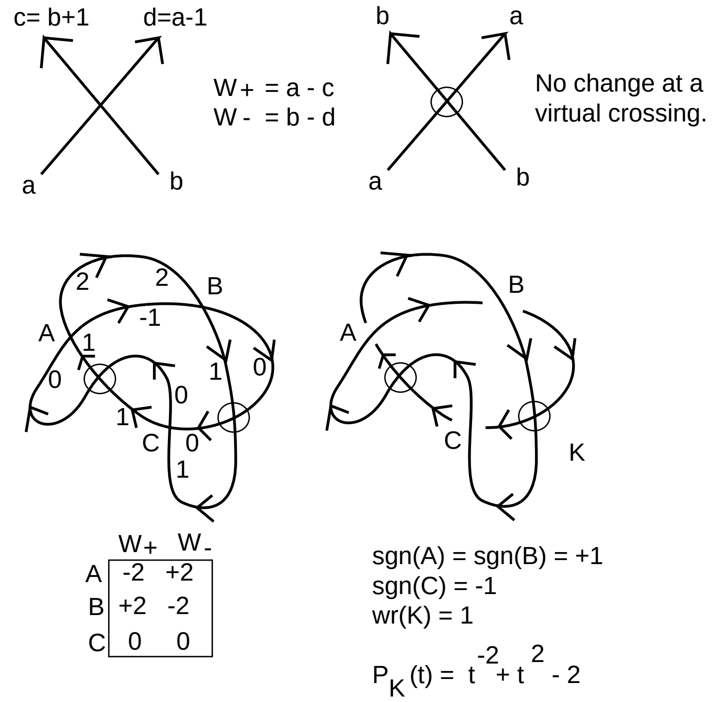

Figure 1. Labeled Flat Crossing and Example 1

2. The Polynomial Invariant

We define a polynomial invariant of of virtual knots by first describing how to calculate the polynomial. We then justify that this definition is invariant under virtual isotopy. Calculation begins with a flat oriented virtual knot diagram (the classical crossings in a flat diagram do not have choices made for over or under). An *arc* of a flat diagram is an edge of the 4-regualar graph that it represents. That is, an edge extends from one classical node to the next in orientation order. An arc may have many virtual crossings, but it begins at a classical node and ends at another classical node. We label each arc c in the diagram with an integer $\lambda(c)$ so that an arc that meets a classical node and crosses to the left increases the label by one, while an arc that meets a classical node and crosses to the right decreases the label by one. See Figure 1 for an illustration of this rule. We prove that such integer labelling can always be done for any virtual or classical link diagram. In a virtual diagram the labeling is unchanged at a virtual crossing, as indicated in Figure 1. One can start by choosing some arc to have an arbitrary integer label, and then proceed along the diagram labelling all the arcs via this crossing rule. We call such an integer labelling of a diagram a *Cheng coloring* of the diagram. The invariant described herein is a generalization the the invariant described by Cheng in [1].

Given a labeled flat diagram we define two numbers at each classical node c: $W_-(c)$ and $W_+(c)$ as shown in Figure 1. If we have a labeled classical node with left incoming arc a and right incoming arc b then the right outgoing arc is labeled

$a-1$ and the left outgoing arc is labeled $b+1$ as shown in Figure 1. We then define

$$W_+(c) = a - (b+1)$$

and

$$W_-(c) = b - (a+1).$$

Note that

$$W_-(c) = -W_+(c)$$

in all cases.

Given a crossing c in a diagram K, we let $sgn(c)$ denote the sign of the crossing. The sign of the crossing is plus or minus one according to the standard convention. The *writhe*, $wr(K)$, of the diagram K is the sum of the signs of all its crossings. For a virtual link diagram, labeled in the integers according to the scheme above, and a crossing c in the diagram, define $W(c)$ by the equation

$$W(c) = W_{sgn(c)}(c)$$

so that $W(c)$ is $W_\pm(c)$ according as the sign of the crossing is plus or minus.

Let K be a virtual knot diagram. Define

$$P_K = \sum_c sgn(c) t^{W(c)} - wr(K).$$

We shall prove that the Laurent polynomial P_K is a highly non-trivial invariant of virtual knots.

In Figure 1 we show the computation of the weights for a given flat diagram and the computation of the polynomial for a virtual knot K with this underlying diagram. The knot K is an example of a virtual knot with unit Jones polynomial. The polynomial P_K for this knot has the value

$$P_K = t^{-2} + t^2 - 2,$$

showing that this knot is not isotopic to a classical knot.

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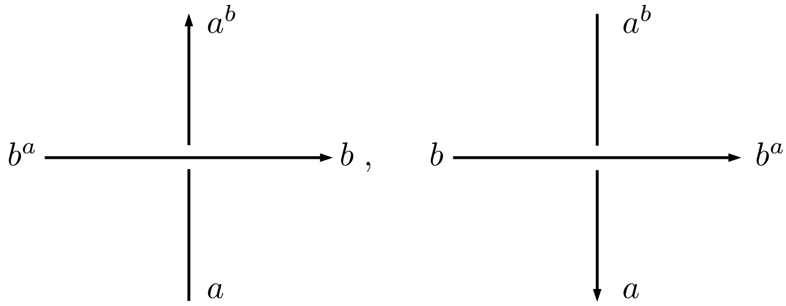
Invariants and homology of biquandle classifying space

Roger Fenn

This talk had the following themes:

1. A new description of the biquandle
2. The idea of a "partial" biquandle
3. Applications to classical knots—in particular discriminating the left and right trefoil

1. The definition of the biquandle can be simplified by the application of



2. Partial biquandles are now defined on *all* pairs—in particular taking the *double* of a biquandle pairs of pairs $\{(ac)(bc)\colon c^a = c^b\}$,

$$(ac)^{(bc)} = (a^b c_b), \quad (bc)_{(ac)} = (b_a c_a).$$

3. In the classifying space of the double of the 3-colour quandle, the left and right trefoil have values $+1$ and -1 respectively.

Injectivity of satellite operators in knot concordance

Tim D. Cochran
(joint work with Chris W. Davis and Arunima Ray)

The satellite construction is a procedure that transforms an oriented knot K in S^3 to another knot. Suppose P is an oriented knot in the solid torus $ST \equiv S^1 \times D^2$, called a *pattern knot.* For any oriented knot K in S^3 we denote by $P(K)$ the (untwisted) **satellite** of K obtained by using P as a **pattern**. Each pattern may thus be viewed as a function $P : \mathcal{K} \to \mathcal{K}$ on the set of isotopy classes of knots. These induce functions, called **satellite operators**, on $\mathcal{K}/\sim$ for other important equivalence relations, in particular on the set of concordance classes of knots. We will establish the injectivity of these functions in some important cases.

The importance of satellite operations extends beyond knot theory. They generalize to operations on 3 and 4-manifolds where they produce very subtle variations while fixing the homology type [5, Sec. 5.1]. Winding number one satellites are closely related to Mazur 4-manifolds which in turn are closely related to Akbulut *corks* [1]. The latter are contractible 4-manifolds that can be used to alter the smooth structure on 4-manifolds (by removing them and reinserting them with a twist). Specifically, a knot K may occur as the attaching circle of a 2-handle in the handlebody description of a 4-manifold. It was shown, for example, in another talk at this conference [2], that, for the simplest strong winding number one operators P, the modification of the handlebody effected by $K \rightsquigarrow P(K)$ can alter the smooth structure on the 4-manifold without altering the homeomorphism type!

We consider four different sets, denoted $\mathcal{C}$, $\mathcal{C}^{ex}$, $\mathcal{C}^{top}$, and $\mathcal{C}^{\frac{1}{n}}$ respectively. Here $\mathcal{C}$ is the (usual) set of **smooth knot concordance** classes. Here $\mathcal{C}^{top}$ is the (usual) set of **topological knot concordance** classes. $\mathcal{C}^{ex}$ is the set where two knots are equivalent if they are smoothly concordant in $S^3 \times [0,1]$ equipped with a possibly exotic smooth structure. This has been called *pseudo-concordance* by some authors. If the smooth 4-dimensional Poincaré Conjecture is true then $\mathcal{C}^{ex} = \mathcal{C}$. Finally, for a fixed non-zero integer n, let $\mathcal{C}^{\frac{1}{n}}$ denote the set of equivalence classes of knots in S^3 where two are equivalent if they cobound a smoothly embedded annulus in a smooth 4-manifold that is $\mathbb{Z}[\frac{1}{n}]$-homology cobordant to $S^3 \times [0,1]$. For odd n it seems to be unknown whether or not $\mathcal{C} = \ mathcal{C}^{\frac{1}{n}}$! For economy we will use the notation $\mathcal{C}^*$ to denote either $* = top$, $* = ex$ or $* = \frac{1}{n}$, reserving the notation $\mathcal{C}$ for the smooth knot concordance group. If $K = 0 = U$ in $\mathcal{C}$ (respectively: $\mathcal{C}^{ex}, \mathcal{C}^{top}, \mathcal{C}^{\frac{1}{n}}$) then K is called a (smooth) **slice knot** (respectively: pseudo-slice, topologically slice, $\mathbb{Z}[\frac{1}{n}]$-slice).

We are interested in whether or not such satellite operators are injective functions (beware they are not homomorphisms). Call an operator **weakly injective** if $P(K) = P(0)$ implies $K = 0$ (here 0 is the class of the trivial knot U). It is a long-standing open problem as to whether or not the Whitehead double operator is weakly injective on $\mathcal{C}$ [7, Problem 1.38]. Considerable effort has been expended in providing evidence for this conjecture (see [6] for a survey and the most recent results). There has recently been speculation that many other "non-trivial" satellite operators are injective on $\mathcal{C}$. In [4] large classes of winding number zero operators called "robust doubling operators" were introduced and evidence was presented for their injectivity. Yet no single "non-trivial" operator is known to be even weakly injective.

Here we have more success for non-zero winding number operators, especially winding-number ± 1 operators. The **winding number** of P is the algebraic intersection number of P with a meridional disk of ST. Let η denote the oriented meridian of ST, $\{1\} \times \partial D^2$. The condition that a pattern P has winding number ± 1 is equivalent to the condition that η generates $H_1(S^3 - P(U))$.

Definition 1. *The pattern P has **strong winding number** ± 1 if the meridian of the solid torus ST normally generates $\pi_1(S^3 - P(U))$.*

The example in Figure 1 has strong winding number one. If $P(U)$ is unknotted then strong winding number one is the same as ordinary winding number one. We show that strong winding number ± 1 patterns are plentiful. Our main theorem is:

Theorem 1. *Suppose P is a pattern with non-zero winding number n. Then*

a. *$P : \mathcal{C}^{\frac{1}{n}} \to \mathcal{C}^{\frac{1}{n}}$ is an injective function.*

Suppose that P is a pattern with strong winding number ± 1. Then

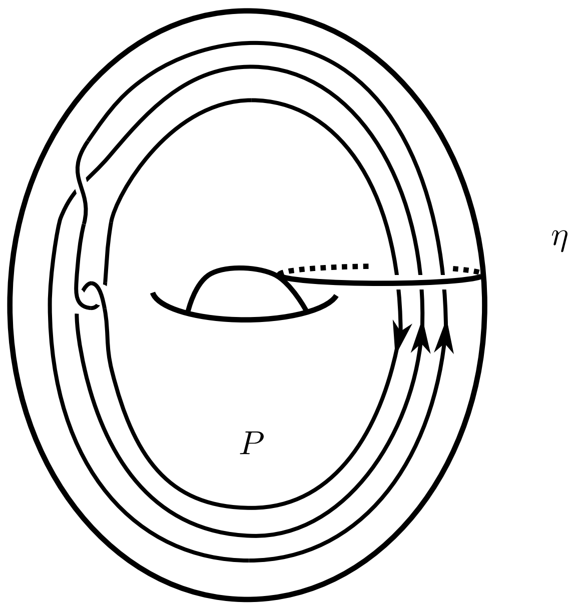


FIGURE 1. A strong winding number one pattern P

b. *$P : \mathcal{C}^{ex} \to \mathcal{C}^{ex}$ is an injective function,*
c. *$P : \mathcal{C}^{top} \to \mathcal{C}^{top}$ is an injective function, and*
d. *if S^4 has a unique smooth structure (up to diffeomorphism) then $P : \mathcal{C} \to \mathcal{C}$ is an injective function.*

This establishes that the sets $\mathcal{C}^*$ admit many natural self-similarities (as conjectured in [4]).

Restricting part a. of the theorem to the case $n = 1$ yields:

Corollary 2. *Suppose P is a pattern with winding number ± 1. Then $P(K)$ is smoothly concordant to $P(J)$ in a smooth homology $S^3 \times [0,1]$ if and only if $K\# -J$ is smoothly slice in a smooth homology B^4.*

Similarly, restricting part a. to cable operations yields:

Corollary 3. *If p and q are coprime positive integers then the (p,q) cable of K is smoothly concordant to the (p,q) cable of J in a smooth $\mathbb{Z}[\frac{1}{p}]$-homology $S^3 \times [0,1]$ if and only if K is smoothly concordant to J in a smooth homology $\mathbb{Z}[\frac{1}{p}]$-homology $S^3 \times [0,1]$.*

The case $p = 2$ of Corollary 3 was proved previously by the third author and indeed was one of the inspirations for the current paper. An important ingredient in our proof is a well-known relationship between concordance of knots and homology cobordism of certain 3-manifolds associated to the knots via surgery. In this regard we also owe a substantial debt to the recent work of Cochran-Franklin-Hedden-Horn [3]. Our techniques are elementary. We use only basic topology and handlebody techniques, except for our use of the 4-dimensional topological Poincaré conjecture.

We also extend some of our results to links.

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A new construction of the Fukaya–Seidel category

Andriy Haydys

Inspired by Seidel's construction [2], I present a new construction of the Fukaya–Seidel category, which is associated to a symplectic Lefschetz fibration.

Let $(M, \omega = d\lambda,\ J)$ be an exact symplectic manifold equipped with an almost complex structure. A symplectic Lefschetz fibration in our setting is a J–holomorphic function $f\colon M \to \mathbb{C}$ with finitely many non–degenerate critical points $m_1, \dots, m_k$. Then one can conjecturally associate to f an A_∞–category $\mathcal{A}_f$ as follows (see [1] for details). The objects of $\mathcal{A}_f$ are critical points of f. To define the morphism spaces choose a pair of critical points $m_\pm \in \{m_1, \dots, m_k\}$ and denote by $\Gamma(m_-, m_+)$ a suitably chosen subspace of $\{\gamma\colon \mathbb{R} \to M \mid \gamma(\pm\infty) = m_\mp\}$. Consider the functional

$$\mathcal{F}\colon \Gamma(m_-, m_+) \longrightarrow \mathbb{R}, \qquad \mathcal{F}(\gamma) = \int_{\mathbb{R}} \gamma^*\lambda + \int_{\mathbb{R}} \operatorname{Im}\left(e^{-i\theta(t)} f \circ \gamma(t)\right) dt,$$

where $\theta = \theta(t)$ is a suitably chosen function. Then, roughly speaking, $hom(m_-, m_+)$ is the Morse–Witten complex of $\mathcal{F}$. Let us describe some details. The critical points of $\mathcal{F}$ are solutions of the problem

$$\dot\gamma + v^t = 0, \qquad \lim_{t \to \pm\infty} \gamma(t) = m_\mp, \tag{1}$$

where $v^t = \cos\theta(t)\ \operatorname{grad}\operatorname{Re} f + \sin\theta(t)\ \operatorname{grad}\operatorname{Im} f$. These freely generate $hom(m_-, m_+)$ as a $\mathbb{Z}/2\mathbb{Z}$–vector space.

Furthermore, choose a pair $\gamma_\pm$ of solutions of (1) and consider the problem

$$(2)\qquad \begin{aligned} &\partial_s u + J\big(\partial_t u + v^t\big) = 0, && u\colon \mathbb{R}^2_{s,t} \to M,\\ &\lim_{t\to\pm\infty} u(s,t) = m_\mp, && \lim_{s\to\pm\infty} u(s,t) = \gamma_\mp(t),\\ &\lim_{t\to\pm\infty} \int_{-\infty}^{+\infty} |\partial_s u(s,t)|\, ds = 0, && \lim_{s\to\pm\infty} \int_a^b |\partial_s u(s,t)|\, dt = 0. \end{aligned}$$

Notice that the first equation is formally the antigradient flow equation for the functional $\mathcal{F}$. Denote the space of solutions of the above equations by $\mathcal{M}(\gamma_-;\gamma_+)$ and put $\hat{\mathcal{M}}(\gamma_-;\gamma_+) = \mathcal{M}(\gamma_-;\gamma_+)/\mathbb{R}$, where $\mathbb{R}$ acts by translations in s–variable. One defines the differential $\partial\colon hom(m_-, m_+) \to hom(m_-, m_+)$ in the usual way, provided points in $\hat{\mathcal{M}}(\gamma_-;\gamma_+)$ can be sensibly counted. The composition $hom(m_p, m_q) \otimes hom(m_q, m_r) \to hom(m_p, m_r)$ as well as the "Massey products" are defined in a similar manner. Conjecturally, these combine to an A_∞–structure.

Theorem 1. *The space $\mathcal{M}(\gamma_-;\gamma_+)$ is the zero locus of a Fredholm section.*

Theorem 2. *Assume critical values $f(m_1), \dots, f(m_k)$ are in convex position and the fibration $f\colon M \to \mathbb{C}$ is trivial in the neighbourhood of infinity. Then the space*

$$\hat{\mathcal{M}}(m_-;m_+) = \bigcup_{\gamma_\pm} \hat{\mathcal{M}}(\gamma_-;\gamma_+)$$

is compact, where the union is taken over all pairs of solutions of (1)*.*

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Naturality of Heegaard Floer homology

András Juhász

(joint work with Peter Ozsváth, Dylan Thurston)

Heegaard Floer homology, defined by Ozsváth and Szabó in [1, 2], is a package of invariants for 3- and 4-manifolds. For concreteness, we will focus on the 3-manifold invariant $\widehat{HF}$ with $\mathbb{Z}_2$-coefficients. Given a closed oriented 3-manifold Y, the construction of [1] assigns to Y the $\mathbb{Z}_2$-vector space $\widehat{HF}(Y)$, well-defined up to isomorphism. There are several choices involved in the construction; most importantly, one has to choose a based Heegaard diagram $(\Sigma, \boldsymbol{\alpha}, \boldsymbol{\beta}, z)$ of Y. Here $\Sigma \subset Y$ is an oriented genus g surface that splits Y into two handlebodies, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are two g-tuples of pairwise disjoint, homologically linearly independent simple closed curves in Σ, and $z \in \Sigma \setminus (\boldsymbol{\alpha} \cup \boldsymbol{\beta})$ is a basepoint.

