

## Tim Cochran

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Tim Cochran (1955–2014), a leading topologist of his generation, made numerous fundamental contributions across geometric topology. His research interests included 3 and 4-manifold theory, with his most influential work related to link theory (in the classical and high dimensions) and to classical knot concordance. Tim probed some of the hardest, deepest problems of his day. A tremendous collaborator, Tim brought a constant stream of new ideas and energy to every project. His infectious enthusiasm for mathematics continues to inspire a tight-knit group of students and postdocs who came to Rice for Tim’s guidance and collaboration.

Tim’s early work on link concordance displayed drive and outstanding vision. His geometric approach to Milnor’s  $\bar{\mu}$ -invariants, culminating in his AMS Memoir, remains influential to this day, particularly through his developing iterative derivatives and antiderivatives of links to unveil the geometric underpinnings of Milnor’s link invariants. With Kent Orr, Tim solved an outstanding problem in link theory by exhibiting links in all odd dimensions that are not concordant to boundary links; in the classical dimension, these went beyond the obstructions already arising from Milnor’s invariants. A later collaboration with Kent Orr and Peter Teichner uncovered a deep filtered structure in the knot concordance group using analytic signature techniques, driving a mainstream movement in current research. During the ensuing decade and a half Tim continued to deepen and extend this vision in collaboration with former students, especially Shelly Harvey, Connie Leidy, Peter Horn, Chris Davis, and Arunima Ray.

Tim employed a remarkable profusion of algebraic, topological, and analytical techniques in these papers. For instance, Tim’s work with Orr and Teichner on knot concordance developed linking theory in iterated abelian knot covers, using modules over non-commutative rings as well as non-commutative localization techniques, detected via analytic signatures. Tim explored these tools in increasingly greater depth under his rubric ‘non-commutative knot theory,’ most notably with Shelly Harvey and Constance Leidy. The demonstration that the *derived* filtration defined in the Cochran–Orr–Teichner papers are non-trivial at *all* levels involved, on the one hand, elaborate topological constructions partially rooted in Tim’s earlier work on derivatives of links, and on the other, analytic machinery derived from the von Neumann  $\rho$ -invariants of Cheeger and Gromov.

The invariants derived in this fashion apply equally to smooth and topological concordance. In the same era, many deep results about smooth concordance were obtained using tools from gauge theory, especially the newly introduced Heegaard Floer theory. Tim's recent papers with Harvey and Horn began integrating these two viewpoints deriving new filtrations of the *smooth* concordance group detected by Floer theoretic invariants. Notably, some of the topological ideas behind this *bipolar filtration* take root in Tim's papers with Bob Gompf, an earlier effort to systematically apply Donaldson's 4-dimensional gauge theory results to get results on smooth knot concordance.

Tim never lost sight of the basic geometric ideas underlying the technical advances that he helped introduce. His recent paper with Chris Davis gave a family of genus one slice knots as counterexamples to a long-standing conjecture of Kauffman's, who conjectured that the Seifert surface of a genus one slice knot should contain a slice knot on which the Seifert form vanishes. His paper with Bridget Franklin, Matt Hedden, and Peter Horn on the relation between concordance and homology cobordism gave subtle counterexamples to long-held beliefs of many topologists.

Tim's earliest success arose from his ability to identify and isolate central problems. His thesis, on embeddings of 4-manifolds, gave necessary and sufficient conditions for the existence of an embedding in the 5-sphere. Although these seemed intractable at first, within a year of graduation, Tim used these conditions to identify non-embeddable 4-manifolds. Another important early contribution to 4-manifold theory was his joint paper with Nathan Habegger, clarifying homotopy-theoretic issues concerned with self-homotopy equivalences of simply connected 4-manifolds that had eluded any number of experts in the area. Tim wrote several strong papers with Paul Melvin on quantum invariants of 3-manifolds; his paper with Amir Gerges and Orr on surgery equivalence plays an important role in determining the structure of Heegaard Floer homology groups.

Reflecting Tim's unparalleled enthusiasm and vitality, his letters and email were invariably festooned with multiply underlined words, all-caps phrasing, and many, *many* exclamation points. His papers shared his exuberant style: his paper on embedding 4-manifolds had a topological argument involving a matrix about which he declared that 'we succeed, almost like the magicians of old, in pulling the matrix  $F$  out of thin air.' Sometimes, Tim's lectures flared with showmanship. At a memorial service shortly following Tim's death his brother, Rob, described Tim as the ringleader who would organize elaborate shows to be performed by his siblings and cousins at family gatherings. These youthful Tim-tales foretell his impulsive, inspired adulthood.

Tim typically arrived at any conference with a soccer ball in his suitcase, and a game soon followed. Tim's competitive nature and fast style of play helped create the reputation of Kirby students as ferocious soccer players. On the field, and in his research, Tim set his standards high. But, especially in his research, he tempered

his competitive nature with kindness and generosity, working with his many students and talented postdocs who came to Rice over the years. In recognition of his devotion, Tim was awarded the Rice Graduate Student Association's Faculty Teaching and Mentoring Award in 2014. Tim strongly supported women in his field, promoting their visibility, highlighting their contributions, and actively recruiting female graduate students and postdocs.

Tim's sudden death has left us bereft, and as colleagues and collaborators, we feel keenly the loss of his mathematical dynamism. His great legacy of beautiful work and ideas now motivates a new generation of students and researchers. We have also lost a loyal and dear friend, with a unique spirit. There really was no one else like Tim!!!

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#### **Ph. D. students and mathematical descendants of Tim Cochran**

- Serguei Sirotine (1995)
- Paul Bellis (1996)
- Simrat Ghuman (1996)
- Amir Gerges (1997)
- Amy Lampazzi (2001)
- Shelly Harvey (2002)
  - Carolyn Otto (2011)
  - Taylor Martin (2013)

- R. Taylor McNeill (2013)
- Katherine Vance (2016)
- Carol Gee (2004)
- Aaron Heap (2004)
- Constance Leidy (2004)
  - John Burke (2011)
- Jamie Jorgensen (2008)
- Peter Horn (2009)
- Andrew Elliott (2010)
- Christopher William Davis (2012)
- Bridget Franklin (2012)
- Arunima Ray (2014)
- Diego Vela (2015)
- Kenan Ince (2016)
- JungHwan Park (2017, expected)
- Anthony Bosman (2017, expected)

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