To obtain the 4-manifold invariants, one has to define maps induced on Heegaard Floer homology by cobordisms, and for that one needs to assign a concrete vector space $\widehat{HF}(Y)$ to a 3-manifold Y. The same is needed to have a diffeomorphism action on $\widehat{HF}$, or to be able to talk about the contact element in Heegaard Floer homology (as opposed to just being able to say whether this element is zero or not). In [2], Ozsváth and Szabó attempted to prove these naturality properties of Heegaard Floer homology. However, as noticed by the author, there is a gap in their proof, they did not account for the way the Heegaard surface Σ is embedded in Y. Furthermore, it is unclear whether $\widehat{HF}$ depends on the choice of basepoint (just like the fundamental group depends on a basepoint). As an analogy, one can consider simplicial homology, which depends on a triangulation, and since manifolds can carry several PL structures, it is a priori unclear how to compare the homology computed from triangulations lying in different PL structures. In that case, singular homology overcomes this problem.

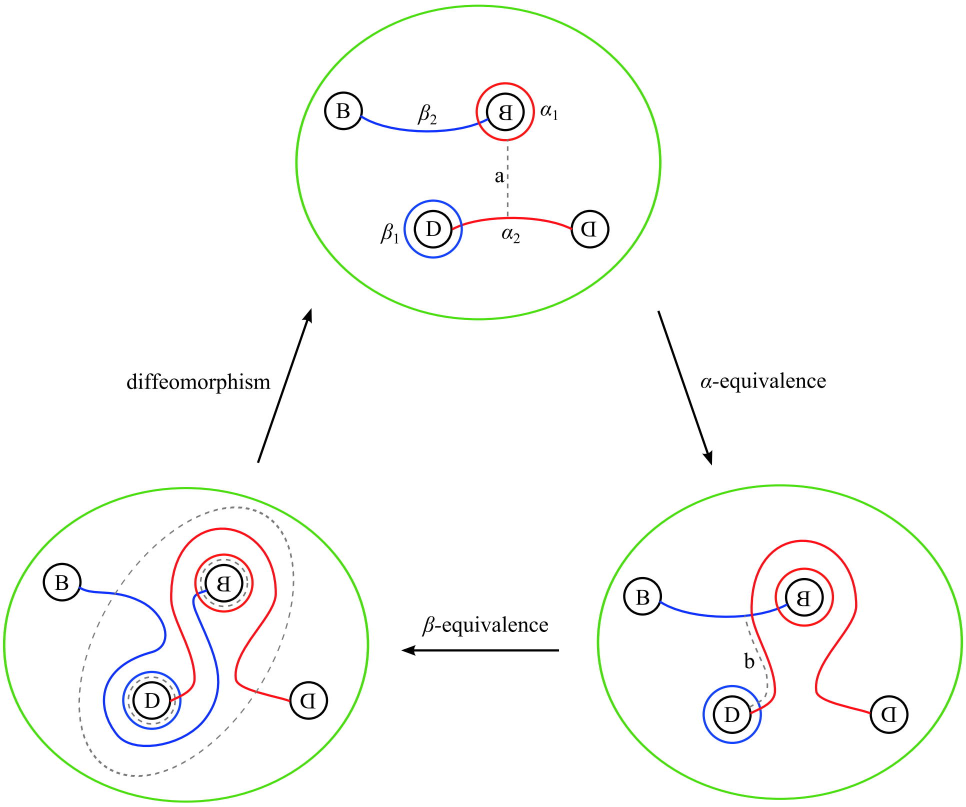
Intuitively, one should think of $\widehat{HF}$ as assigning a $\mathbb{Z}_2$-vector space to every based diagram $(\Sigma, \boldsymbol{\alpha}, \boldsymbol{\beta}, z)$ with $\Sigma \subset Y$. Hence, $\widehat{HF}$ is a $\mathbb{Z}_2$-vector bundle over the "space of diagrams". A priori, this might be a non-trivial bundle, one could move Σ around a non-trivial loop and get a non-trivial automorphism of $\widehat{HF}(\Sigma, \boldsymbol{\alpha}, \boldsymbol{\beta}, z)$. This would mean that, a priori, the Goeritz group of the Heegaard splitting (Y, Σ) might act non-trivially on $\widehat{HF}(\Sigma, \boldsymbol{\alpha}, \boldsymbol{\beta}, z)$.

As we show in [3], Heegaard Floer homology is in fact natural and is equipped with a diffeomorphism action. If D and D' are both based diagrams of the based 3-manifold (Y, p), then we construct an isomorphism $\Phi_{D,D'} \colon \widehat{HF}(D) \to \widehat{HF}(D')$. These satisfy the property that $\Phi_{D',D''} \circ \Phi_{D,D'} = \Phi_{D'',D}$. Then the vector space $\widehat{HF}(Y, p)$ is defined to be

$$\coprod_D \widehat{HF}(D)/\sim,$$

where the disjoint union is taken over all based diagrams D of (Y, p) with $z = p$, and the equivalence relation $\sim$ is defined by requiring that $x \in \widehat{HF}(D)$ and $x' \in \widehat{HF}(D')$ are equivalent if and only if $x' = \Phi_{D,D'}(x)$. Given a diffeomorphism $d \colon (Y, p) \to (Y', p')$, we define the diffeomorphism map $d_* \colon \widehat{HF}(Y, p) \to \widehat{HF}(Y', p')$ as follows: Let D be a diagram of (Y, p), and $D' = d(D)$ the corresponding diagram of (Y', p'). Then d_* maps the equivalence class of $x \in \widehat{HF}(D)$ to the equivalence class of $d_*(x) \in \widehat{HF}(D')$, where d_* on the chain level is simply given by the map d induces from $\mathbb{T}_\alpha \cap \mathbb{T}_\beta$ to $\mathbb{T}_{\alpha'} \cap \mathbb{T}_{\beta'}$.

In fact, we provide a set of axioms for arbitrary invariants of Heegaard diagrams that should be satisfied in order to give rise to a functorial invariant of 3-manifolds. The proof proceeds by studying the bifurcations of generic 2-parameter families of gradient vector fields on 3-manifolds. A Heegaard diagram arises from a generic gradient on Y, while bifurcations of generic 1-parameter families of gradients correspond to certain generalized Heegaard moves. We assign an isomorphism to every classical Heegaard move, and then show that no loop of moves produces a non-trivial monodromy, where a move can be a diffeomorphism $\Sigma \to \Sigma'$ that is isotopic to the identity in Y (without this, it is impossible to compare $\widehat{HF}$ for


B
β_2
B
α_1
a
β_1
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α_2
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diffeomorphism
α-equivalence
β-equivalence
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D
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b

Figure 1. A simple handleswap

different Heegaard surfaces). From this loop of moves, we construct a loop of gradient vector fields, parametrized by S^1. Then we extend this to D^2. The link of a codimension-2 bifurcation gives a loop of diagrams, and we simplify these loops until consecutive diagrams are related by the standard Heegaard moves and diffeomorphisms isotopic to the identity. Finally, we show the monodromy is trivial along each loop in the simplification. It turns out that, in addition to the loops checked in [2], one only has to verify that $\widehat{HF}$ has no monodromy for one specific type of loop that we call a simple handleswap, see Figure 1.

There are simple examples of multi-pointed Heegaard diagrams where moving the basepoints produces non-trivial automorphisms of the Floer homology. We conjecture that this is also the case for $\widehat{HF}(Y,p)$, but not for the other flavors of Heegaard Floer homology. The latter claim is motivated by the fact that the other flavors are isomorphic to the various versions of monopole Floer homology, whose construction does not depend on the choice of basepoint.

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Free Knots and Parity in Low-Dimensional Topology

Vassily Olegovich Manturov

The main objective of my talk is the concept of parity. Assume for some topological theory objects are encoded by *diagrams* modulo *moves*. Assume further that diagrams have some combinatorial data (crossings, vertices, singularities, etc) such that there is a natural rule of decorating this combinatorial data with elements from some finite set (usually, $\{0, 1\}$) in such a way that if two diagrams are connected by one move then their combinatorial data is transformed in a good way.

Then this allows one to do the following.

(1) Construct functorial mappings between objects.
(2) Reduce problems about objects to questions about their representatives.
(3) Prove minimality theorems.
(4) Refine many well-known invariants.
(5) Localize the non-triviality: a non-triviality of an object can follow from the existence of *odd data*.

Our main domain is *virtual knot theory* [1] with its closed relatives: free knots, flat knots, graph-links etc [3, 4, 5, 7, 9]. We do not distinguish between virtual knot diagrams that differ by a detour move.

By a *parity* for the knot theory $\mathcal{K}$ we mean a rule for associating 0 or 1 with every (classical) crossing of every diagram K from the theory $\mathcal{K}$ in a way such that:

(1) For every Reidemeister move $K \to K'$ the corresponding crossings have the same parity;
(2) For each of the three Reidemeister moves the sum of parities of crossings taking part in this move is zero modulo two.

A *flat knot* is an equivalence class of virtual knots modulo crossing switches. A *free knot* is an equivalence class of virtual knots modulo crossing switches and virtualizations.

The main example of parity is the *Gaussian parity*, i.e., the parity for virtual knots (free knots, flat knots) which associates with every classical crossing 0 if and only if the corresponding chord of the Gauss diagram is *even*, i.e., it is linked with evenly many chords [2]. Nevertheless, many results work for arbitrary parity.

One of the striking examples [3] is the *parity bracket* $[\cdot]$ which takes free knots to $\mathbb{Z}_2$-linear combinations of framed 4-graphs modulo the second Reidemeister moves. In the case when all crossings are odd, the bracket allows one to reduce questions about free knots to questions about their representatives.

We mention the following results based on *parity*.

(1) **Counterexample to Turaev's conjecture**. In [11], Turaev conjectured all free knots to be trivial. In [3], infinitely many examples of non-trivial free knots were constructed.
(2) **Cobordisms.** It was proved [4] that free knots admit non-trivial cobordism classes: there are free knots not spannable by discs with typical 3-dimensional singularities.
(3) **Reducing problems about knots to problems about representatives**. In [6], it was first proved that the minimal virtual crossing number of virtual knots can grow quadratically with respect to the minimal classical crossing number. This result was obtained by applying parity argument and the fact that the minimal crossing number of 4-valent graphs grows quadratically with respect to the number of vertices.

    By using the bracket $[\cdot]$, one easily shows that free knots are generally not invertible and that long free knots generally do not commute.
(4) **Projection.** For every virtual knot diagram K whose underlying genus is not minimal in the given knot class, there exists a diagram K' obtained from K by making some classical crossings virtual and having the same knot type as K.

    There exists a projection from the set of virtual knots to the set of classical knots: for every virtual knot diagram K there exists a classical knot diagram K'' obtained by making some classical crossings of K virtual.

    As a consequence, we see that minimal classical crossing number of a virtual knot can be achieved only on classical diagrams (for virtual diagrams which are not detour-equivalent to any classical one, it is strictly greater).
(5) By using free knots, one constructs a virtual knot with the unit Jones polynomial which can not be undone by Reidemeister moves and virtualizations. This disproves the corresponding conjecture from [8].

Questions for further research:

(1) **Categorification of the bracket.** The bracket $[K]$ of a 4-valent graph with an opposite half-edge structure is a sum of graphs which correspond to states at *even* crossings of the initial graph. For each crossings, the two local states A and B are not ordered, so, it there is no straightforward way to generalize Khovanov's construction. How to construct a complex whose Euler characteristic is the parity bracket (with all graphs treated, say, as gradings)?
(2) **Higher dimensions.** How to construct the parity theory for higher-dimensional objects? For example, for 2-surfaces in 4-space and which analogs of properties listed above can be obtained in this case? Is there any analog of the bracket in higher dimensions? A hint can be taken from [4].
(3) **Singular knots.** It is well known [10] that there is no non-trivial parity for classical knots. The main reason behind that is that the plane has no

homology unlike closed 2-surfaces of positive genera, virtual knots live in. Probably, there should be some parity for singular knots in 3-space.

(4) The same question about (classical) Legendrian knots.

(5) The same question about classical knots when we restrict ourselves to the situation when Reidemeister moves between the two knots are not arbitrary but with some restriction, e.g., when study fix a homotopy class of a path in the space of knots.

(6) **Group Theory.** Homotopy classses of curves on 2-surfaces can be considered as conjugacy classes of elements in the fundamental group of the surface; on the other hand, they can be considered as euqivalence classes of diagrams modulo Reidemeister moves. Which other groups can be studied as similar equivalence classes? When can one locate the information about groups at crossings? How to define the *bracket* for the group?

(7) **Arbitrary 3-manifolds.** Is there any parity theory for knots in arbitrary 3-manifolds?

(8) Are there applications of parity in link homotopy theory?

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Mutant knots with symmetry

Hugh R. Morton

We compare knots K and K' made up from oriented 2-tangles A, B and C as in figure 1.

When the tangles A and B are symmetric under the half-twist in figure 2 the knots K and K' are examples of mutants in the sense of Conway.

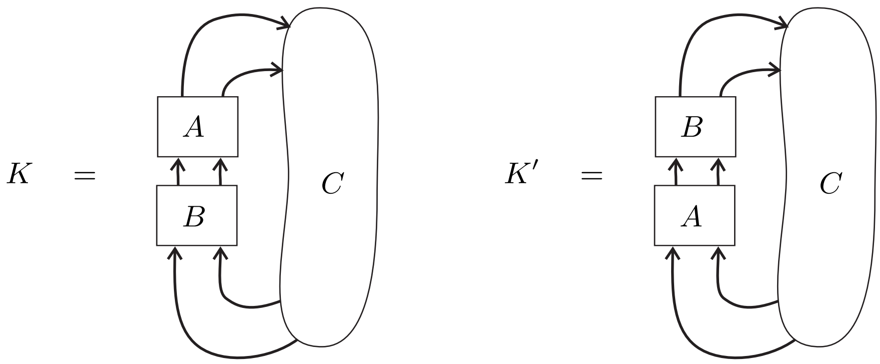

FIGURE 1

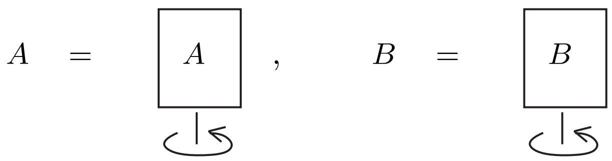

FIGURE 2

We look at ways of distinguishing such knots K and K' by comparing the Homfly polynomials of their satellites.

Examples of knots K and K' include pretzel knots such as those in figure 3.

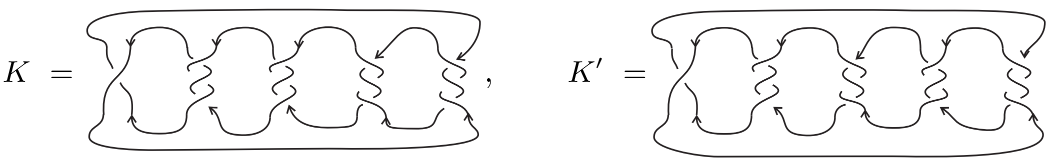

FIGURE 3

Mutants are never distinguished by their Homfly polynomial nor that of any 2-string satellite.

We show that, in contrast to the general case, the directed m-string satellites of any knots K and K' made up of symmetric tangles as above share the same Homfly polynomial for all $m \leq 5$. In addition the satellites of K and K' based on the (m, n) torus knot pattern, where m and n are coprime, have the same Homfly polynomial for *all* values of m.

However m-string satellites of K and K' other than the true (m, n) cables can have different Homfly polynomials when $m > 5$.

It can be shown explicitly that the 6-parallels of the two pretzel knots in figure 3 have different Homfly polynomials.

The proofs make use of the relation between the quantum $sl(N)$ invariants of K and K' and the Homfly polynomials of their satellites. In particular the calculation for the difference of the Homfly polynomials of the two 6-parallels depends on a comparison of the quantum $sl(3)$ invariants of the two pretzel knots based on the irreducible 27 dimensional $sl(3)$ module with partition $4, 2$.

A detailed account of these results appears in [1].

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Cork twisting exotic Stein 4-manifolds

Kouichi Yasui
(joint work with Selman Akbulut)

We discuss how to construct exotic 4-manifolds using corks ([1], [2], [3]). In particular, from any 4-dimensional compact oriented handlebody X without 3- and 4-handles and with $b_2(X) \geq 1$, we construct arbitrary many compact Stein 4-manifolds which are all homeomorphic but mutually non-diffeomorphic, so that their topological invariants coincide with those of X.

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Towards a categorification of the universal $sl(2)$ link invariant

Anna Beliakova

Witten-Reshetikhin-Turaev invariants of any homology 3-sphere at all roots of unity are dominated by a certain generating function - called a unified invariant, which has its values in the Habiro ring. This ring is a cyclotomic completion of the polynomial ring in one variable with integral coefficients.

In the talk we provide evidence to the fact that the unified invariants are more natural objects for categorification than the original invariants. A categorification program for unified invariants is based on a categorification of the universal $sl(2)$ link invariant, which is a generalization of the colored Jones polynomial taking values in the universal enveloping algebra of the corresponding quantum group.

Together with K. Habiro we recently made a crucial step towards a categorification of the universal R-matrix. We constructed an unbounded bicomplex which belongs to the Drinfeld center of the Khovanov-Lauda 2-category, whose Euler characteristic is the ribbon element of quantum $sl(2)$.

Corks and Exotic Smooth Structures of 4-Manifolds

Cagri Karakurt

It is known that any two simply connected homotopy equivalent closed smooth 4-manifolds differ by a surgery along a contractible codimension 0 sub-manifold so-called cork. Understanding gauge theoretical properties of corks play a crucial role in smooth classification of 4-manifolds. In this talk I will present a joint work with S. Akbulut on calculation of relative Ozsváth-Szabó invariants of an infinite family corks.

Unlinking numbers of links and their families

Slavik Jablan

The main topic of this talk is the Bernhard-Jablan Conjecture [2, 8, 9] and recent results related to it. The term "link" will be used for both knots and links. In this setting we have the following definition:

Definition 1.

a) *The* unlinking number $u(D)$ *of a link diagram* D *is the minimal number of crossing changes on the diagram required to obtain a diagram representing an unlink.*
b) *The* $u_M(L)$ *of a link* L *in* R^3 *is the minimum of* $u(D)$ *over all minimal crossing number diagrams* D *representing* L*.*
c) *The* unlinking number $u(L)$ *of a link* L *in* R^3 *is the minimum of* $u(D)$ *over all diagrams* D *representing* L*.*

Computing unlinking number is usually very difficult and complex problem. Therefore we define BJ-unlinking number which is computable due to the algorithmic nature of its definition.

Definition 2. *For a given crossing* v *of a diagram* D *representing link* L *let* D_v *denote the link diagram obtained from* D *by switching crossing* v*.*

a) *The* BJ-unlinking number $u_{BJ}(D)$ *of a diagram* D *is defined recursively in the following manner:*
   (1) $u_{BJ}(D) = 0$ *iff* D *represents an unlink.*
   (2) $u_{BJ}(D) = 1 + \min_{D_v} u_{BJ}(D_v)$ *where the first minimum is taken over all crossings* v *of* D *and the second minimum is taken over all minimal diagrams of a link represented by* D_v *for which the value is already defined.*
b) *The* BJ-unlinking number $u_{BJ}(L)$ *of a link* L $u_{BJ}(L) = \min_{D} u_{BJ}(D)$ *where the minimum is taken over all minimal diagrams* D *representing* L*.*

Bernhard [2] in 1994 and independently Jablan in 1995, conjectured that for every link L we have that $u(L) = u_{BJ}(L)$.

Definition 3. *Let S denote the set of numbers in the unreduced*[1] *Conway symbol $C(L)$ of a link L* [3]. *Given $C(L)$ and an arbitrary (non-empty) subset $\tilde{S} = \{a_1, a_2, \ldots, a_m\}$ of S, the family $F_{\tilde{S}}(L)$ of knots or links derived from L is constructed by substituting each $a_i \in \tilde{S}$, $a_i \neq 1$ in $C(L)$ by $sgn(a)(|a|+n)$, for $n \in N^+$.*

For even integers $n \geq 0$ this construction preserves the number of components, i.e., we obtain (sub)families of links with the same number of components. If all parameters in a Conway symbol of a knot or link are 1, 2, or 3, such a link is called *generating.*

For alternating knots, signature can be computed by using a combinatorial formula derived by P. Traczyk [14]. We will use this formula, proved by J. Przytycki, in the following form, taken from [13], Theorem 7.8, Part (2):

Theorem 1. *If D is a reduced alternating diagram of an oriented knot, then*

$$\sigma_D = -\frac{1}{2}w + \frac{1}{2}(W - B) = -\frac{1}{2}w + \frac{1}{2}(|D_{s+}| - |D_{s-}|),$$

where w is the writhe of D, W is the number of white regions in the checkerboard coloring of D, which is for alternating minimal diagrams equal to the number of cycles $|D_{s+}|$ in the state $s+$, and B is the number of black regions in the checkerboard coloring of D equal to the number of the cycles $|D_{s-}|$ in the state $s-$.

Introducing orientation of a knot, every n-twist (chain of digons) becomes *parallel* or *anti-parallel.*

Lemma 2. *By replacing n-twist ($n \geq 2$) by $(n+2)$-twist in the Conway symbol of an alternating knot K, the signature changes by -2 if the replacement is made in a parallel twist with positive crossings, the signature changes by $+2$ if the replacement is made in a parallel twist with negative crossings, and remains unchanged if the replacement is made in an anti-parallel twist.*

Theorem 3. *The signature σ_K of an alternating knot K given by its Conway symbol is*

$$\sigma_K = \sum_P -2\left[\frac{n_i}{2}\right] c_i + 2c_0,$$

where the sum is taken over all parallel twists n_i, $c_i \in \{1, -1\}$ is the sign of crossings belonging to a parallel twist n_i, and $2c_0$ is an integer constant which can be computed from the signature of the generating knot.

The proof of this theorem follows directly form the preceding Lemma, claiming that only additions of twists in parallel twists in a Conway symbol result in the change of signature, and that by every such addition, signature changes by $-2c_i$. Notice that the condition that we are making twist replacements in the standard Conway symbols, i.e., Conway symbols with the maximal twists, is essential for computation of general formulae for the signature of alternating knot families.

[1]The Conway notation is called *unreduced* if 1's denoting elementary tangles in vertices are not omitted in symbols of polyhedral links.

K. Murasugi [11] proved the lower bound for the unknotting number of knots, $u(K) \geq \frac{|\sigma_K|}{2}$. Using this criterion, for many (sub)families of knots we can confirm that their BJ-unknotting numbers, i.e., unknotting numbers computed according to Bernhard-Jablan Conjecture, represent the actual unknotting numbers of these families.

Let be given the Conway symbol of a knot family F with parameters $p_1, p_2, \ldots, p_n$ denoting twists ($p_i > 1$, $i = 1, \ldots, n$). The polynomial with variables $p_1, p_2, \ldots, p_n$ and integer coefficients, obtained as the evaluation of the Conway polynomial of the family F for $x = 2i$, where i is the imaginary unit, will be called the *critical polynomial*.

Conjecture 4. *For every knot or link, the maximal degree of every variable in its critical polynomial is 1.*

If Conjecture 4 is true, following the changes of the sign of critical polynomial, we will be able to obtain general formulae for the signature of particular subfamilies of nonalternating knots and links and confirm their unknotting numbers estimated according to BJ-conjecture.

Except the above mentioned problems, we will consider a few other topics related to unknotting: ascending numbers [12, 7], knot distances [4], smoothing numbers (band unknotting numbers) [1], pseudodiagrams and their trivializing and knotting numbers [5, 6], as well as unknotting numbers related to virtual knots and links [10, 6].

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Detecting tightness through open book decompositions

Andy Wand

Let M be a closed, oriented 3-manifold. An open book decomposition of M is a fibration of the complement of an embedded link B (the *binding*) in M over S^1, such that each fibre (the *page*) is the interior of a Seifert surface for B. Open book decompositions of 3-manifolds provide a very useful topological framework for studying properties of contact structures. In particular, work of Giroux ([1]) has shown a 1-1 correspondence between isotopy classes of contact structures on M and stabilization-equivalence classes of open book decompositions. The starting point of this talk is the problem of determining how various properties of a contact structure are reflected in arbitrary corresponding open book decompositions.

The structure of the talk is as follows: we begin by describing how, given the data of an open book decomposition along with a collection of disjoint, properly embedded arcs in a page, to define a necessary condition for tightness of the supported contact structure, in some sense a generalization of the "right-veering" condition of Honda, Kazez, and Matić ([2]). We further show that, in contrast to right-veering, our condition is invariant under stabilization and destabilization, and composition of the monodromy of the open book with arbitrary positive Dehn twists. It follows then that the condition is indeed a sufficient condition for tightness, thus providing the first known algorithm for determining tightness from an arbitrary open book decomposition.

A particular application is that tightness of a closed contact 3-manifold is preserved under Legendrian surgeries on that manifold. This provides a much sought-after link between intrinsically 3-dimensional contact phenomena and those which are induced as the boundaries of 4-dimensional manifolds with particular geometric (e.g. Stein or symplectic) structures.

We also indicate an interpretation of the property as the vanishing of a certain homology. Conjecturally, this should be isomorphic to the 0-level filtration of the embedded contact homology of Hutchings (see the appendix of [3]), and should give some indication of how the grading on the embedded contact homology differential carries over to the various homology theories (Heegaard Floer and Monopole Floer in particular) to which it is isomorphic.

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The 2-variable Jones polynomial and the invariants from the Yokonuma–Hecke algebras

Sofia Lambropoulou

(joint work with Maria Chlouveraki)

We compare the knot invariant defined using the Juyumaya trace on the Yokonuma-Hecke algebras with the 2-variable Jones polynomial, and we show that they do not coincide except in a few trivial cases.

1. The 2-variable Jones or HOMFLYPT polynomial

Let B_n denote the Artin braid group of type A. Closing a geometric braid β with simple arcs gives rise to an oriented knot or link $\widehat{\beta}$ and, by the classical Markov theorem, oriented knot or link types are in bijective correspondence with equivalence classes of braids in $\cup_n B_n$ under the moves:
(i) Conjugation in each B_n: $\alpha\beta \sim \beta\alpha$;
(ii) Positive and negative stabilization: $\alpha \sim \alpha{\sigma_n}^{\pm 1}$, $\alpha \in B_n$.

Let $q \in \mathbb{C} \setminus \{0\}$. The *Iwahori-Hecke algebra* $\mathcal{H}_n(q)$ of type A is a $\mathbb{C}$-associative algebra with generators $G_1, G_2, \ldots, G_{n-1}$, which can be defined as the quotient of the algebra $\mathbb{C}B_n$ over the quadratic relations $G_i^2 = (q-1)G_i + q$ for all i. The algebras $\mathcal{H}_n(q)$ support the *Ocneanu trace* with parameter ζ [3, Theorem 5.1], a unique linear Markov trace $\tau : \bigcup_{n\geq 0} \mathcal{H}_n(q) \longrightarrow \mathbb{C}[\zeta]$. Another characterization of the Ocneanu trace can be given by its values on the basis elements G_{w_μ}, $\tau(G_{w_\mu}) = \zeta^{\ell(w_\mu)}$, where w_μ is a minimal length representative of the conjugacy class of $\mathfrak{S}_n$ parametrized by the partition μ of n [2], and $\ell(w_\mu)$ denotes the length of w_μ. After re-scaling and normalizing the Ocneanu trace according to the Markov theorem, Jones constructed the *2-variable Jones* or *HOMFLYPT polynomial* [3], an isotopy invariant of oriented links:

$$P(\widehat{\alpha}) = (D_{\mathcal{H}})^{n-1} (\sqrt{\lambda_{\mathcal{H}}})^{\epsilon(\alpha)}\, (\tau \circ \pi)\, (\alpha)$$

where $\alpha \in B_n$, for any $n \in \mathbb{N}$, $\pi : \mathbb{C}B_n \longrightarrow \mathcal{H}_n(q)$ is the natural algebra epimorphism that maps the braid generator σ_i to the algebra generator G_i, $\epsilon(\alpha)$ is the algebraic sum of the exponents of the braid generators in the braid word α, $\lambda_{\mathcal{H}} := \frac{\zeta+(1-q)}{q\zeta}$ and $D_{\mathcal{H}} := -\frac{1-\lambda_{\mathcal{H}} q}{\sqrt{\lambda_{\mathcal{H}}}(1-q)} = \frac{1}{\zeta\sqrt{\lambda_{\mathcal{H}}}}$.

2. Link invariants from the Yokonuma-Hecke algebras

The *Yokonuma-Hecke algebra* [7], $\mathrm{Y}_{d,n}(u)$, for $d \in \mathbb{N}$ and $u \in \mathbb{C}\backslash\{0\}$, is a $\mathbb{C}$-associative algebra with generators $g_1, \ldots, g_{n-1}, t_1, \ldots, t_n$, satisfying: the braid relations for the g_i's, the relations $t_i t_j = t_j t_i$ for all i, j, the modular relations $t_j^d = 1$ for all j, the mixed relations $t_j g_i = g_i t_{s_i(j)}$ for all i, j, where s_i denotes the transposition $(i, i+1)$, and the quadratic relations:

$$g_i^2 \quad = \quad 1 + (u-1)\, e_i + (u-1)\, e_i\, g_i \qquad \text{for all } i$$

where $e_i := \frac{1}{d}\sum_{s=0}^{d-1} t_i^s t_{i+1}^{-s}$, idempotents in $\mathrm{Y}_{d,n}(u)$.

The algebras $\mathrm{Y}_{d,n}(u)$ support the *Juyumaya trace* with parameters $z, x_1, \ldots, x_{d-1}$ in $\mathbb{C}\backslash\{0\}$ [4, Theorem 12], a unique linear Markov trace $\mathrm{tr} : \bigcup_{n\geq 0} \mathrm{Y}_{d,n}(u) \longrightarrow \mathbb{C}[z, x_1, \ldots, x_{d-1}]$. Trying to normalize and re-scale the Juyumaya trace according to the Markov braid equivalence, in order to obtain a link invariant, turns out to be impossible, basically because $\mathrm{tr}(\alpha e_n) \neq \mathrm{tr}(e_n)\mathrm{tr}(\alpha)$, unless the trace parameters $x_1, \ldots, x_{d-1}$ satisfy the so-called E-*system* [5, 6]:

$$\sum_{s=0}^{d-1} x_{m+s} x_{d-s} = x_m \sum_{s=0}^{d-1} x_s x_{d-s} \qquad (m = 1, \ldots, d-1)$$

where the sub-indices on the x_j's are regarded modulo d and $x_0 := 1$. As it is shown in [5] (in the Appendix by Paul Gérardin), the solutions of the E-system are parametrized by the non-empty subsets S of $\mathbb{Z}/d\mathbb{Z}$. The case $E := \mathrm{tr}\,(e_i) = 1$ leads to the "trivial" solution: x_1 a d-th root of unity and $x_m = x_1^m$ $(m = 1, \ldots, d-1)$.

Given a solution $X_S = \{\mathrm{x}_1, \ldots, \mathrm{x}_{d-1}\}$ of the E-system, a link isotopy invariant Δ_S was defined in [6], depending on the variables u, z:

$$\Delta_S(\widehat{\alpha}) = D_{\mathrm{Y}}^{n-1}(\sqrt{\lambda_{\mathrm{Y}}})^{\epsilon(\alpha)}\,(\mathrm{tr} \circ \delta)\,(\alpha),$$

where $\alpha \in B_n$, for any $n \in \mathbb{N}$, $\delta : \mathbb{C}B_n \longrightarrow \mathrm{Y}_{d,n}(u)$ is the natural algebra homomorphism that maps the braid generator σ_i to the algebra generator g_i, $\epsilon(\alpha)$ is the sum of the exponents of the braid generators in the braid word α, $\lambda_{\mathrm{Y}} := \frac{z+(1-u)E}{uz}$, and $D_{\mathrm{Y}} := \frac{1-\lambda_{\mathrm{Y}} u}{\sqrt{\lambda_{\mathrm{Y}}}(1-u)E} = \frac{1}{z\sqrt{\lambda_{\mathrm{Y}}}}$.

3. Comparing P and Δ_S

It is natural to ask how the invariants P and Δ_S compare. Computational data so far do not indicate that one invariant is stronger than the other. Thus, it is possible that the two invariants are topologically equivalent. For comparing the two invariants we would like to introduce a given solution of the E-system as early in the construction as possible. Therefore, in [1] we first consider the *specialization map* $\theta : \mathbb{C}[z, x_1, \ldots, x_{d-1}] \longrightarrow \mathbb{C}[z]$, given by $z \mapsto z$ and $x_m \mapsto \mathrm{x}_m$ $(m = 1, \ldots, d-1)$, for $\mathrm{x}_1, \ldots, \mathrm{x}_{d-1} \in \mathbb{C} \setminus \{0\}$. Then we consider the *specialized Juyumaya trace* $\theta \circ \mathrm{tr}$ with parameter z and we show that, as in the case of the Ocneanu trace, this is uniquely determined by its values on the elements g_{w_μ}, $\mathrm{tr}(g_{w_\mu}) = z^{\ell(w_\mu)}$. We show this by considering the linear map $\varphi : \bigcup_{n\geq 0} \mathrm{Y}_{d,n}(u) \longrightarrow \bigcup_{n\geq 0} \mathrm{Y}_{d,n}(u)$ defined inductively on $\mathrm{Y}_{d,n}(u)$, for all $n \in \mathbb{N}$, by the following rules:

$$\begin{array}{rcl} \varphi(1) & = & 1 \\ \varphi(w_n g_n g_{n-1} \cdots g_i t_i^k) & = & g_n \varphi(w_n g_{n-1} \cdots g_i t_i^k) \\ \varphi(w_n t_{n+1}^k) & = & \mathrm{x}_k \varphi(w_n) \end{array}$$

where $w_n \in \mathrm{Y}_{d,n}(u)$, $\mathrm{x}_1, \ldots, \mathrm{x}_{d-1} \in \mathbb{C} \setminus \{0\}$, $\mathrm{x}_0 = 1$ and $k \in \mathbb{Z}/d\mathbb{Z}$. Then we show that $\mathrm{tr} \circ \varphi = \theta \circ \mathrm{tr}$ and that $\varphi(\mathrm{Y}_{d,n}(u))$ is the $\mathbb{C}$-linear subspace of $\mathrm{Y}_{d,n}(u)$ spanned by the elements g_{w_μ}.

Mapping now $g_i \mapsto G_i$ and $t_i^m \mapsto \mathrm{x}_m$ $(m = 1, \ldots, d-1)$ does not define an algebra homomorphism between $\mathrm{Y}_{d,n}(u)$ and $\mathcal{H}_n(u)$, unless we are in the case $E = 1$. In this case we can then show that P and Δ_S coincide.

The map φ provides us with the earliest possible specialization to a solution of the E-system during the construction of Δ_S. So, we proceed in [1] with comparing the invariants P and Δ_S as maps on isotopy classes of knots and links. Forcing $P = \Delta_S$ on the identity braid 1 in each B_n we deduce $D_{\mathcal{H}} = D_{\mathrm{Y}}$. Then, taking $\alpha = \sigma_1^2$, $\alpha = \sigma_1^3$ and $\alpha = \sigma_1\sigma_2^2\sigma_1\sigma_2^2$ we find necessary conditions for the maps P and Δ_S to coincide. Namely, when $u = 1$ or $q = 1$ or $E = \mathrm{tr}(e_i) = 1$. Then we prove that these are also sufficient conditions for the maps P and Δ_S to coincide on any braid α. Our proof is by induction on $\nu(\alpha) := |\,$sum of all negative exponents of the braid generators in $\alpha\,|$. For the first step ($\nu(\alpha) = 0$), we proceed by induction on n on the braid monoid B_n^+, see [1] for details. To recapitulate, in [1] we prove the following.

Theorem 1. *Let X_S be a solution of the* E*-system. Let* tr *be the Juyumaya trace on $\mathrm{Y}_{d,n}(u)$ with parameters z, X_S, and let τ be the Ocneanu trace on $\mathcal{H}_n(q)$ with parameter ζ. Let $E = \mathrm{tr}(e_i)$ for all $i = 1, \ldots, n-1$. Then $P = \Delta_S$ or, more generally, P is a scalar multiple of Δ_S if and only if we are in one of the cases portrayed in the following tables:*

Case	q	ζ	u	z	E
1	1	z	1	$\mathbb{C}^*$	*any*
2	1	$-z$	1	$\mathbb{C}^*$	*any*
3	$\mathbb{C}^*$	q	1	1	*any*
4	$\mathbb{C}^*$	q	1	-1	*any*
5	$\mathbb{C}^*$	-1	1	1	*any*
6	$\mathbb{C}^*$	-1	1	-1	*any*
7	1	E	$\mathbb{C}^*$	$-E$	*any*

Case	q	ζ	u	z	E
8	1	$-E$	$\mathbb{C}^*$	$-E$	*any*
9	$\mathbb{C}^*$	q	$\mathbb{C}^*$	-1	1
10	$\mathbb{C}^*$	q	$\mathbb{C}^*$	u	1
11	$\mathbb{C}^*$	-1	$\mathbb{C}^*$	-1	1
12	$\mathbb{C}^*$	-1	$\mathbb{C}^*$	u	1
13	u	z	$\mathbb{C}^*$	$\mathbb{C}^*$	1
14	$1/u$	$-z/u$	$\mathbb{C}^*$	$\mathbb{C}^*$	1

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Seiberg-Witten maps and stable homotopy

Stefan Bauer

Consider the space of all compact non-linear perturbations of a fixed linear Fredholm operator L satisfying a boundedness condition: Preimages of bounded sets are supposed to be bounded. Suitably topologized, this space will have the weak homotopy type of the infinite loop space

$$\Omega^\infty \Sigma^\infty (S^{-ind(L)}).$$

The Seiberg-Witten map for a closed four-manifold satisfies the necessary boundedness conditions and thus defines a map of spectra: From the Thom spectrum of the virtual index bundle of the Dirac operator over the Picard group of the manifold to the sphere spectrum. In this way one obtains a lift of what is known as the Bauer-Furuta invariant [1] from the homotopy category of spectra to the category of spectra. An extension of this construction to compact four-manifolds with boundary a finite union of rational homotopy three-spheres was announced. From this extension one can conclude that the 11/8 conjecture holds for spin four-manifolds with finite fundamental group.

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Recognizing trivial links in polynomial time

Chad Musick

We let a link be a family of circles disjointly embedded in $\mathbb{R}^3 \cup \{\infty\}$. Trivial links are those links embedded such that the 3-dimensional space may be continuously deformed so that the resultant link is embedded as a subset of $\mathbb{R}^2$. We give a method to decide, for any link L, whether or not L is trivial. This method is polynomial in complexity.

We proceed by developing two new measures of complexity, the *crumple* and the *sentence length*. To define these, we restrict our attention to links embedded in a restricted fashion as described below.

Let L be a link. If we may find a finite set S of concentric spheres of positive integer radius such that the closure of the portion of L lying in $(\mathbb{R}^3 - S) \cup \{\infty\}$ is a set of disjoint line segments each lying along a line through the common center of the members of S, we say that the link is a *tar link*.

Given a tar link L with a set S of spheres as described above, we let the crumple of L be the sum of the lengths of the portions of L lying in the complement of S. By choosing a tar link in which all of the members of the intersection of L and the complement of S lie in the plane $\mathbb{R}^2$, we may write a descriptive sentence of the embedding of L. The length of this sentence will depend upon the number of line segments in the complement of S and on the number of mutual intersections between S, L, and $\mathbb{R}^2$.

We give a method to construct a sentence from any link diagram. The initial length of this sentence will be a linear multiple of the number of crossings in the diagram. By its construction, the crumple of the resulting link will be at worst quadratic on the number of crossings. As well, the number of crossings in any diagram of a sentence will be at most quadratic on the length of the sentence. This gives us an upper bound on the complexity of the diagrams as a function of the length of the descriptive sentence.

As a Lemma, we show that given a trivial link and a descriptive sentence for a tar link equivalent to this link – and so also trivial – there is a simple isotopy occurring entirely within one of the spheres associated to the link such that the resultant link has a strictly smaller crumple. This gives an immediate algorithm to determine whether or not a link is trivial.

Knowing this Lemma, we then prove that there is a strictly monotonic sequence on a 5-tuple describing a link. Each of the measures of this tuple is at worst quadratic on the number of crossings of the diagram used to generate the initial sentence, and so on the number of crossings of the original diagram. We show that there are a polynomial number of attempts needed before a move may be found that results in a strictly lower tuple from a trivial link. Taken together, these moves constitute an algorithm to decide whether or not a link is trivial. Those links that are trivial will produce a sequence of diagrams proceeding stepwise to a trivial diagram. Those links that are not trivial will terminate when none of the polynomial number of possible moves produces an improvement in the complexity tuple.

We demonstrate the use of the algorithm on two trivial knot diagrams and give a complete description of the construction of the sentence and of the algorithm.

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How to Fold a Manifold

J. Scott Carter

A classical result of J.W. Alexander states that any $k+2$ dimensional oriented compact connected manifold can be obtained as a simple branched cover of the $(k+2)$-sphere with branch set an embedded k-dimensional manifold. Other important results include those of Hilden and Montesinos which indicate that a 3-dimensional manifold is a simple 3-fold branched cover of the 3-sphere, and that of Iori and Piergallini that states that a 4-dimensional manifold can be obtained as a simple 5-fold branched cover of the 4-sphere branched along am embedded surface in the 4-sphere.

Our result (Carter and S. Kamada) is that we can embedded 2-fold simple branched covers of the 2,3, and 4-sphere branched along a linked oriented manifold in the sphere times a 2-disk in such a way that the projection to the sphere is the covering map. Similarly, we can immerse the 3-fold branched covers in the same space. Covers with higher branching indices can often be embedded. In this way, we are initiating a theory of braids for knotted 3-manifolds in 5-space and knotted 4-manifolds in 6-space.

Gluck twisting 4-manifolds with odd intersection form

Selman Akbulut

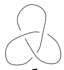
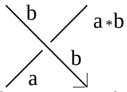
Given an imbedding of $F_g \subset M^4$, where F_g is a surface of genus g, I discussed the question of when (and if) you get an exotic copy of M by twisting M along F_g (when $g = 0$ this operation is called Gluck twisting). In particular, I will discuss a recent theorem about Gluck twisting proved jointly with Yasui, which says that Gluck twisting 4-manifold M with odd intersection form, along a 2-sphere S does not change the smooth structure of M under mild conditions on the homology class of S (e.g. when it is homologous to zero).

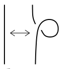
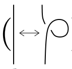
Distributivity versus associativity in the homology theory of algebraic structures

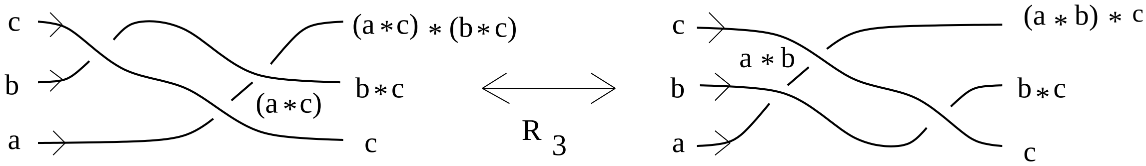
Józef H. Przytycki

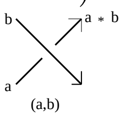
1. Introduction

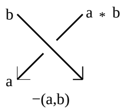
While homology theory of associative structures, such as groups and rings, has been extensively studied in the past beginning with the work of Hurewicz, Hopf, Eilenberg, and Hochschild, the non-associative structures, such as racks or quandles, were neglected until recently. The distributive structures have been studied for a long time and even C.S. Peirce in 1880 emphasized the importance of (right) self-distributivity in algebraic structures. However, homology for such universal algebras was introduced only between 1990 and 1995 by Fenn, Rourke, and Sanderson. We develops theory in the historical context and propose a general framework to study homology of distributive structures. We outline potential relations to Khovanov homology and categorification, via Yang-Baxter operators. We use here the fact that Yang-Baxter equation can be thought of as a generalization of self-distributivity.

1.1. **Invariants of arc colorings.** Consider a link diagram D, say , and a finite set X. We may define a diagram invariant as the number of colorings of arcs of D by elements of X, $col_X(D)$. Even such a naive definition leads to a link invariant $col_X(L) = min_{D\in L} col_X(D)$, where $D \in L$ means that D is a diagram of L.[1]. More sensible approach would start with magma $(X;*)$ with the coloring convention 

, Again, let for a finite X, $col_X(D)$ denote the number of colorings of arcs of D by elements of X, according to the above convention at every crossing. We can define an oriented link invariant by considering $col_X(L) = min_{D\in L} col_X(D)$. Alternatively, we can minimize $col_X(D)$ over minimal crossing diagrams of L only. Such an invariant would be very difficult to compute so it is better to look for properties of $(X;*)$ so that $col_X(D)$ is invariant under Reidemeister moves: R_1 , gives idempotency relation $a * a = a$. R_2 () forces $*$ to be an invertible operation and the third move is illustrated in detail below:



The magma $(X;*)$ satisfying all three conditions is called a quandle, the last two – a rack, and only the last condition – a shelf or RDS. Thus, if $(X;*)$ is a quandle then $col_X(D)$ is a link invariant. We can do more (after Carter-Kamada-Saito). We can sum over all crossings pairs $\pm(a,b)$ according to the convention 



; the investigation of invariance of $\sum \pm(a,b)$ under Reidemeister moves was a hint toward construction of (co)homology of quandles.

Let us now compare homology for associative structures (semigroups) with that for distributive structures (shelves).

1.2. **Group homology of a semigroup.** Let $(X,*)$ be a semigroup. We define a chain complex $\{C_n, \partial_n\}$ as follows: $C_n(X) = \mathbb{Z}X^n$ and $\partial_n = \sum_{i=0}^{n}(-1)^i d_i$ (alternating sum of face maps). Where $d_0(x_1,...,x_n) = (x_2,...,x_n)$,
$d_i(x_1,...,x_n) = (x_1,...,x_{i-1}, x_i * x_{i+1}, x_{i+2},...,x_n)$, for $0 < i < n$ and
$d_n(x_1,...,x_n) = (x_1,...,x_{n-1})$.
We check that $d_i d_j = d_{j-1} d_i$ for $i < j$. The sequence of groups C_n $(n \geq 0)$ with maps $d_i : C_i \to C_{i-1}$, which satisfies equalities $d_i d_j = d_{j-1} d_i$ for $0 \leq i < j \leq n$ is called a presimplicial group (or presimplicial Z-module).

1.3. **Hochschild homology of a semigroup.** Let $(X;*)$ be a semigroup. We define a Hochschild chain complex $\{C_n, \partial_n\}$ as follows $C_n(X) = \mathbb{Z}X^{n+1}$ and the Hochschild boundary $\partial_n = \sum_{i=0}^{n}(-1)^i d_i$, where $d_i(x_0,...,x_n) = (x_0,...,x_{i-1}, x_i *$

[1]One can say that it is a nonsense but an invariant nontrivial: $col_X(L) = |X|^{2cr(L)+t(L)}$, where $cr(L)$ is the crossing number of L and $t(L)$ the number of trivial components in L.

$x_{i+1}, x_{i+2}, ..., x_n)$, for $0 \le i < n$ and $d_n(x_0, ..., x_n) = (x_n * x_0, ..., x_{n-1})$.
Again, (C_n, d_i) is a presimplicial module. If $(X, *)$ is a monoid, one can define $n+1$ homomorphisms $s_i : C_n \to C_{n+1}$, called degeneracy maps, by $s_i(x_0, ..., x_n) = (x_0, ..., x_i, 1, x_{i+1}, ..., x_n)$ (similarly, in the case of group homology of a monoid, we put, $s_i(x_1, ..., x_n) = (x_1, ..., x_i, 1, x_{i+1}, ..., x_n))$. We check that in both cases the following conditions hold:

$$
\begin{aligned}
&(1) \quad d_i d_j = d_{j-1} d_i \ for \ i < j, \\
&(2) \quad s_i s_j = s_{j+1} s_i, \quad 0 \le i \le j \le n, \\
&(3) \quad d_i s_j = \begin{cases} s_{j-1} d_i & \text{if } i < j, \\ s_j d_{i-1} & \text{if } i > j+1, \end{cases} \\
&(4) \quad d_i s_i = d_{i+1} s_i = Id_{C_n}.
\end{aligned}
$$

(C_n, d_i, s_i) satisfying conditions (1)-(4) above is called a simplicial module (Z-module in our case). If we replace (4) by a weaker condition $d_i s_i = d_{i+1} s_i$ we deal with a weak simplicial module, the concept useful in the theory of homology of distributive structures (spindles or quandles).

1.4. **Homology of distributive structures.** Recall that a shelf $(X; *)$ is a set X with a right self-distributive binary operation $* : X \times X \to X$ (i.e. $(a * b) * c = (a * c) * (b * c)$). We define a (one-term) distributive chain complex $C_n, \partial_n)$ as follows: $C_n = \mathbb{Z}X^{n+1}$ and the boundary operation $\partial_n^{(*)} = \sum_{i=0}^{n} (-1)^i d_i$, where $d_i(x_0, ..., x_n) = (x_0 * x_i, ..., x_{i-1} * x_i, x_{i+1}, ..., x_n)$.
(C_n, d_i) is a presimplicial module. If we define $s_i : C_n \to C_{n+1}$ by $s_i(x_0, ..., x_n) = (x_0, ..., x_{i-1}, x_i, x_i, x_{i+1}, ..., x_n)$, then (C_n, d_i, s_i) satisfies conditions (1)-(3) of simplicial module, and if $(X; *)$ is a spindle ($a * a = a$), then (C_n, d_i, s_i) is a weak simplicial module and degenerate homology (not necessarily trivial) can be defined.

If $(X; *)$ is a rack then the complex $(C_n^{(*)}, \partial^{(*)})$ is acyclic, but in the general case of a shelf or spindle homology can be nontrivial with nontrivial free and torsion part (joint work with A. Crans, K. Putyra and A. Sikora [1, 5, 6]).

1.5. **Multi-term distributive homology.** One generalize one term distributive homology as follows. Let X be a set and $Bin(X)$ the set of all binary operations on X (in fact a monoid with composition $a *_1 *_2 b = (a *_1 b) *_2 b$ and $a *_0 b = a$). We say that $S \subset Bin(X)$ is a distributive set if for any pair $*_1, *_2 \in S$ we have $(a *_1 b) *_2 c = (a *_2 c) *_1 (b *_2 c)$. If $(*_1, ..., *_k)$ is a distributive set, we define a multiterm chain complex by taking any linear combination of $\partial^{(*_i)}$, that is $(C_n, \partial^{(a_1,...,a_k)})$ is defined by $C_n = ZX^{n+1}$ and $\partial^{(a_1,...,a_k)} = a_1 \partial^{(*_1)} + ... + a_k \partial^{(*_k)}$, where $a_i \in Z$. We computed with K. Putyra [5] various multiterm homology, including that for finite distributive lattices (including Boolean algebras). Also multiterm homology based on the distributive embedding of G to $Bin(G)$, given by Greg Mezera, where $g \to *_g$ with $a *_g b = ab^{-1}gb$ is of great interest.

1.6. **From distributivity homology to Yang-Baxter homology.** For a given Yang-Baxter operator we attempt to find presimplicial module using graphical presentation of the (co)presimplicial category Δ_{pre}^{op}, from which homology will be derived. The figure below illustrate various graphical interpretation of the generating morphism d_i of a presimplicial category Δ^{op}_{pre}. They are loosely related to homology of set-theoretical Yang-Baxter equation of Carter-Kamada-Saito and Fenn, and to homology of Yang-Baxter equation of Eisermann [2, 3]. We should also acknowledge stimulating observations by Ivan Dynnikov.

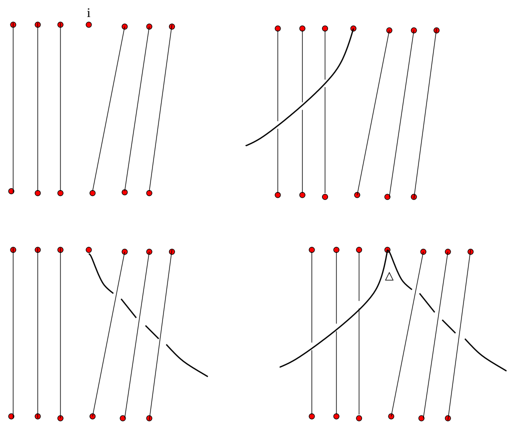


Figure 1. Various interpretation of the graphical face map d_i

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Spinal open books and algebraic torsion in contact 3-manifolds

Chris Wendl

(joint work with Janko Latschev, Samuel Lisi, Jeremy Van Horn-Morris)

It is a long-standing conjecture[1] in 3-dimensional contact topology that if (M, ξ) is any tight contact manifold and (M', ξ') is obtained from (M, ξ) by Dehn surgery along a Legendrian knot with framing -1 relative to the canonical contact framing, then (M', ξ') is also tight. Since Legendrian surgery can be viewed as the

[1]During the same workshop, Andy Wand announced a proof of this conjecture using open book decompositions and a new characterization of tightness in terms of the monodromy map.

attachment of a Weinstein 2-handle and thus gives rise to a Stein cobordism from (M, ξ) to (M', ξ'), it is immediate that it always preserves symplectic fillability; the latter implies tightness, but the converse is not true. The following joint result with Janko Latschev shows that there is a great deal of interesting structure within the class of contact manifolds that are tight but not fillable.

Theorem 1 ([1]). *There exists a nested infinite sequence of classes of closed contact 3-manifolds*

$$\{\textit{all}\} = \Xi_0 \supset \Xi_1 \supset \Xi_2 \supset \ldots \supset \Xi_\infty \supset \{\textit{symplectically fillable}\}$$

with the following properties:

(1) *For each $k = 0, 1, 2, \ldots, \infty$, Ξ_k is preserved by Legendrian surgery,*
(2) *Ξ_1 contains all closed contact 3-manifolds that are tight.*

It is possible that Ξ_1 in this result may be *precisely* the class of tight contact 3-manifolds: this would be equivalent to the statement that a closed contact 3-manifold has vanishing contact homology if and only if it is overtwisted. That would of course imply a new proof of the conjecture about surgery and tightness, but as yet no one knows how to prove that vanishing contact homology implies overtwistedness. Regardless, one can interpret the theorem as saying that there is an infinite hierarchy of "degrees of tightness," in which some tight contact manifolds are tighter than others. There are examples to show that the inclusions $\Xi_k \hookrightarrow \Xi_{k-1}$ are all proper (see below). There are also candidates that might belong to Ξ_∞ without being fillable, but no proof of this as yet.

The classes Ξ_k are defined in terms of a contact invariant that lives in the *Symplectic Field Theory* outlined by Eliashberg, Givental and Hofer. The version of SFT we are interested in is, roughly speaking, the homology of a chain complex

$$H_*^{\mathrm{SFT}}(M, \xi) := H_*\left(\mathcal{A}[[\hbar]], D_{\mathrm{SFT}}\right),$$

where $\mathcal{A}$ is a free graded commutative algebra with unit, with generators q_γ corresponding to closed Reeb orbits γ on (M, ξ), $\hbar$ is a formal variable, and

$$D_{\mathrm{SFT}} : \mathcal{A}[[\hbar]] \to \mathcal{A}[[\hbar]]$$

is a linear operator that encodes a count of rigid pseudoholomorphic curves in the symplectization of (M, ξ), with cylindrical ends asymptotic to closed Reeb orbits. This homology is functorial with respect to exact symplectic cobordisms: in particular, whenever there exists an exact cobordism from (M_-, ξ_-) to (M_+, ξ_+), it induces an $\mathbb{R}[[\hbar]]$-module morphism

$$H_*^{\mathrm{SFT}}(M_+, \xi_+) \to H_*^{\mathrm{SFT}}(M_-, \xi_-)$$

which maps $[\hbar^k] \mapsto [\hbar^k]$ for all $k \geq 0$. This property is especially useful in light of the following hypothetical example: suppose that for some choice of suitable data, the symplectization of (M, ξ) contains only one rigid pseudoholomorphic curve, which has genus 0, no negative ends and positive ends approaching a set of k distinct Reeb orbits $\gamma_1, \ldots, \gamma_k$. Then it turns out that

$$D_{\mathrm{SFT}}\left(q_{\gamma_1} \ldots q_{\gamma_k}\right) = \hbar^{k-1},$$

hence the element $[\hbar^{k-1}]$ vanishes in $H_*^{\mathrm{SFT}}(M,\xi)$. Among other things, this implies that (M,ξ) cannot have an exact symplectic filling, as $[\hbar^k]$ is nontrivial in $H_*^{\mathrm{SFT}}(\emptyset)$ for every $k \geq 0$.

With this example in mind, we define a numerical invariant, called the *order of algebraic torsion*, by

$$\mathrm{AT}(M,\xi) = \sup\{k \geq 0 \mid [\hbar^{k-1}] \neq 0 \in H_*^{\mathrm{SFT}}(M,\xi)\} \in \{0,1,2,\ldots,\infty\},$$

and then define (M,ξ) to be in Ξ_k if and only if $\mathrm{AT}(M,\xi) \geq k$. It follows immediately from the properties described above that if $(M,\xi) \in \Xi_k$ and there is an exact cobordism from (M,ξ) to (M',ξ'), then $(M',\xi') \in \Xi_k$; in particular this is true whenever the latter is obtained from the former by Legendrian surgery, since Stein cobordisms are always exact.

To find interesting examples of contact 3-manifolds with varying orders of algebraic torsion, one needs a source of existence and uniqueness results for holomorphic curves. One such source is the relationship between Lefschetz fibrations and a certain generalization of open book decompositions. To motivate the idea, consider a Lefschetz fibration $\Pi : E \to \Sigma$ whose base and generic fiber are both compact oriented surfaces with nonempty boundary. The boundary of E then has two smooth faces $\partial E = \partial_h E \cup \partial_v E$, which each inherit fibrations

$$\partial_h E := \bigcup_{z\in\Sigma} \partial E_z \to \Sigma$$

$$\partial_v E := E|_{\partial\Sigma} \to \partial\Sigma \cong S^1 \sqcup \ldots \sqcup S^1.$$

In the simplest example, if Σ is a disk, then the resulting decomposition of ∂E can be viewed as an open book decomposition, with $\partial_h E$ as a tubular neighborhood of the binding.

More generally, we define a *spinal open book* on a closed oriented 3-manifold M to be a decomposition of the form $M = M_\Sigma \cup M_P$, where the two pieces (called the *spine* and *paper* respectively) both come with fibrations: the fibers of $M_P \to S^1$ are surfaces with boundary, the connected components of which are called *pages*, while fibers of $M_\Sigma \to \Sigma$ are circles, with Σ being a compact oriented surface with boundary, the connected components of which are called *vertebrae*. We require additionally that the boundary of every page should be a disjoint union of fibers of the spine. We then say that a contact structure ξ is *supported by* the spinal open book if it admits a contact form α such that $d\alpha$ is positive on the interiors of all pages and the fibers on M_Σ are closed Reeb orbits.

It's important to note that in these definitions, neither Σ nor the fibers of $M_P \to S^1$ need be connected, so there can be multiple vertebrae and multiple families of pages with varying topological types, though in the specific example of the boundary of a Lefschetz fibration, this does not happen. Indeed, most spinal open books cannot be boundaries of Lefschetz fibrations, and this observation becomes very powerful in light of the following joint result with Sam Lisi and Jeremy Van Horn-Morris:

Theorem 2 ([2]). *Suppose (M,ξ) is supported by a spinal open book containing a page of genus 0. Then the symplectic fillings of (M,ξ) (up to symplectic deformation equivalence) are in one-to-one correspondence with the Lefschetz fibrations (up to diffeomorphism) that restrict to the given spinal open book at their boundaries.*

This generalizes a result in [3] which covers the case $\Sigma = \mathbb{D}^2$. The main reason such results hold is that spinal open books always give rise to holomorphic curves: in general, one can choose data such that each genus g page of a spinal open book lifts to a holomorphic curve of index $2-2g$ in the symplectization, so for the case $g=0$, these curves have the "expected" dimensions and can be extended to a family of holomorphic curves foliating any given filling. Alternatively, the genus 0 pages in the symplectization can be counted in order to compute SFT, giving rise to the following:

Theorem 3 ([2]). *Suppose (M,ξ) is supported by a spinal open book containing a page of genus 0 with $k+1$ boundary components, and also a page of positive genus. Then $[\hbar^k] = 0 \in H_*^{SFT}(M,\xi)$.*

The simplest examples are S^1-invariant contact structures on manifolds of the form $S^1 \times \Sigma$, where Σ is a closed oriented surface. In this case the surfaces $\{*\} \times \Sigma$ are convex, and the above theorem applies for instance if the dividing set consists of $k+1$ curves cutting Σ into two pieces, of which one has genus 0 and the other does not. It is shown in [1] that this contact manifold satisfies AT $= k$, hence it belongs to $\Xi_k \setminus \Xi_{k+1}$.

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Vassiliev invariants for virtual knots

Heather A. Dye

I construct degree one, finite type invariants for virtual knots. This method focuses on the idea of partitioning the crossings in a virtual knot diagram into sets. The sets of crossings are not invariant under the Reidemeister moves, but the sum of the signs of the crossings in the set (the signed cardinality of the set) is an invariant of the knot.

Let K_+ denote a knot diagram with a positive crossing c. Correspondingly, let K_- denote a knot diagram identical to K_+ except at a crossing c – where the over passing edge is switched to form a negative crossing. Let $K_\bullet$ denote a knot diagram identical to K_+ except that the crossing c is singular. A finite type invariant $\mathcal{V}$ [4] [2] has the property that

$$\mathcal{V}(K_\bullet) = \mathcal{V}(K_+) - \mathcal{V}(K_-).$$

More generally, let X be a tuple of k singular crossings in the knot diagram K. Let $\sigma(X)$ denote a resolution of each crossing as a positively or negatively signed crossing and let $|\sigma|$ denote the sum of negative crossings in $\sigma(X)$. Summing over all possible resolutions:

$$\mathcal{V}(K_X) = \sum_{\sigma} (-1)^{|\sigma|} \mathcal{V}(K_{\sigma(X)}).$$

The invariant $\mathcal{V}$ is said to have finite type $\leq n$ if $\mathcal{V}(K_X) = 0$ for all tuples of length $n+1$ or greater.

Henrich introduced a degree one, finite type invariant called the gluing invariant in [5]. Let K be a virtual knot diagram. Let K_c denote the virtual knot diagram obtained from K by gluing the crossing c (making the crossing a singularity) and let K_cO denote the diagram obtained by attaching a small unknot near c with a singularity. Let $F(K)$ denote the flat projection of the knot K. The gluing invariant is a formal sum of equivalence classes of flat virtual knots with one singular crossing.

$$\mathcal{G}(K) = \sum_{c \in K} [F(K_c) - F(K_cO)].$$

(Note that if the crossing c is involved in a Reidemeister I move, then $[F(K_c) - F(K_cO)]$ is the empty diagram.) By extending the flat classical and virtual Reidemeister moves to include one singular crossing, the formal sum $\mathcal{G}(K)$ becomes an invariant of virtual knots and a universal, degree one, finite type invariant.

We compute the weight of a chord in a Gauss diagram. The weights partition the chords into sets which can be used to define finite type invariants of the Gauss diagrams. A Gauss diagram G is a clockwise oriented circle with a collection, C, of oriented, signed chords. Equivalence classes of Gauss diagrams are determined by three types of moves: 1) the introduction or deletion of a single isolated chord, 2) the introduction of two adjacent (feet and head), oppositely signed chords, and 3) the (2,1) and (3,0) triangle moves with appropriately selected signs. (Although my orientation conventions are slightly different, the set of chord diagram moves is introduced in [8]. In this paper, Turaev determines the minimum set of moves required to generate the equivalence classes of Gauss diagrams.) We construct a map $P\colon C \to Z$. Let c denote a chord and let N_c be the set of chords that intersect c. The oriented intersection number of the chord c with the chord x is denoted $int_c(x)$. Then

$$p(c) = \sum_{x \in N_c} int_c(x) sgn(x).$$

The value of $p(c)$ is the weight of the chord. For each integer i, we define $A_i(G) = \{c \,|\, p(c) = i\}$, the set of chords with weight i. For a positive integer, we define $V_j(G) = \{c \,|\; |p(c)| = j\}$, the set of chords with weight j or $-j$. The signed cardinality of these sets are integers that are denoted as $|A_i(G)|$ and as $|V_j(G)|$ respectively.

$$|A_i(G)| = \sum_{c \in A_i(G)} sgn(c) \text{ and } |V_j(G)| = \sum_{c \in V_j(G)} sgn(c).$$

We prove that for any non-zero integer i and any positive integer j, $|A_i(G)|$ and $|V_j(G)|$ are invariant under the Gauss diagram moves. The proof can be briefly sketched as follows. Chords involved in a single chord move have weight zero. Chords involved in a two chord move have the same weight, but have opposite signs. Their net contribution to the signed cardinality of a set is zero. The weight of chords is not changed by the triangle moves.

Virtual knots are equivalence classes of virtual knot diagrams that are in one to one correspondence with equivalence classes of Gauss diagrams [6]. Let K be a virtual knot diagram and let G_K be the corresponding Gauss diagram. We define $V_j(K)$ ($A_i(K)$) to be the set of crossings that correspond to chords in $V_j(G_K)$ that have parity j or $-j$ (respectively i). We define the gluing invariant $\mathcal{G}_j(K)$ to be a formal sum of flat virtual knot diagrams with one singular crossing. However, instead of gluing all crossings, we restrict our attention to crossings with particular weights.

$$\mathcal{G}_j(K) = \sum_{c \in V_j(K)} sgn(c)[F(K_c) - F(K_c O)].$$

This formal sum of flat diagrams with one singular crossing $\mathcal{G}_j(K)$ (modulo the slightly expanded Reidmeister move set) is a degree one Vassiliev invariant.

We can obtain a numerical invariant equivalent to $|V_j(K)|$ by mapping the formal sum to the integers and each non-empty diagram class, $[F(K_c) - F(K_c O)]$, is mapped to 1. (The value $|A_i(G_K)|$ is an invariant of virtual knots, but it is not a finite type invariant.) We remark that

$$\mathcal{G}(K) = \sum_{i=1}^{\infty} \mathcal{G}_i(K).$$

Taking products of the $\mathcal{G}_i(K)$ produces finite type invariants of higher degree. (The weights can also be extracted from the polynomial invariant introduced by Henrich in [5].)

Finite type invariants can be obtained from the Jones polynomial ($F_K(A)$) of the virtual knot K [6]. We expand $F_K(e^x)$ as a Taylor series centered at 0. The coefficient of x^k is a finite type invariant of degree k. We illustrate this with the following calculation. Let $K+$ ($K-$) denote a knot diagram with a positive (respectively negative) crossing. Let K_V and K_H denote a knot diagram with a vertical and horizontal smoothing respectively. Now $\langle K+ \rangle = A\langle K_H \rangle + A^{-1}\langle K_V \rangle$. After normalization:

$$F_{K+}(A) = -A^{-2} F_{K_V} - A^{-4} F_{K_H},$$
$$F_{K-}(A) = -A^{2} F_{K_V} - A^{4} F_{K_H}.$$

Hence

$$F_{K_\bullet}(A) = (-A^{-2} + A^2)(F_{K_V} - (-A^2 + A^{-2})F_{K_H}).$$

We see that the expansion of $F_{K_{\bullet\bullet}}(e^x)$ has the form $x^2 g(x)$ and and the coefficient of x is zero. Thus the coefficient of x^k is a Vassilliev invariant of degree k [6].

The normalized arrow polynomial, $W_K(A, K_i)$, [3] is an enhanced version of the Jones polynomial that *tracks* the planarity of a knot diagram using the variables K_i. The normalized arrow polynomial $W_{K_\bullet}$ factors in a similar fashion to $F_K(A)$ and the planarity variables K_i are invariant under Reidemeister moves. As a result, if we form the Taylor series of $W_K(e^x, K_i)$ at centered at zero, we can obtain Vassiliev invariants of degree 1 and higher. The coefficients of the monomials with the form $x^i K_j$ in the arrow polynomial $W_K(e^x)$ expanded as a Taylor series at 0 are Vassiliev invariants of degree i. Computations suggest that there is a relationship between the coefficients of $W_K(A, K_i)$ and $|V_i(K)|$.

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Surface bundles, Lefschetz fibrations, and their (multi)sections

Refik Inanc Baykur

Surface bundles and Lefschetz fibrations over surfaces constitute a rich source of examples of smooth, symplectic, and complex manifolds. Their sections and multisections carry interesting information on the smooth structure of the underlying four-manifold. In this talk we will discuss several problems and recent results on surface bundles, Lefschetz fibrations, and their (multi)sections, which we will tackle, for the most part, using various mapping class groups of surfaces. Conversely, we will use geometric arguments to obtain some structural results for mapping class groups.

On a cohomology theory for colored tangles

Carmen Caprau

1. Introduction

In [6] Khovanov constructed a cochain complex associated to an oriented framed link whose components are labelled by irreducible representations of $U_q(sl(2))$. The graded Euler characteristic of the homology of this complex is the colored Jones polynomial. Specifically, [6] provides a categorification of the colored Jones polynomial by interpreting the defining formula for the polynomial

$$J_n(K) = \sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} (-1)^i \left(\begin{array}{c} n-i \\ i \end{array} \right) J(K^{n-2i}),$$

where K^j is the j-parallel cable of the knot K, as the Euler characteristic of a complex whose objects require the original Khovanov homology [5] of the cablings K^{n-2i}, for $i = 0, \dots, \lfloor \frac{n}{2} \rfloor$. This construction works over $\mathbb{Z}_2$, due to the sign ambiguity in the functoriality property of the original Khovanov homology.

A similar approach to constructing homology theories for colored links was proposed by Mackaay and Turner [7], and independently by Beliakova and Wehrli [2], by exploring ideas of Bar-Natan [1]. In both constructions one needs to juggle with the sign ambiguity in the functoriality property of the employed Khovanov-type homology theories.

The goal of this talk is two-folded. First we improve the existing categorifications of the colored Jones polynomial by giving a clean definition of the colored invariant of a knot. For that, we employ the universal $sl(2)$ *foam cohomology theory* using foams (seamed 2-cobordisms) constructed by the author in [4] (see also [3]). Moreover, we construct a *local colored cohomology theory*, in that it is built with colored tangles in mind, and which "composes" well under tangle composition, leading to efficient computations of the colored invariants of a knot.

The construction in [4] is defined over the ring $R = \mathbb{Z}[i, a, h]$, and it involves *webs* and dotted *foams* modulo local relations. The resulting Khovanov-type cohomology theory is properly functorial with respect to tangle or link cobordisms with no sign indeterminacy. Denote by $Kof_{/h}$ the homotopy category of complexes over dotted foams modulo certain local relations. Given a diagram D of an oriented tangle T, the author constructed a formal complex $[D]$ whose isomorphism class in $Kof_{/h}$ is an invariant of T. If $C \subset \mathbb{R}^3 \times [0, 1]$ is a tangle cobordism between tangles T_1 and T_2, then there is an induced map $[T_1] \to [T_2]$, well-defined under ambient isotopy of C (rel. boundary). The category $Kof_{/h}$ has a natural structure of an oriented planar algebra, and $[\,\cdot\,]$ is a planar algebra morphism from the planar algebra of oriented tangles modulo the three Reidemeister moves to $Kof_{/h}$.

2. Colored tangle-cohomology

We restrict our attention to oriented framed tangles T without closed components, whose strands are colored by the same natural number n, or equivalently, by the $(n+1)$-dimensional irreducible representation of quantum $sl(2)$. Let D be a diagram of T whose blackboard framing agrees with the given framing of T. The binomial coefficient $\begin{pmatrix} n-k \\ k \end{pmatrix}$ equals the number of ways of selecting k pairs of neighbors from n dots placed on a line, such that each dot appears in at most one pair. A *dot-row* s is a set of n dots on a line in which some adjacent dots are paired. Denote by $p(s)$ the number of pairs in s.

Let Γ_n be the oriented graph whose vertices are all possible dot-rows s corresponding to n. Two vertices s and s' of Γ_n are connected by an edge $e\colon\ s \to s'$ if and only if all pairs in s are pairs in s', and $p(s') = p(s) + 1$. The *height of a vertex* s is equal to $p(s)$, and the edges are oriented towards increasing heights. In Figure 1 we show such a graph for $n = 5$. To a dot-row s attach the cable diagram $D_s := D^{n-2p(s)}$, formed by taking the $(n-2p(s))$-parallel cable of D. When forming an m-parallel cable of D we enumerate the strands in a cross-section of D^m from left to right by 1 to m, and orient the parallel cable-strands such that adjacent strands receive opposite orientations, where we give strand 1 the original orientation of D. To an edge $e\colon\ s \to s'$ we attach the cobordism $S_e : D_s \to D_{s'}$ given by contracting the neighboring strands in D_s corresponding to the pair in s' but not in s. That is, S_e is the cobordism with two inputs and no output for these two strands, and the identity otherwise (see Figure 1).

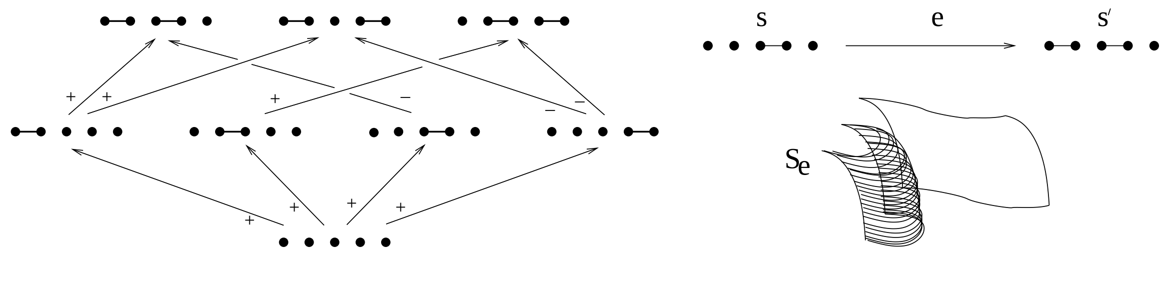

Figure 1. Graph Γ_5 and cobordism S_e

We sprinkle the edges of the resulting graph of tangle diagrams and tangle cobordisms with some minus signs. Let $o(s, s')$ represent the number of pairs in s to the right of the only pair in $s' \setminus s$, and multiply each cobordism S_e by $(-1)^{o(s,s')}$.

To the latter graph we apply now the morphism $[\,\cdot\,]$ constructed in the $sl(2)$ foam cohomology, and form the complex $C_n(D)$ for the colored tangle theory. The cochain objects are given by

$$C_n^i(D) := \bigoplus_s [D_s],$$

where the sum is over all dot-rows s such that $p(s) = i$. The i-th differential $d^i : C_n^i \to C_n^{i+1}$ is a formal sum of all morphisms $[S_e]$ corresponding to edges e at height i, where $[S_e]\colon\ [D_s] \to [D_{s'}]$.

Theorem 1. *The isomorphism class of the cochain complex $C_n(D)$ is an invariant of the colored framed link T.*

To obtain a cohomology theory we apply a functor to switch from the geometric picture to an algebraic one. Specifically, we employ the Bar-Natan type functor $\mathcal{F}$ constructed in the $sl(2)$ foam cohomology theory, switching from the category of foams (rel. local relations) to the category of R-modules. This yields an ordinary complex $\mathcal{F}C_n(D)$, and we can take its cohomology. The isomorphism class of the cohomology group $H_n(D) := H(\mathcal{F}C_n(D))$ is a triply-graded invariant of the colored framed tangle T. Moreover, if the tangle T is a knot, the total graded Euler characteristic of $\mathcal{F}C_n(D)$ is the colored Jones polynomial of the knot.

We consider now two colored tangle diagrams D_1 and D_2, whose components are colored by n. Moreover, we suppose that the vertical tangle composition $D_1 \circ D_2$ is defined, and that D_1, D_2 and $D_1 \circ D_2$ have no circle components. Then there exists a binary operation $*$ defined on the homotopy category of complexes over $Kof_{/h}$, such that $C_n(D_1) * C_n(D_2) = C_n(D_1 \circ D_2)$.

Denote by $(\mathcal{C}^i, \phi^i) := C_n(D_1) * C_n(D_2)$. Then $\mathcal{C}^i = \bigoplus_{s,p(s)=i} ([D_{1,s}] \otimes_R [D_{2,s}])$ and

$$\phi^i(v_1 \otimes v_2) := \sum_e [S_{1,e}](v_1) \otimes [S_{2,e}](v_2), \text{ for all } v_1 \otimes v_2 \in [D_{1,s}] \otimes_R [D_{2,s}]$$

where the latter sum is over all edges e with tail s. The formal tensor product here is the "gluing" operation of formal complexes coming from the $sl(2)$ foam cohomology.

Theorem 2. *$(\mathcal{C}^i, \phi^i)$ is a cochain complex, and $(\mathcal{C}^i, \phi^i) = C_n(D_1 \circ D_2)$, up to homotopy.*

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Seiberg-Witten theory and four-dimensional homology cobordisms

Nikolai Saveliev

I will talk about an invariant introduced in our joint work with Tom Mrowka and Daniel Ruberman with an eye toward the study of homology cobordisms in dimension four. The definition of the invariant uses Seiberg–Witten gauge theory on homology $S^1 \times S^3$ and some index theory on manifolds with periodic ends. I will define the invariant and describe its properties and calculations available up to date.

Torus bundles, and the lower central series of metabelian groups

Kent E. Orr

(joint work with Gilbert Baumslag and Roman Mikhailov)

Abstract: We address the following question: Given a residually nilpotent, solvable group G, what do the lower central series quotients tell us about G?

Our approach is motivated by knots and three manifolds, group closure and localization, and the theory of birational equivalence of affine algebraic sets. We present some classification results arising in this problem, with answers framed in terms of classical number theoretical ideas. We briefly consider applications to torus bundles over the circle.

Description of the talk: The talk arises from a sequence of papers with Gilbert Baumslag and Roman Mikahilov, most still in preparation, and including *A new look at finitely generated metabelian groups.* The latter will appear in *Contemporary Mathematics: Combinatorial and Computational Group Theory with Cryptography.*

We study this question:

Question: Given a residually nilpotent, finitely presented, metabelian group, G, what can I conclude about the group G from its lower central series quotients?

In the talk we discussed briefly the relevance of this question to some central questions in combinatorial group theory, knot theory, and four manifolds, including the *Isomorphism Problem* for finitely generated metabelian groups, the *Parafree Conjecture*, the topological disk embedding problem, the problem of defining transfinite Milnor link invariants, and homology cobordism of manifolds.

Definition 1. *A group H is para-G if H is residually nilpotent, finitely generated, and metabelian, and if there is a homomorphism $G \to H$ inducing an isomorphism on lower central series quotients.*

Given a metabelian group G with a short exact sequence

$$1 \to B \to G \to A \to 1$$

where A and B are abelian, then B has a $\mathbb{Z}[A]$-module structure. We call the ring $R_G = \mathbb{Z}[A]/Ann(B)$ the *coordinate ring of* G. This ring depends only on the group G and not on the choice of exact sequence above.

Theorem 1. *Let $S = 1 + \ker\{\epsilon\colon \mathbb{Z}[A] \to Z\}$. If H is para-G, then;*

(1) *The coordinate rings of G and H are isomorphic.*
(2) $S^{-1}[G,G] \cong S^{-1}[H,H]$.

Definition 2. *We say that a finitely generated, residually nilpotent, metabelian group H is para-equivalent to G if H is para-G and G is para-H.*

We think of the coordinate ring R_G of G and the R_G-module $S^{-1}[G,G]$ as the primary invariants of the para-equivalence class of G.

Our secondary invariant is our *telescope of G*.

To define this, suppose G is a metabelian group, classified by the second cohomology class

$$k(G) \in H^2(G_{ab}; [G,G]).$$

We define $\overline{G}$, our *telescope of G*, to be the group determined by the element

$$k(\overline{G}) \in H^2(G_{ab}; S^{-1}[G,G])$$

where the latter is the image of $k(G)$ under the coefficient induced homomorphism $[G,G] \to S^{-1}[G,G]$.

We emphasize that the telescope of G is not new, and is a version of a well known construction due to J. P. Levine that he called the *algebraic closure of G.*

Theorem 2 (Telescope Theorem)**.** *There is a filtration by subgroups*

$$G = G_1 \subset G_2 \subset \cdots \subset \cup_k G_k = \overline{G}$$

such that;

(1) *Each group $G_k \cong G$.*
(2) *The inclusion of any $G_k \subset G_\ell$ is a para-equivalence, $k < \ell$.*
(3) *H is para-G if and only if $\overline{G} \cong \overline{H}$.*

This has many corollaries, a few enumerated below.

Corollary 3. *If H is para-G and finitely generated, then H and G are para-equivalent and G is finitely generated.*

Corollary 4. *If G is poly-cyclic and H is para-G then G and H are para-equivalent. Furthermore, G contains an isomorph of H of finite index, and H contains an isomorph of G of finite index.*

Corollary 5. *Suppose $G \cong M \rtimes \mathbb{Z}$. If the coordinate ring of G is a principal ideal domain, then every finitely generated para-G group is isomorphic to G.*

The Lamplighter group is among numerous examples covered by this last corollary.

We prove a classifying theorem for para-equivalence of finitely generated, residually nilpotent, metabelian groups.

Definition 3. *We call a submodule $M \subset S^{-1}[G,G]$ an S-fractional submodule if this inclusion induces an isomorphism $S^{-1}M \cong S^{-1}[G,G]$.*

Theorem 6 (Classification Theorem)**.** *Define the* Ideal Class Monoid of G *as follows:*

$$C\ell(G) = \frac{\{S\text{-}fractional\ submodules\ of\ S^{-1}[G,G]\}}{Aut(\overline{G})}.$$

Isomorphism classes of groups para-equivalent to G lie in one-to-one correspondence to elements of $\mathcal{C}\ell(G)$.

Here $Aut(\overline{G})$ acts on the module $S^{-1}[G,G]$ by conjugation, and thus on the set of S-fractional sub-modules.

We apply this result in the talk to compute para-equivalence classes of torus bundles over S^1. Homeomorphism classes of such bundles correspond to a special class of real quadratic extensions of the field of rational numbers. We proved:

Theorem 7. *For any 3-dimensional torus bundle M such that $\pi_1(M)$ is residually nilpotent, there are at most finitely many torus bundles with groups para-equivalent to $\pi_1(M)$.*

Numerous examples were given. For instance, for the torus bundle with group the semi-direct product $\mathbb{Z}[\sqrt{82}] \rtimes \mathbb{Z}$, there are precisely four para-equivalent torus bundles. Here the quotient group $\mathbb{Z}$ acts on $\mathbb{Z}[\sqrt{82}]$ by multiplication by $9 + \sqrt{82}$.

Curves on class VII surfaces. A gauge theoretical approach for proving existence of a cycle

Andrei Teleman

Let M be a 4-manifold with $b_1 = 1$ and $b_+ = 0$ (negative definite intersection form). Suppose for simplicity $\pi_1(M, x_0) = \mathbb{Z}$, and put $b := b_2(M)$. Donaldson's first theorem: q_M is standard, so H^2 has a basis $(e_i)_{1 \leq i \leq b}$ with $e_i e_j = -\delta_{ij}$. We put $I_0 := \{1, \ldots, b\}$, for $I \subset I_0$ put $\bar{I} := I_0 \setminus I$, $e_I := \sum_{i \in I} e_I$. The characteristic elements with vanishing SW expected dimension have the form $c_I := e_I - e_{\bar{I}}$.

The Seiberg-Witten invariants are not well defined [1]: A generic SW moduli space contains a circle of reductions and finitely many regular irreducible points. What can try to define an invariant by counting only the irreducible points (by taking into account the signs defined by the fixed orientation data). But one can see that, deforming the parameters, some irreducible points might become reducible and then disappear, so the algebraic cardinality of the irreducible part of the moduli space will jump. One has a countable chamber structure in the parameter space, and the SW "invariant" obtained by counting algebraically irreducible points can take infinitely many values [1].

Idea: forget about SW and use ideas from original GW theory: count only rational curves in the given class. The corresponding Gromov-Witten moduli space of rational curves will be: $\mathcal{M}^{GW}_{e_i}(\hat{H}) = \{E_i\}$, so this approach produces apparently a well defined "invariant" of the complex structure (which takes the values 1), but this invariant *is not constant in deformations:*

Example: $\mathcal{X} \to D$ with $X_t =$ blown up Hopf surface for $t \neq 0$ and X_0 a so called Kato surface (a known minimal class VII surface). X_0 does not contain any effective divisor (irreducible or not) in the classes e_i. A natural question is: What happens with the exceptional curves $E_{i,t}$ as $t \to 0$? Why do they vanish? The answer is: Explosion of area. The area of $E_{i,t}$ tends to infinity as $t \to 0$! A more sophisticated answer is given in a recent paper with G. Dloussky: "Infinite bubbling".

About Kato surfaces: they are the known minimal class VII surfaces with $b_2 > 0$. Any Kato is a deformation of blown up Hopf surfaces and has (*intriguing*) b_2 rational curves. So the total number of rational curves is constant in the *known* deformations, but the classes which are represented by these curves are not. Curves appear in classes with negative expected dimension… and we have no tools to count all rational curves in all possible homology classes (including those with negative GW expected dimension).

2. Class VII surfaces (not classified yet) $VII \supset VII_{\min}^{b_2>0} \supset$ Kato surfaces (the known surfaces in the class).

Conjecture 1. *If $X \in VII_{\min}^{b_2>0}$ then X has b_2 rational curves. This would imply that X is Kato, so it would complete classification. It is equivalent to the GSS conjecture.*

Conjecture 2. *If $X \in VII_{\min}^{b_2>0}$ then X has a cycle of rational curves. This would imply that X is a deformation of blown up Hopf surfaces, so it would complete classification up to deformation equivalence.*

3. Using Donaldson theory: E Hermitian 2-bundle $D := \det(E)$ fix $a \in \mathcal{A}(D)$. Put $\mathcal{M}_a^{\mathrm{ASD}}(E) := \{A \in \mathcal{A}_a(E) \mid (F_A)_0^+ = 0\}/\Gamma(SU(E))$. The obvious pimorphism $\rho : H_1(M, \mathbb{Z}) \to \{\pm 1\}$ defines a flat line bundle L_ρ. Important $\otimes L_\rho$ defines an involution on $\mathcal{M}_a^{\mathrm{ASD}}(E)$: its fixed points are *twisted reductions* (whose pull-back on the double cover $\tilde{M}_\rho$ associated with ρ are split) and its quotient is the usual moduli space of $PU(2)$ instantons on $\bar{P}_E$.
Example: $b = 2$. Take E with $c_2(E) = 0$, $c_1(E) = e_1 + e_2$. One has $-p_1(\bar{P}_E) = \Delta(E) = 4c_2(E) - c_1^2(E) = 2 \ \leq 3$, so the corresponding moduli space is compact. It contains two circles of reductions corresponding to the topological splittings $E = L_0 \oplus L_{e_1+e_2}$, $E = L_{e_1} \oplus L_{e_2}$. Any circle $\mathcal{C}_i$ has a neighborhood which is a fibration over $\mathcal{C}_i$ with fibre cone over $\mathbb{P}^1$ $(= D^3)$ so smooth! If $\pi_1(M) \simeq \mathbb{Z}$, one can count also the twisted reductions and gets two isolated twisted reductions. For generic metric one gets a smooth 4-manifold. Its signature $\sigma_M := \sigma(\mathcal{M})$ is a $\mathcal{C}^\infty$-invariant of M.

We will prove: If M is the underlying differentiable manifold of an *unknown* minimal class VII surface with $b_2 = 2$ then $\mathcal{M} \simeq S^4$, and *using this fact, we will prove Conjecture 2 for $b_2 = 2$.* Therefore the moduli space $\mathcal{M}$ can be accurately described (although the information we have on the base manifold M is very vague) using complex geometry. The main tool will be the KH correspondence, relating instantons to stable bundles. This correspondence has been used by Donaldson to compute Donaldson invariants, so he used complex geometry to solve gauge

theoretical problems. This time we use gauge theory to solve complex geometric problems.
Dichotomy: Either the signature invariant σ is always trivial (which will substantially simplify the proof of Conjecture 2 in the case $b_2 = 2$) or is not trivial, which would be interesting from a topological point of view.
Remark: Using a standard cobordism argument and the Donaldson class $\mu(h)$, where h is a generator of $H_1(M, \mathbb{Z})$, it follows that the two circles $\mathcal{C}_i$ belong to the same component of the moduli space.

Let now X be a minimal class VII surface with $b_2 = 2$, and $\mathcal{M} := \mathcal{M}_a^{\mathrm{ASD}}(E) = \mathcal{M}_{\mathcal{K}}^{\mathrm{pst}}(E)$ (the Kobayashi-Hitchin correspondence) is compact, has a structure of smooth complex surface in the complement of the reductions (on the stable part). Known pieces of $\mathcal{M}$: (1) 2 circles of reductions $\mathcal{C}_1$, $\mathcal{C}_2$, (2) 2 twisted reductions $\mathcal{B}_1$, $\mathcal{B}_2$, (3) four strata of stable extensions $\mathcal{M}_I^{\mathrm{st}}$, $I \subset \{1,2\}$. $\mathcal{M}_I^{\mathrm{st}}$ consists of extensions of the form $0 \to \mathcal{L} := \mathcal{K} \otimes \mathcal{M}^\vee \to \mathcal{E} \to \mathcal{M} \to 0$ with $c_1(\mathcal{M}) = c_I$. $\mathcal{M}_{\{1,2\}}^{\mathrm{st}}$ is a $\mathbb{P}^1$-bundle over a punctured disk, $\mathcal{M}_{\{1\}}^{\mathrm{st}}$, $\mathcal{M}_{\{2\}}^{\mathrm{st}}$ are punctured disks, and $\mathcal{M}_\emptyset^{\mathrm{st}} = \{\mathcal{A}, \mathcal{A}' := \mathcal{A} \otimes \mathcal{L}_{\mathcal{R}}\}$, where $\mathcal{A}$ is the central term of a non-trivial extension:

$$0 \to \mathcal{K} \to \mathcal{A} \to \mathcal{O} \to 0. \tag{1}$$

Step 1: $\mathcal{M}_0 := \mathcal{C}_1 \cup \mathcal{C}_2 \cup \{\mathcal{B}_1, \mathcal{B}_2\} \cup (\cup_{I \neq \emptyset} \mathcal{M}_I^{\mathrm{st}})$ is a connected component of $\mathcal{M}$. Using the fact that the two circles of reductions belong to the same component, one can see that this component (the *known* component of the moduli space) is isomorphic to S^4.
Dichotomy: Either $\mathcal{A}$ belongs $\mathcal{M}_0$ (the known component of the moduli space space), or it belongs to a smooth compact surface $Y \subset \mathcal{M}^{\mathrm{st}}$.
Remark: If $\mathcal{A}$ belongs to $\mathcal{M}_0$ then X has a cycle of rational curves! [1]This can be proved easily: Suppose for instance that $\mathcal{A}$ belonged to one of the strata $\mathcal{M}_I^{\mathrm{st}}$, $I \neq \{1,2\}$. In this case the bundle $\mathcal{A}$ (which was defined as the central term of the nontrivial extension (1)) could be written as an extension in a different (new) way. If this was the case, composing the epimorphism of the exact sequence (1) with the monomorphism of the new exact sequence we obtain a line bundle morphism, which is neither trivial nor isomorphism. The zero locus of this composition is a non-empty effective divisor, and one can prove that it is a cycle.
Step 2: The appearance of a new compact connected component Y in the moduli space leads to a contradiction.
For this statement we have two proofs: for the first one, which uses the classification of complex surfaces, we refer to [3]. A new proof is based on very recent results of Bismut, namely on a refinement of the GRR theorem in the non-Kähler case, which gives the computation of the Chern character of the total direct image

[1] A cycle of rational curves is en effective divisor of the form $C = \sum_{i \in \mathbb{Z}_k} C_i$, where either $k = 1$ and C_0 is a rational curve with a simple singularity, or $k \geq 2$ and $C_i \cdot C_{i+1} = 1$, $C_i \cdot C_j = 0$ for $i - j \neq \pm 1$.

in Bott-Chern cohomology. *This proof might be generalized to arbitrary* b_2 *(work in progress).*

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Functorial maps and weak parities

Igor Nikonov

Parity theory introduced by V.O. Manturov [1] has many interesting applications. One of them is the construction of functorial maps [2, 3]. Functorial map is defined as transformation of knot diagrams that replaces classical crossings of diagrams with virtual ones in a way compatible with Reidemeister moves: if two diagrams differ with a Reidemeister move then the functorial map transforms them into two diagrams that differ with a Reidemeister move. So any functorial map determines a well-defined map from knots to knots and thus allows to get new knot invariants as compositions of the functorial map with known knot invariants.

Weak parities defined by V.O. Manturov give another description of functorial maps. Among weak parities there is a distinguished one — the maximal nontrivial weak parity. In the talk we describe the maximal nontrivial weak parity for knots in a given surface and show that all the weak parities (and functorial maps) on the classical knots are trivial.

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Recent progress on topologically slice knots

Matthew Hedden

In this talk I discussed work with several collaborators, including Se-Goo Kim, Paul Kirk, Charles Livingston, and Daniel Ruberman. The focus of the talk was on the concordance group of knots in the 3-sphere, with special emphasis paid to the distinction between the groups defined in the smooth and topological categories.

To set the stage, recall that two knots in the 3-sphere are called *concordant* if they arise as the boundary of a smooth and properly embedded cylinder in the 3-sphere times an interval. Concordance is clearly an equivalence relation. Modulo this relation, the set of knots forms an abelian group $\mathcal{C}$, with the role of addition played by connected sum and identity given by the class of the unknot. The inverse is obtained by considering the mirror image, with reversed orientation. The focus of this talk was on the fact that one can also define *topological concordance* by merely requiring the cylinder above to be *flatly embedded*. Roughly, this is a continuous embedding of the cylinder which extends to a continuous embedding of the "thickened cylinder" i.e. the cylinder times a 2-disk. Using this definition, one obtains a topological concordance group, denoted $\mathcal{C}^{top}$. There is an obvious homomorphism

$$\phi : \mathcal{C} \longrightarrow \mathcal{C}^{top}.$$

I'll denote the kernel of this homomorphism by $\mathcal{C}_{TS}$, and call it the concordance group of *topologically slice knots*. This name comes from the fact that it is the subgroup of the concordance group generated by knots which bound flatly embedded disks, so-called topologically slice knots. Until recently, very little was known about this group and there are many interesting open questions. In terms of its size, a result of Endo [1] showed that $\mathcal{C}_{TS}$ is quite large: he showed that it contains an infinitely generated free abelian subgroup. Endo's result used non-abelian gauge theory, exploiting work of Furuta [3] that built on Fintushel-Stern's $SO(3)$ gauge theory for pseudo-free orbifolds [2].

More recently, combined work of Manolescu-Owens [9], and Livingston [8] showed that $\mathcal{C}_{TS}$ splits off a free abelian group of rank 3; that is,

$$\mathcal{C}_{TS} \cong \mathbb{Z}^3 \oplus G,$$

for some group G. These results used a combination of Heegaard Floer homology [12, 11] and Rasmussen's s-invariant [13] coming from Khovanov homology [7].

My talk discussed three recent results concerning the structure of $\mathcal{C}_{TS}$. The first concerns satellite operations. It is a well-known observation to those working with concordance that the satellite operation from knot theory descends to yield self-maps on the concordance groups. Given a pattern knot $P \subset S^1 \times D^2$, one obtains a map

$$P : \mathcal{C} \longrightarrow \mathcal{C}$$

which takes the concordance class of a knot K to that of $P(K)$ (the satellite knot of K defined via P). Such an operation is also defined on $\mathcal{C}^{top}$. While not homomorphisms in general, these maps have been extremely useful in the study of concordance. Perhaps the most famous satellite operation in this context is

"Whitehead doubling", which I'll denote by D. A well-known conjecture of Kirby asserts that the Whitehead double of a knot K is slice if and only if K is slice. This can be stated succinctly as

Conjecture 1. *$D^{-1}(O) = O$.*

Our first result is that the doubling operator has infinite rank. What makes this result interesting is that the doubling operator has rank zero in the topological category. This follows from a fundamental result of Freedman that implies the Whitehead double of any knot is topologically slice, hence Im $D \subset \mathcal{C}_{TS}$. Let $T_{2,2^n+1}$ denote the $(2, 2^n + 1)$ torus knot. We have

Theorem 2 (with Kirk [5])**.** *The set $\{D(T_{2,2^n+1})\}_{n=1}^{\infty}$ freely generates a subgroup isomorphic to $\mathbb{Z}^\infty \lhd \mathcal{C}_{TS}$.*

The proof uses a refinement of the Furuta and Fintushel-Stern technique, together with calculations of the Chern-Simons invariants of flat $SO(3)$ connections on the branched double cover of $D(T_{2,2^n+1})$. This is the first example of a satellite operation whose image has infinite rank in the smooth category but finite rank in the topological category.

Like all previous results on $\mathcal{C}_{TS}$, Theorem 2 relies on Freedman's result which says that a knot whose Alexander polynomial is equal to 1 is topologically slice. An interesting question was whether Freedman's result captures the difference between $\mathcal{C}$ and $\mathcal{C}^{top}$; that is, whether $\mathcal{C}_{TS}$ is generated by knots which are smoothly concordant to Alexander polynomial one knots. Let

$$\mathcal{C}_\Delta = \langle \{K \mid K \text{ is concordant to a knot with Alexander polynomial } 1\} \rangle .$$

Thus $\mathcal{C}_\Delta \lhd \mathcal{C}_{TS}$ is the subgroup generated by Freedman's theorem. The next result I discussed is the following:

Theorem 3 (with Livingston and Ruberman [6])**.**

$$\mathbb{Z}^\infty \lhd \mathcal{C}_{TS}/\mathcal{C}_\Delta.$$

The proof of this result uses Heegaard Floer homology, in the form of the "correction terms" [11] of the branched double cover of certain explicitly constructed knots, together with surgery formulae relating knot Floer homology invariants to the invariants of 3-manifolds obtained by Dehn surgery. A key tool is a formula for the knot Floer homology of the Whitehead double of a knot [4].

Until now, all results about $\mathcal{C}_{TS}$ showed that if a class was non-trivial, then it had infinite order. The result which I discussed in most detail is also the most recent:

Theorem 4 (with Se-Goo Kim and Livingston [10])**.**

$$(\mathbb{Z}/2\mathbb{Z})^\infty \lhd \mathcal{C}_{TS}.$$

The proof is similar in spirit to that of Theorem 3. An explicit family of knots is constructed which are fully amphichiral and can be seen to be topologically slice via Freedman's theorem. Thus they have order two in $\mathcal{C}$ and lie in $\mathcal{C}_{TS}$. Then an explicit examination of the correction terms of their branched double covers is performed, again using surgery formulae for the Floer invariants. The calculations involved in the proof of Theorem 3 were quite delicate, and complicated by the fact that the manifolds involved do not arise as surgery on a knot in the 3-sphere. They do, however, arise as surgery on a two-component link.

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Combinatorial Spatial Graph Floer Homology

Shelly L. Harvey

Knot Floer homology, introduced by Ozsváth and Szabó [6], and independently by Rasmussen [8], is an invariant of knots in S^3 that categorifies the Alexander polynomial. Knot Floer homology is widely studied because of its many applications in low-dimensional topology. For example, it detects the unknot [5], detects whether a knot is fibered [1, 4], and detect the genus of a knot [5]. The theory was generalized to links in [7]. We extend their theory for *balanced* spatial graphs in S^3.

Originally, knot Floer homology was defined as the homology of a chain complex obtained by counting certain holomorphic disks in a $2g$-dimensional symplectic manifold with some boundary conditions that arose from a (doubly pointed) Heegaard diagram for S^3 compatible with the knot. As such, the chain *groups* were combinatorial but one could not, a priori, compute the boundary map for a particular example. However, Sucharit Sarkar discovered an criterion that would ensure that the count of certain holomorphic disks was combinatorial. This idea was used by Manolescu, Ozsváth, and Sarker in [2] to give a combinatorial description of link Floer homology using grid diagrams. Using this description, in [3], Manolescu, Ozsváth, Szábo, and D. Thurston gave a self-contained, combinatorial proof that link Floer homology is an invariant.

We generalize the combinatorial description and proof in [2, 3] for certain spatial graphs. The class of spatial graphs that we work with are embeddings of oriented graphs with a transverse disk at each vertex splitting up the incoming and outgoing edges. Morevoer, we require that there must be the same number of incoming edges as outgoing edges at each vertex. We call these spatial graphs balanced.

To define the chain complex, we first introduce the notion of a graph grid diagram for a balanced spatial graph. Roughly, a grid diagram for a spatial graph is an $n \times n$ grid of squares each of which is decorated with an X, an O, or is empty and that satisfies some conditions. Like for links, there is precisely one O per row and column. However, there may be many Xs in each row and column but they must be grouped around a single O and be in L-formation. An O grouped with multiple Xs corresponds to a vertex of the spatial graph. Note, for this definition, we do not need to have the same number of incoming edges as outgoing edges. See Figure 1 for an example.

	O		**X**	
		O	**X**	
			O	**X**
X				**O**
O	**X**	**X**		

FIGURE 1. Example of a graph grid diagram

We associate an oriented spatial graph to a grid diagram by connecting the Xs to the Os vertically and the Os to the Xs horizontally. We also use the convention that the vertical strands go over the horizontal strands. See Figure 2 for an example.

We prove that any two representative for the same spatial graph are related by a sequence of graph grid moves.

Theorem 1. *If $f : G \to S^3$ is a balanced graph then there is grid diagram $gr(f)$ representing f. Moreover, if $gr(f)$ and $gr'(f)$ are two grid diagrams representing*

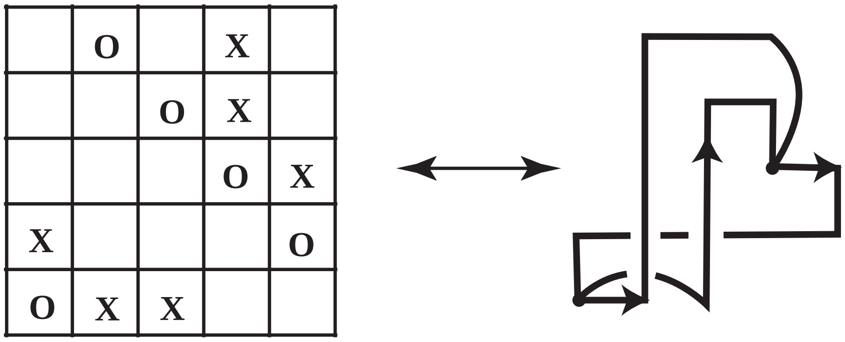

FIGURE 2. Associating a spatial graph to a graph grid diagram

f then the grid diagrams are related by a finite sequence of the following moves: cyclic permutation, commutation', and stabilization.

We remark that the only move that differs from that of links is commutation'.

Next, we define the chain complex $(C^-(gr(f)), \partial^-)$ associate to a graph grid diagram $gr(f)$. The chain groups consist of free modules over $\mathbb{F}[U_1, \ldots, U_V]$ where $\mathbb{F} \cong \mathbb{Z}/2\mathbb{Z}$ is the ring with two elements and V is the number of vertices in the graph. Like in link Floer homology, the generators of the chain groups are unordered tuples of intersections between the horizontal and vertical curves in the grid. The Maslov grading is defined exactly as in [3]. Note that this is possible since it only depends on the set of Os on the grid. For links, the Alexander grading lives in $\mathbb{Z}^m$. For spatial graphs, we define an Alexander grading that has values in $H_1(S^3 \setminus f(G))$ which can be identified with $\mathbb{Z}^m$ after choosing a basis. To define this, for each point in the lattice of the grid, we define an element of $H_1(S^3 \setminus f(G))$, called the generalized winding number. It is defined so that if you can get from one point to another by passing under an edge of $f(G)$ then the difference between their values is the homology class of the meridian of that edge. The Alexander grading of a generator is defined by taking the sum of the generalized winding numbers of the elements in the set. Each U_i is associated with a vertex O and we define the Alexander grading so that multiplication by U_i corresponds to lowering the Alexander grading by the element of $H_1(S^3 \setminus f(G))$ represented by a meridian of the vertex corresponding to U_i. The ∂^- map is defined by counting empty rectangles in the (toroidal) grid that do not contain an X. We show that $\partial^- \circ \partial^- = 0$ and hence this gives a well-defined bigraded homology group for each graph grid diagram. Moreover, one can show that the homology is independent of the choice of grid.

Theorem 2. *If $gr(f)$ and $gr'(f)$ are two graph grid diagrams representing $f : G \to S^3$ then $(C^-(gr(f)), \partial^-)$ and $(C^-(gr'(f)), \partial^-)$ are quasi-isomorphic as $\mathbb{F}[U_1, \ldots, U_V]$-modules.*

To prove this, we show that the quasi-isomorphism type of the chain complex is preserved under the three graph grid moves. This gives a bigraded homology module over $\mathbb{F}[U_1, \ldots, U_V]$ associated to each balanced spatial graph.

One can set all the U_i's to zero and take the bigraded Euler characteristic of the homology (or chain complex). This will give a multivariable polynomial associated to the spatial graph. We show that this polynomial is equal to the torsion of a certain chain complex associated to the spatial graph.

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Parity Biquandles

Aaron Kaestner

(joint work with Louis H. Kauffman)

We use crossing parity to construct a generalization of biquandles for virtual knots which we call Parity Biquandles. These structures include all biquandles as a standard example referred to as the even parity biquandle. Additionally, we find all Parity Biquandles arising from the Alexander Biquandle and Quaternionic Biquandles. Examples are provided showing that using this method we can find examples of parity biquandles which are distinct from the associated even parity biquandle. Furthermore we discuss some related results and additional directions for this research.

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Floer theoretically essential tori in rational blowdowns

Yanki Lekili
(joint work with Maksim Maydanskiy)

We compute the Floer cohomology of monotone tori in the Stein surfaces obtained by a linear plumbing of cotangent bundles of spheres, also known as the Milnor fibre associated with the complex surface singularity of type A_n. We next study some finite quotients of the A_n Milnor fibre which coincide with the Stein surfaces that appear in Fintushel and Stern's rational blowdown construction. We show that these Stein surfaces have no exact Lagrangian submanifolds by using the already available and in depth understanding of the Fukaya category of the A_n Milnor fibre coming from homological mirror symmetry. On the contrary, we find Floer theoretically essential monotone Lagrangian tori, finitely covered by the monotone tori that we studied in the A_n Milnor fibre. We conclude that these Stein surfaces have non-vanishing symplectic cohomology.

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Minimal Diagrams of Free Knots

Allison Henrich
(joint work with Tomas Boothby and Alexander Leaf)

An *irreducibly odd graph* is a graph such that each vertex has odd degree and for every pair of vertices, a third vertex in the graph is adjacent to exactly one of the pair. This family of graphs was introduced recently by Manturov in relation to free knots. Manturov proved that if a free knot diagram is associated to an irreducibly odd graph (given a certain canonical association), then the diagram is a minimal crossing representation of the free knot it represents. In our work, we begin to classify irreducibly odd graphs so they may give us insight into the classification of free knots. In particular, we show that every graph is the induced subgraph of an irreducibly odd graph. We also prove that all irreducibly odd graphs must contain a particular minor called the *3-morningstar.* In addition, we introduce a family of permutation graphs that correspond to minimal diagrams of free knots. This family is of particular interest since it provides many more minimal diagrams of free knots.

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Markov Theorem for Free Links

Hang Wang
(joint work with Vassily O. Manturov)

Definition 1. *A* free link *is an equivalence class of framed* 4*-valent graphs modulo the following three transformations:*

(1) The first Reidemeister move *being an addition/removal of a loop.*
(2) The second Reidemeister move *being an addition/removal of a bigon formed by a pair of edges which are adjacent (not opposite) at each of the two vertices.*
(3) The third Reidemeister move *being a triangle move involving three vertices.*

When projecting a free link on a plane, we obtain a free link diagram*, with both flat and virtual crossings.*

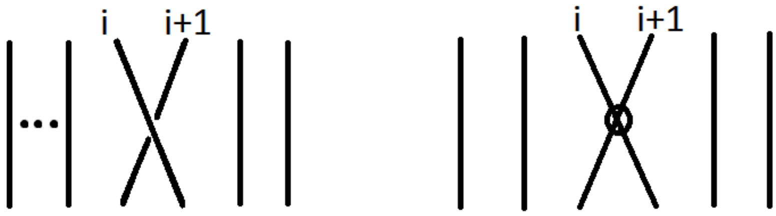

FIGURE 1. The classical crossing σ_i and the virtual crossing ζ_i

Free knots were first considered by V. G. Turaev who conjectured these knots to be all trivial. Using parity, a natural invariant for free knots, V. O. Manturov disproved this conjecture [5]. Free knots are intimately related to flat virtual knots in the following sense.

Proposition 1 ([5]). *Two representatives of free links represent the same equivalence class if and only if the corresponding virtual link diagrams are the same modulo a combination of the following transformations:*

(1) *The generalized Reidemeister moves for virtual knot theory.*
(2) *Crossing switches that make a diagram flat.*
(3) *Virtualization move.*

In classical knot theory, knots and links can be represented as equivalence classes of braids modulo Markov moves. The Markov theorem is powerful in constructing their invariants [7]. Our motivation is to seek invariants for free links, by investigating representations of the corresponding braid groups. We define the *n-strand free braid group fB_n* as the quotient of the n-strand virtual braid group (cf. [2]) by two more relations: the cross-switching and virtualization.

Definition 2 ([6]). *The set of* the n-strand free braids fB_n *is a group with* $2n-2$ *generators* $\sigma_1, \sigma_2, \ldots, \sigma_{n-1}, \zeta_1, \zeta_2, \ldots, \zeta_{n-1}$*, see Fig. 1, subject to the following relations:*

- *(Relations for classical braids)*
  - $\sigma_i\sigma_j = \sigma_j\sigma_i$, *for all* $|i-j| > 1, 1 \leq i, j \leq n-1$;
  - $\sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1}$, *for* $1 \leq i \leq n-2$.
- *(Additional relations for virtual braids)*
  - $\zeta_i\zeta_j = \zeta_j\zeta_i$ *and* $\zeta_i\sigma_j = \sigma_j\zeta_i$, *for all* $|i-j| > 1$;
  - $\zeta_i\zeta_{i+1}\zeta_i = \zeta_{i+1}\zeta_i\zeta_{i+1}$ *and* $\sigma_i\zeta_{i+1}\zeta_i = \zeta_{i+1}\zeta_i\sigma_{i+1}$ *for* $1 \leq i \leq n-2$;
  - $\zeta_i^2 = 1$.
- *(Additional relations for free braids)* $\sigma_i\zeta_i = \zeta_i\sigma_i$, $\sigma_i^2 = 1$ *for all* $1 \leq i \leq n-1$.

In [3] L. Kauffman and S. Lambropoulou proved the Alexander's theorem for virtual links. The Alexander for free links follows easily as a corollary.

Theorem 2 ([6]). *For any free link, there exists a free braid whose closure is isotopic to the given link.*

In [3] Louis Kauffman and Sofia Lambropoulou using the L-move method to give a local version of the Markov theorem for virtual braids. Markov theorem for flat virtual links follows immediately from the argument of [3]. Together with Proposition 1, we proved the following main theorem. The most important thing in proving it is to deal with the virtualization moves. See [6] Lemma 3.4.

Theorem 3 (Markov theorem for free links [6]). *Two oriented free links are isotopic if and only if two corresponding free braids differ by a finite sequence of free braid isotopy and the following moves and their inverses:*

(1) *Flat conjugation.*
(2) *Right virtual L-moves.*
(3) *Right flat L-moves.*
(4) *Right and left threaded L-moves.*

Remark 1. *There is also a type of knot theory defined by virtual knots modulo virtualizations and we have the corresponding Markov theorem (*[6] *Theorem 3.7).*

The main motivation for our interest to Markov's theorem for free links is that we would like to construct invariants for free knots out of free braids. We start tackling such problems by using classical objects, such as the Yang-Baxter equation. Let V be a vector space of dimension n and $R : V\otimes V \to V\otimes V$ be a linear transformation. We denote by R^{ab}_{cd} the $(a\otimes b, c\otimes d)$-th matrix entry, where a, b, c and d belong to a set of the basis of V.

Definition 3. *Let* Id *be the identity map of V, then* the Yang-Baxter equation *is an equation on $V^{\otimes 3}$ given by*

$$(R\otimes \mathrm{Id})(\mathrm{Id}\otimes R)(R\otimes \mathrm{Id}) = (\mathrm{Id}\otimes R)(R\otimes \mathrm{Id})(\mathrm{Id}\otimes R). \tag{1}$$

Remark 2. *Note that (1) is considered over some field $\mathbb{K}$. However, a similar calculation of solutions to (1) can be performed on a module V, over a noncommutative ring $\mathbb{K}$ or a ring $\mathbb{K}$ with zero divisors.*

Let us take the following representation of the free braid group fB_n on $V^{\otimes n}$

$$\sigma_i \to \mathrm{Id}\otimes\cdots\otimes\mathrm{Id}\otimes \underbrace{R}_{i\mathrm{th},(i+1)\mathrm{st}} \otimes\mathrm{Id}\otimes\cdots\otimes\mathrm{Id}, \tag{2}$$

where R corresponds to the i-th and $(i+1)$-st factors in $V^{\otimes n}$ and satisfies the following conditions:

$$R^{ab}_{cd} = R^{ba}_{dc}, \text{ for all } a, b, c, d \text{ being in the set of the basis of } V; \tag{3}$$

$$R^2 = I_{n_2}\ (I_{n_2} \text{ is the identity matrix of size } n^2). \tag{4}$$

The first step in obtaining an invariant for free knots is to take the trace of the representation, and this trace is automatically invariant under flat conjugations. Nevertheless, the problem of finding such representations for which the trace is invariant under all L-moves is more complicated because we have two sorts of "stabilization moves" corresponding to the first classical Reidemeister move and the first virtual Reidemeister move. Here, we restrict ourselves to an example.

Example 1 ([6]). *Let V be a 2-dimensional vector space/module over a field/ring $\mathbb{K}$, with a basis $\{e_0, e_1\}$. We solve R satisfying (1), (3) and (4). We restrict to the case of the eight-vertex model, that is, $R = \begin{pmatrix} a & 0 & 0 & b \\ 0 & c & d & 0 \\ 0 & d & c & 0 \\ b & 0 & 0 & a \end{pmatrix}$ and obtain the following result:*

- *If $\mathbb{K}$ is commutative with no zero divisors, we obtain the complete set of solutions of (a, b, c, d) by solving the set of equations: $(1, 0, 0, \pm 1)$, $(1, 0, 1, 0)$, $(-1, 0, 0, \pm 1)$, $(-1, 0, -1, 0)$, $(0, 1, 1, 0)$ and $(0, -1, -1, 0)$.*
- *If $\mathbb{K}$ is a ring with zero divisors, then there are more interesting solutions. For example, when $\mathbb{K} = \mathbb{Z}_{12}$, a solution to (a, b, c, d) is $(4, 3, 3, 4)$.*

Remark 3. *It is interesting to study the representations of free braid groups when $\mathbb{K}$ as a ring has zero divisors or is noncommutative, and then use it to study quantum invariants for free knots and links. Other possible ways of constructing free knot invariants may consist of constructing biquandles similar to* [1, 4].

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Graph-links and Bouchet graphs

Denis Ilyutko

It is well known that classical and virtual knots can be represented by Gauss diagrams, and the whole information about the knot and its invariants can be read out of any Gauss diagram encoding it. Whenever a Gauss diagram does not describe any *embedded 4-valent graph* in $\mathbb{R}^2$ (just because the corresponding Gauss code is not planar), one gets a virtual knot, where generic immersion points of intersections of edges are encircled.

It turns out that some information about the knot can be obtained from a more combinatorial data: the intersection graph of a Gauss diagram. The intersection graph is a *simple graph*, i.e. a graph without loops and multiple edges, whose vertices are in one-to-one correspondence with chords of the Gauss diagram of the knot (the latter are, in turn, in one-to-one correspondence with classical crossings of the knot). Two vertices of the intersection graph are *adjacent* whenever the corresponding arrows of the Gauss diagram are *linked*, see Fig. 1. Each vertex of the intersection graph is endowed with the local writhe number of the corresponding crossing. Even more, if we forget about the writhe number information and only have the structure of opposite edges, we shall get non-trivial objects (modulo Reidemeister's moves).

However, sometimes the Gauss diagram can be obtained from the intersection graph in a non-unique way, and some graphs (shown in Fig. 2) cannot be represented by chord diagrams at all.

Probably, the simplest evidence that one can get some information out of the intersection graph is the number of circles one gets in a certain state after a smoothing, see Fig. 3. The circuit-nullity formula allows one to count the number of circles in Kauffman's states out of the intersection graph. In particular, this means that graphs not necessarily corresponding to any knot admit a way of generalising the Kauffman bracket, which coincides with the usual Kauffman bracket when the graph is realizable by a knot.

Likewise virtual knots appear out of non-realizable Gauss code and thus generalize classical knots (which have realizable Gauss codes), graphs-links come out of intersection graphs: We may consider graphs which realize chord diagrams, and, in turn, virtual links, and pass to arbitrary simple graphs which correspond to some mysterious objects generalizing links and virtual links.

Traldi and Zulli [7] constructed a self-contained theory of "non-realizable knots" (the theory of *looped interlacement graphs*) possessing lots of interesting knot theoretic properties by using Gauss diagrams. These objects are equivalence classes of (decorated) graphs modulo "Reidemeister moves".

The author and V. O. Manturov suggested another way of looking at knots and links and generalizing them (the theory of *graph-links*): whence a Gauss diagram corresponds to a transverse passage along a knot, one may consider a *rotating circuit* which never goes straight and always turns right or left at a classical crossing. One can also encode the type of smoothing (Kauffman's A-smoothing or Kauffman's B-smoothing) corresponding to the crossing where the circuit turns right or left and never goes straight, see Fig. 4. We note that chords of the diagrams are naturally split into two sets: those corresponding to crossings where two opposite directions correspond to *emanating* edges with respect to the circuit and the other two correspond to *incoming* edges, and those where we have two consecutive (opposite) edges one of which is incoming and the other one is emanating.

After the two theories were constructed, some questions arose. The first question is whether or not every graph is Reidemeister equivalent (each theory has own Reidemeister moves) to the intersection graph of a virtual knot diagram. The

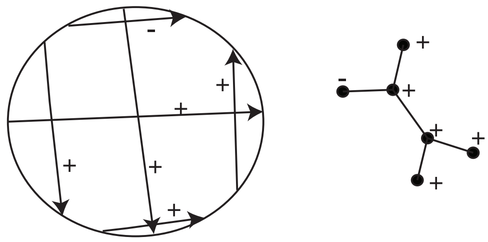

Figure 1. A Gauss diagram and its labeled intersection graph.

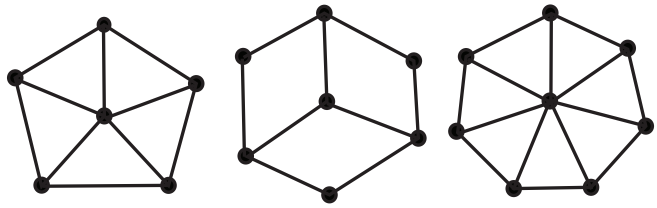

Figure 2. Non-realizable Bouchet graphs.

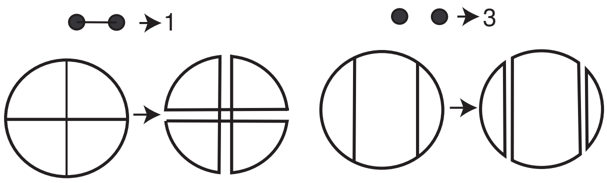

Figure 3. Resmoothing along two chords yields one or three circles

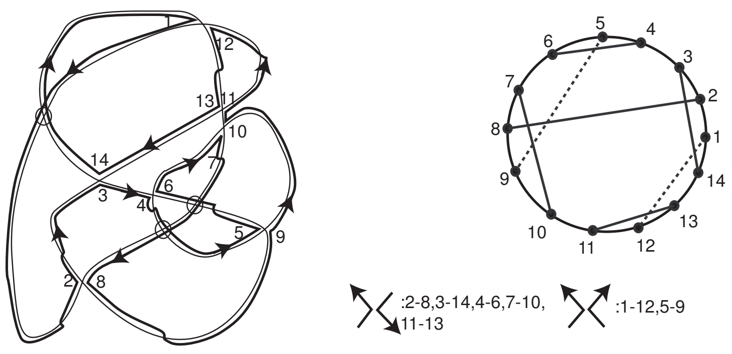

Figure 4. Rotating circuit shown by a thick line; chord diagram

second question is related to the existence of an equivalence between two theories. Other questions concern invariants.

The first question was resolved by using parity theory introduced by Manturov in [5]. It was shown that in the theory of looped interlacement graphs there were graphs being not equivalent to intersection graphs of Gauss diagrams of knots by Reidemeister moves. The same situation is for graph-links. The equivalence of these two theories (the theory of looped interlacement graphs and the theory of graph-knots) was proved in [1]. Also, some invariants were constructed, see [2, 3, 4, 7].

Most of obtained results are related to graphs encoding knots, not links! Because a Gauss diagram represents a knot. If we have a free-link with many components, then the situation is more complicated. We can endow a graph-link with orientation. Therefore, in this case, we can get more invariants.

The theory of graph-links (looped interlacement graphs) is interesting for various reasons:

a) in some cases it exhibits purely combinatorial ways of extracting invariants for knots (see, e.g. [6]);

b) in some cases it produces heuristic approaches to new "knot theories";

c) it highlights some "graphical" effects which are hardly visible in usual or virtual knot theory.

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On Seifert fibered 4-manifolds

Weimin Chen

In this talk we discussed a new approach to study the topology of Seifert fibered 4-manifolds. The method relies on two technical inputs: Thurston's Geometrization of 3-manifolds/orbifolds and Rips-Sela's theory of $\mathbb{Z}$-splittings of single-ended finite presented groups. The first fruit of this approach is a finiteness theorem which asserts that there are at most finitely many distinct Seifert fibered 4-manifolds realizing a given finitely presented group of infinite center. Our research also suggests that the differentiable structure of a Seifert fibered 4-manifold is determined by its underlying topological structure.

Stable Generalized Polyak Groups: An Approach to Finite-Type Invariants of Virtual Knots

Micah W. Chrisman

There are two notions of finite-type invariant for virtual knots. The first is the natural notion of Vassiliev invariants for virtual knots [15]. The second type is due to Goussarov, Polyak, and Viro (called GPV finite-type) [11]. There is no known universal Vassiliev finite-type invariant of virtual knots. However, there is a universal order one finite-type invariant [13], a universal GPV finite-type invariant [11], and a universal finite-type invariant for knots in oriented thickened surfaces [2, 1]. This suggests the following strategy:

Strategy: Find the universal finite-type invariant of virtual knots by finding a combinatorial description of the universal finite-type invariant of knots in thickened surfaces. Then "stabilize" this group.

For each compact connected oriented surface Σ, we construct an analogue of the Polyak groups of degree n. This *generalized Polyak group of degree n* is denoted $\mathcal{GP}_n(\Sigma)$. If Σ_1, Σ_2 are compact connected oriented surfaces and $h : \Sigma_1 \to \Sigma_2$ is any orientation preserving map, then there is an induced group homomorphism $\mathcal{GP}_n(h) : \mathcal{GP}_n(\Sigma_1) \to \mathcal{GP}_n(\Sigma_2)$. In fact, $\mathcal{GP}_n$ is a functor.

Let $\mathcal{K}(\Sigma)$ denote the ambient isotopy classes of oriented knots in $\Sigma \times I$.

Theorem 1. *For all natural numbers n, there exists a map $I_n[\Sigma] : \mathcal{K}(\Sigma) \to \mathcal{GP}_n(\Sigma)$ which is a Vassiliev invariant of oriented knots in $\Sigma \times I$ of degree $\leq n$.*

The groups $\mathcal{GP}_n(\Sigma)$ may be "stabilized" to give finite-type invariants of virtual knots. The stabilization is motivated by the stabilization of abstract knots as given in [14]. Set $GP_n = \bigoplus_\Sigma \mathcal{GP}_n(\Sigma)$. We define a set of relations R_n on GP_n: If $x_1 \in \mathcal{GP}_n(\Sigma_1)$, $x_2 \in \mathcal{GP}_n(\Sigma_2)$, there exists a compact connected oriented surface Σ and orientation preserving embeddings $h_1 : \Sigma_1 \to \Sigma$, $h_2 : \Sigma_2 \to \Sigma$, and $\mathcal{GP}_n(h_1)(x_1) = \mathcal{GP}_n(h_2)(x_2)$, then $x_1 - x_2 \in R_n$. Define:

$$\mathcal{SGP}_n := \frac{GP_n}{\langle R_n \rangle}$$

Let $\pi_n[\Sigma] : \mathcal{GP}_n(\Sigma) \to GP_n \to \mathcal{SGP}_n$ denote the map which is inclusion followed by projection. Let $\mathcal{VK}$ denote the set of virtual knots, considered up to Reidemeister and detour moves. For a virtual knot K, let τ_K be its band-pass presentation with surface Σ_K.

Theorem 2. *For every n, the map $\mathcal{SI}_n : \mathcal{VK} \to \mathcal{SGP}_n$ defined for all $K \in \mathcal{VK}$ by:*

$$\mathcal{SI}_n(K) = \pi_n[\Sigma_K] \circ I_n[\Sigma_K](\tau_K)$$

is a Vassiliev invariant of virtual knots of degree $\leq n$.

We show an example which proves that the invariant is non-trivial. Moreover, the groups $\mathcal{SGP}_n$ are not isomorphic to the Polyak groups of [11].

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